MANAGEMENT OF SLUDGE FROM PUERTO RICO'S REGIONAL INDUSTRIAL WASTEWATER TREATMENT PLANT: PHASE 2 - DEWATERING

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

During the course of this work, the most important variables influencing the performance of vacuum filters were studied for the particular case of dewatering the primary and digested sludges from the Barceloneta Regional Wastewater Treatment Plant.

Jar tests were performed to determine the optimum coagulant requirements for both types of sludge. Buchner funnel tests were used to measure their specific resistances at an applied vacuum of fifteen inches of mercury. The specific resistance for the digested sludge was found to be significantly higher than for the primary sludge.

A model was developed, based on Darcy's law, to correlate the loading to a vacuum filter with operational parameters such as applied pressure, solids deposited per unit volume of filtrate, and form time. Other parameters included in the model were the viscosity of the filtrate and the specific resistance of the sludge.

Experimental data were obtained through filter leaf tests to evaluate the empirical parameters of the model. The experimental results were correlated using a non-linear multiple regression program. A statistical analysis of the regression results led us to conclude that the model developed was a satisfactory representation of the behavior of a vacuum filter and, therefore, could be used for the design of a full-scale facility for the dewatering of the sludges from the Barceloneta Regional Treatment Plant.

Some preliminary results were obtained on dewatering primary and digested sludge using a batch centrifuge. Primary sludge seemed to be easier to dewater by this method than digested sludge.

INTRODUCTION

The treatment and handling of wastewater from industrial plants and of municipal use constitutes in the present a serious problem which, with time, becomes more difficult and complex. Strict arrangements and legal norms exist which impose conditions on the levels of discharge. These come from local agencies as well as from the Federal Government such as the National System for the Elimination of Polluting Discharges, which controls the levels of emission permissible from plants whose effluents discharge to surface waters; USEPA, by means of the act of 1977 (PL 95-217) Section 405, which specifies the use and disposition of the solids; the Resource Conservation and Recuperation Act (RCRA) of 1976 (PL 94-580); the Hazardous Substances Control Act of 1976 (PL 94-469), which authorizes the examination of the results of production of industries, concerning selected chemical substances, by the USEPA; and the National Environmental Policy Act of 1976 which authorizes the Federal Government to take action in order to protect the environment. As these arrangements and norms become more strict the problems for designers and plant operators become more difficult since high levels of treatment result in the generation of large quantities of solids complicating its management and final disposition. Inasmuch, the solid treatment system will not only process bigger quantities of material but it will have to be more effective. Solids which are not trapped will be recirculated to the wastewater treatment system with the problem that they may degrade the quality of the effluent, frustrating many purposes of the law. These limitations make the treatment and handling of solids more important, difficult and expensive.

This sombre panorama is motivating more investigators from institutions in charge of looking after the salubrity and quality of the environment and from universities to find, in the near future, more effective and realistic alternatives in order to be able to control the problem.

As part of an extensive research project formulated by the Chemical Engineering Department of the University of Puerto Rico (Mayagüez Campus), the following work provides technical information of the primary and digested sludges from the Barceloneta Regional Treatment Plant which may serve as basis for the design of a new filtration system or for optimizing a system which is presently operating, and for the study of the method of incineration as an alternative for the disposition of solids. In the present, the plant in Barceloneta discharges its effluent to the sea and the solids are disposed of in the areas surrounding the plant in a program of experimental forestation.

OBJECTIVES

The first objective of this work was to select the polymers (chemical conditioners) that will flocculate the primary and digested sludges from the Barceloneta Regional Treatment Plant and establish the optimum dose of the polymer which will offer the best filtration characteristics for both types of sludges.

The second objective was to determine the specific resistance of the primary and digested sludges from the Barceloneta Regional Treatment Plant using the Buchner funnel method for the laboratory tests and, with the results, compare the filtration characteristics of both sludges.

The third objective was to develope a model, based on Darcy's Law, to relate the solid loading to a vacuum filter with operational parameters such as applied pressure, mass of solid per unit volume of filtrate and form time for the primary and digested sludges from the Barceloneta Regional Wastewater Treatment Plant (BRWTP) using the filter leaf method for the laboratory tests.

PREVIOUS WORK

The theoretical development of the filtration of residual sludges is based on Darcy's(1) equation for flow through a porous medium. Starting from this fundamental equation, Gale(2) derived the classic equation for sludge filtration.

Karr and Keinath(3) summed up the most important factors that affect the dewatering characteristics of the sludge. Some of the factors mentioned were cellulose content, pH, particle load, organic content, filtrate viscosity, alkalinity, solid concentration, nitrogen content, chemical conditioning, mixing, compressibility factor, mechanical force of the particles, porosity, biological degradation and particle size. They concluded that the factor which affects the most these characteristics is the particle size.

The Carman-Kozeny(4)-(6) equation, in which the filtrate flow is influenced by the size of the particle through the specific superficial area term, served as theoretical support for this hypothesis. Lapple(7) confirmed that, of all the physical properties of a suspension, the particle size is the characteristic which affects more notably its behavior. Bargman, et al.(8) concluded that particle size is one of the variables which affects the most the dewatering of digested sludges. They compared numerous sludges obtained from different treatment plants in California and demonstrated that the yield of vacuum filters could be related to the percent of the particles of the sludge that passed through a sieve Nº 200. Coackley and Allos(9) fractioned samples of sludge in various size intervals and found that the resistance to filtration increased as the particle size was reduced.

The digestion process affects the particle size distribution of the sludge. Previous work on the effects of digestion on the dewatering characteristics of sludge present a confusing image. Brooks, et al.(10) demonstrated that the sludge digested anaerobically had a lower specific resistance than the activated sludge or mixtures of activated and primary sludge. A manual of the United States Environmental Protection Agency (USEPA)(11) affirms that the anaerobic digestion of primary sludge improves its dewatering characteristics. Haug, et al.(12) and Kini and Nayak(13) reached similar conclusions. Lawler, et al.(14) concluded that the process of anaerobic digestion, when operating correctly, removes preferably small particles improving the dewatering properties of the sludge. On the other hand, other authors such as Pearson and Buswell(15), Morris(16) and Kar and Keinath(3) affirm that resistance to filtration of the sludge increases with anaerobic digestion.

The development of the dewatering technology of sludge in full scale has progressed more as an art than as a science(17). The filters are chosen and designed based on past experiences. Environmental regulations, which are more strict everytime, require a final product appropriate for compost or incineration.

There exists relatively little data appropriate for the design of equipment in full scale. Sludges which are apparently similar, exhibit very dissimilar dewatering characteristics, complicating the problem even more. Therefore, each new installation requires specific tests on samples of the sludge to be processed. Many recent efforts have been directed toward the use of experimental data of specific resistances (or related parameters) in order to predict results in full scale.

Mininni, et al.(18) developed a model that predicts the flow of filtrate and concentration of the cake in a filter press based on the characteristics of the sludge (including specific resistance) and on operational and equipment design variables. Pietila and Joubert(19) used a multiple regression analysis to relate the yield of a filter press with variables such as coagulant requirements, initial solid concentration, filtration time and specific resistance of the sludge.

During the last three years, investigators in Engineering of the University of Puerto Rico have dedicated their efforts in defining the most adequate alternative for the handling of the sludge from the BRWTP. This, not only because the disposition of the sludge generates a problem in the environment but because the available terrain is being restricted. Because of what has formerly been exposed and the existing norms coming from agencies controlling the environment, the identification of a handling process for these solids in a more acceptable manner has become imperative.

Morell and Benítez(20) used rice harvest residues to compost the sludge from Barceloneta. The nutrients content of the product was evaluated on soils specially prepared for ornamental plants. They also tested the lime dosage required to stabilize the sludge, demonstrating that it is feasible.

Díaz(21) studied anaerobic digestion with mixtures of digested sludge from the BRWTP and the beer yeast remnants of a nearby pharmaceutical plant. She established reaction kinetics using a continuous reactor with adequate agitation varying the retention time for different sludge mixture proportions.

Maldonado(22) performed experiments on soil columns in order to determine the percolation of heavy metals on sludge under the effects of rain. He considered different types of typic soils of Puerto Rico varying soil to sludge proportions at different water proportions attaining an important comprehension of the soil-sludge cation interaction.

Rodríguez(23) studied the anaerobic digestion of both primary and digested sludge from the plant in Barceloneta. Presently, these sludges are only partially stabilized by means of aerobic digestion previous to final disposition.

THEORY

Vacuum filtration is a process used for the separation of solids from wastewater in which the liquid phase is removed by applying a vacuum through a porous medium that will retain the solids. The filtering medium may be cloth, nylon, dacron or a steel mesh.

The compatibility of the solids separation process with the final disposition technique is shown in Table 1.

In vacuum filtration operations, a rotating cylinder passes through a tank that contains the sludge. The solids are retained in the cylinder to which vacuum is applied. The immersion of the cylinder in the tank that contains the sludge may vary from 10 to 60%. During this period the solids are retained in the filtering medium and the water is removed by filtration. This filtering period is known as the form time. Afterwards, the cylinder emerges from the tank and the solids deposited on it are dried by the suction which follows when vacuum is applied, this period is called drying time. During the last period, the cake deposited on the cylinder is removed by scraping with a knife to a conveyor for its final disposition. The filtering medium is washed before renewing the cycle by means of a spray. A vacuum filtration operation is shown in Figure 1.

The variables that affect the separation process of solids are: the initial concentration of solids, viscosity of the sludge and the filtrate which is generally the same as that of

TABLE 1. DEWATERING PROCESS COMPATIBILITY WITH SUBSECUENT TREATMENT OR ULTIMATE DISPOSAL TECHNIQUES

Dewatering Process	Incineration	Compost	Agricultural Land Application	Landfill
Basket Centrifuge			X	X
Solid Bowl Centrifuge	X	X	X	X
Belt Filter Press	X	X	X	X
Vacuum Filter	X	X	X	X
Filter Press	X	X	X	X
Drying Beds		X	X	X
Sludges Lagoons		X	X	X

From: "Sludge Dewatering." Manual of Practice N° 20, Water Pollution Control Federation, Washington, D.C., 1983

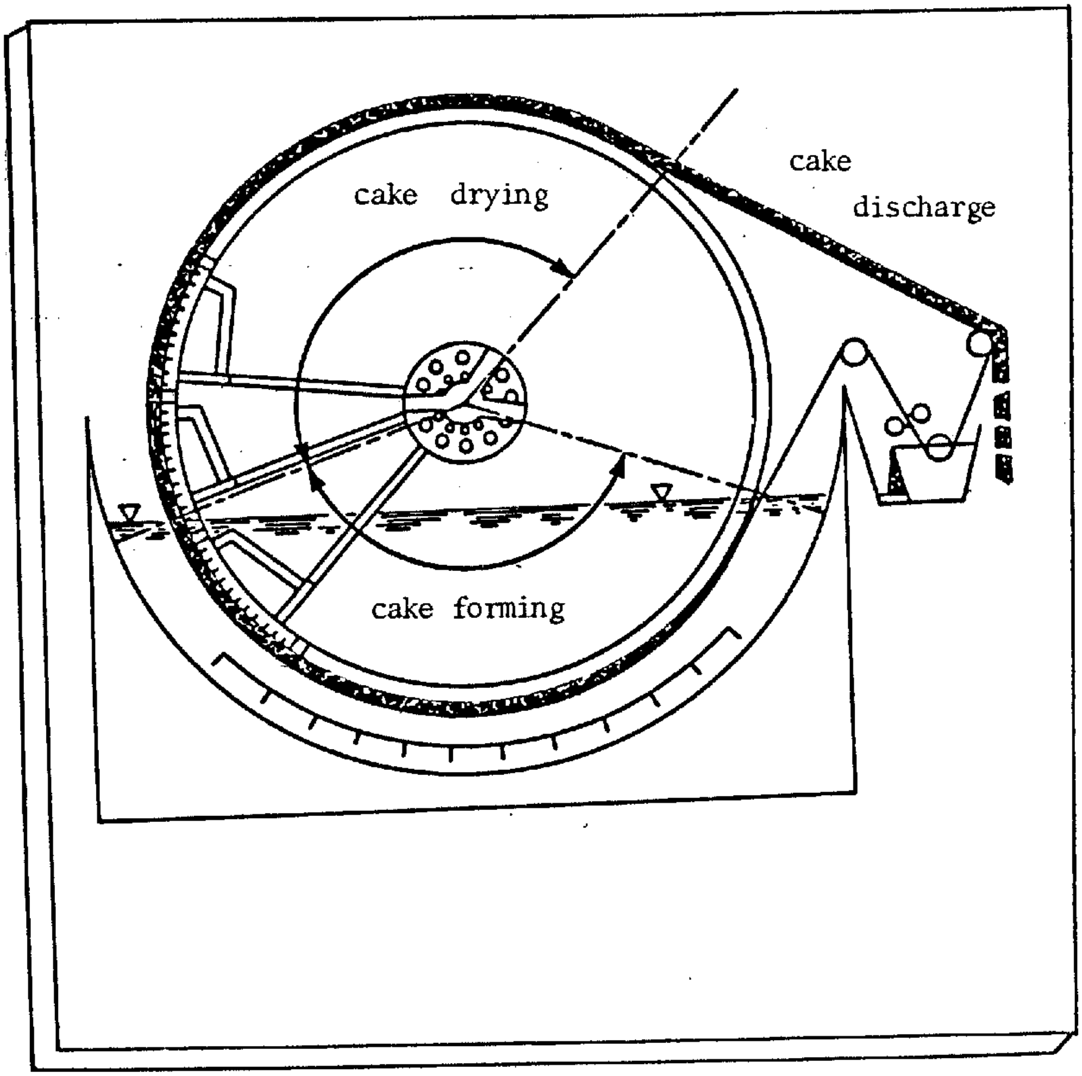

Figure 1 Rotary Vacuum Filter

water at similar temperatures, the compressibility of the sludge which depends on the nature of the particles present in it, the physical and chemical composition including size and form of the particles and the water content of the sludge.

The operation variables of the filter are: the vacuum applied, usually varies between 10 to 20 inches of mercury; cylinder immersion which varies from 10 to 60% (high porosity sludges require more immersion time); the conditioning of the sludge (many sludges require the addition of coagulants such as lime, polyelectrolytes or ferric chlorides) in order to trap small particles and obtain a better texture of the cake; the porosity and type of the filtering medium (a high porosity medium may bring as a result a high filtration velocity).

Conditioners are chemical agents added to the water or wastewater in order to flocculate the colloidal and suspended solids.

There are two types of sludge conditioners: physical such as diatomaceous earth, caustic soda and ashes and chemical such as metallic polyvalent ions and polymers.

The required dose of conditioner is determined by the characteristics of the sludge (pH, alkalinity, concentration of polyvalent cations and ionic force, concentration of suspended solids, superficial load and particle size).

The basic filtration equation originates from Darcy's law, derived for fluid motion through a porous medium.

Darcy[1] related the volumetric flow to the energy lost, medium length and hydraulic conductivity. Equation (1) is an expression of Darcy's law:

$$Q = \frac{KA\,(h_1 - h_2)}{l} \qquad [1]$$

where: Q = volumetric flow

K = permeability

A = area

l = thickness

h_1, h_2 = liquid heights

Another form of Darcy's equation for an incompressible fluid is:

$$\frac{dV}{dt} = \frac{P}{\mu}\,\frac{AK}{l} \qquad [2]$$

where: $\frac{dv}{dt}$ = volumetric flow

P = pressure difference

μ = viscosity

The resistance, R, is defined as: $R=\frac{1}{k}$, then:

$$\frac{dV}{dt} = \frac{P}{\mu}\,\frac{A}{lR} \qquad [3]$$

The effect of resistance on filtration depends on the resistance of the filtering medium and the cake.

$$\frac{dV}{dt} = \frac{P}{\mu}\,\frac{A}{(lR + Rm)} \qquad [4]$$

where R_m is the resistance of the filtering medium.

The volume of the cake is expressed as lA=vV, where v is the cake volume per unit volume of filtrate, and V is total volume of filtrate. Substituting for l in equation (4):

$$\frac{dV}{dt} = \frac{PA^2}{\mu(vVR + RmA)} \qquad [5]$$

Poiseville modified Darcy's equation in terms of dry weight, c, instead of cake volume, and the resistance per unit weight, r, instead of per unit volume:

$$\frac{dV}{dt} = \frac{PA^2}{\mu(crV + RmA)} \cdot \qquad [6]$$

where: c= weight of solids per unit volume of filtrate,

r= specific resistance of the cake.

Integrating equation (6) and re-arranging:

$$\frac{t}{V} = \frac{\mu rc}{2PA^2} \cdot V + \frac{\mu Rm}{PA} \qquad [7]$$

A plot of $\frac{t}{v}$ vs V should be a straight line with slope equal to $\frac{\mu rc}{2PA^2}$ and intercept equal to $\frac{\mu R_m}{PA}$. Defining the slope of the line as b.

$$b = \frac{\mu rc}{2PA^2} \qquad [8]$$

Therefore, the specific resistance of the cake is:

$$r = \frac{2PA^2b}{\mu c} \qquad [9]$$

The specific resistance is primarily used to compare filtration characteristics of different sludges, and to determine the optimum coagulant dose for a particular sludge. The Buchner funnel test is used to generate data necessary to determine specific resistance.

The analysis and design of full-scale filtration processes of compressible sludges is based on a modification of Darcy's law for which the specific resistance and filtration velocity are functions of the pressure gradient through the cake. Tenney(24) proposed the following relation between specific resistance and applied pressure:

$$r = r_o P^s \qquad \text{Tenney}^{(24)} \qquad [10]$$

where: r_o = specific resistance for an applied pressure of 1" Hg,

s = compressibility coefficient.

Substituting in Darcy's law, for small R_m(24):

$$\frac{dV}{dt} = \frac{PA^2}{\mu c r_o P^s V} \qquad [11]$$

Integrating during the form time of the cake, t_f:

$$\frac{cV}{At_f} = \left[\frac{2P^{(1-s)}}{\mu r_o t_f}\right]^{\frac{1}{2}} = L \qquad [12]$$

where L is the solids loading, mass of solids removed per unit filter area per unit time.

Equations (12) may be expressed as:

$$L = K \left[\frac{cP^{(1-s)}}{\mu Rot_f} \right]^{1/2} \qquad [13]$$

where L is given in lb/ft^2 - hr; P in inches of Hg, c in %, R^o = r_o x 10^{-8} cm/g, t_f in minutes, in g/cm.s, and K is a proportionality constant.

The amount of solids removed per unit volume of filtrate is determined by:

$$c = \frac{1}{[(100-Ci)/Ci - (100-Cf)/Cf]} \qquad [14]$$

where: C_i = initial sludge solids concentration, %,

C_f = solids concentration in the cake, %.

A modification of equation (13) is used to predict filtration performance for irregular solid particles:

$$L = K \left[\frac{P^{(1-s)}}{\mu Ro} \right]^{1/2} \left[\frac{c^m}{t_f{}^n} \right] \qquad [15]$$

Once the polymer dose has been established, filter leaf tests may be used to obtain data for the evaluation of the empirical coefficients s, m, n, and r_o. These coefficients are specific for each sludge.

EXPERIMENTAL WORK

The tests on vacuum filtration done in the laboratory were used to evaluate the sludge conditioner (polymer) using jar tests, the specific resistance was determined using the Buchner Funnel method and then, the design parameters were determined using the filter leaf method. Some preliminary results were also obtained for sludge dewatering through centrifugation.

A. Jar tests

The standard jar test was modified for the polymer evaluation tests, for the Buchner Funnel tests and for the filter leaf tests. The equipment used is shown in Figure 2.

The polymer evaluation tests were done with 200 ml. samples of both primary and digested sludges 15 ml. of the polymer were added to each sample (initial dose recommended by Calgon). They were stirred for 30 sec. at 100 rpm, then they were stirred slowly for 20 min. at 10 rpm. If flocculation of the samples had not been obtained (in some cases flocculation was reached within the first 5 to 10 ml.) more polymer was added, in 5 ml. aliquots, until the samples of both primary and digested sludges had flocculated. Otherwise, the polymer would be discarded. By means of the jar tests 18 polymers were tested and 6 were selected.

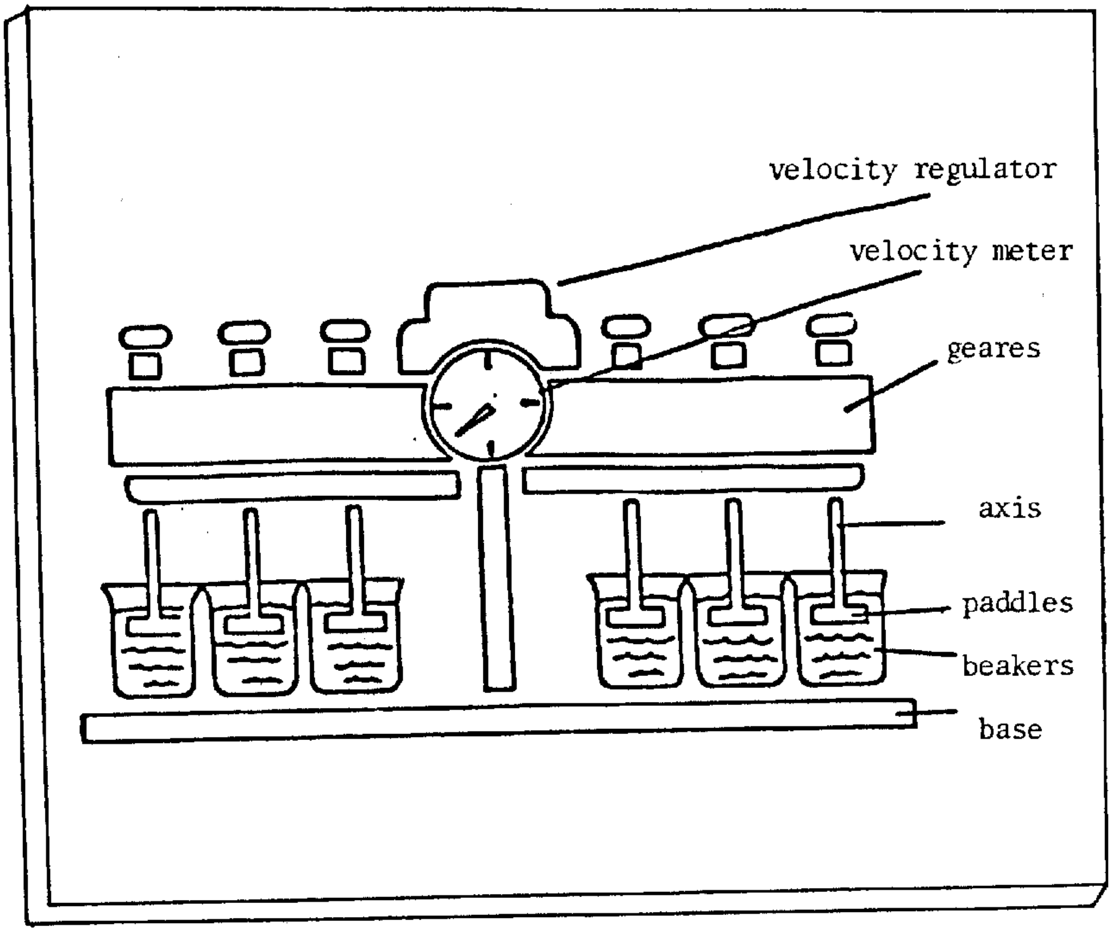

Figure 2 Jar Test Assembly

B. Buchner Funnel tests

Buchner Funnel tests were used to establish the suggested polymer dose and to determine the specific resistance of the primary and digested sludge.

To establish the optimum polymer dose, the modified jar test described in part A was used with each of the 6 selected polymers. Each of these polymers were added in different quantities to the primary and digested sludges to be mixed and flocculated with the jar test. Later, duplicate samples of both sludges were filtered at a vacuum pressure of 15 in. of Hg (typical operation pressure for vacuum filters) in a Buchner Funnel such as the one shown in Figure 3 measuring in each case the time for 25, 50, 75, 100, 125 and 150 ml. of filtrate volume. This way, the filtration velocity for each polymer dose was determined. If 50 ml. of filtrate in 30 sec. or 150 ml. in 3 min. were not obtained the experiment was stopped and the polymer dose was changed until satisfactory results were obtained.

The tests for the determination of the specific resistance of primary and digested sludges from the BRWTP were done using the jar test described in part A. Duplicate samples of both sludges were filtered in accordance to the procedure for the polymer test.

C. Filter Leaf Test

Filter leaf tests were used to determine the coefficients m, n, s, and R_o of equation (15) relating the solid load to the

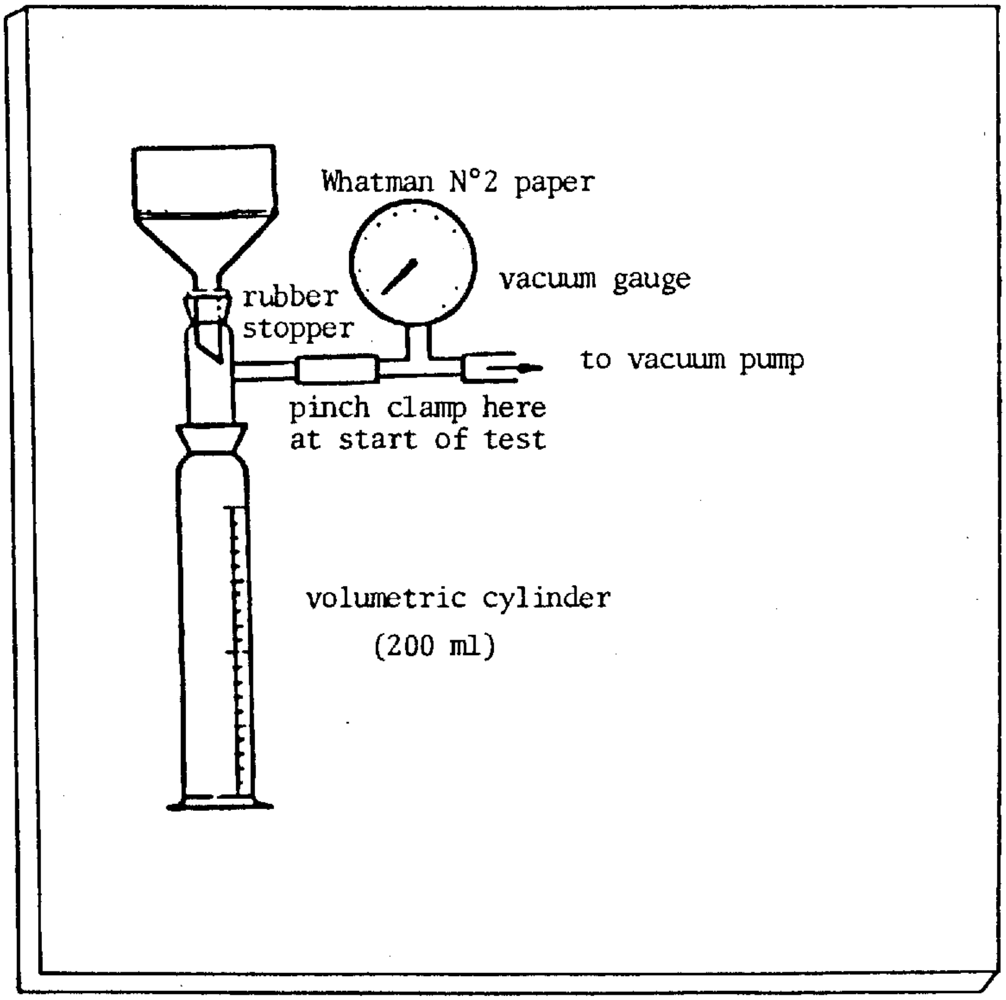

Figure 3 Buchner Funnel Assembly

form time, t_f, the solid concentration, c, and the pressure, P. Sufficient tests were done to evaluate the mentioned parameters for a specific sludge at a given pressure, form time, and drying time, trying to duplicate operation conditions for full scale filters, using equipment such as the one shown in Figure 4.

The samples were filtered using the EIMCO POPR-859 leaf, following the recommendation of the manufacturers of the polymers used. To each of the seven 2,000 ml. samples of primary and digested sludge were added 150 ml. of polymer (suggested dose) in order to mix and flocculate them by means of the jar test. The filter leaf was submerged in the sample already flocculated during the selected form time at a determined vacuum pressure. Once the form time had elapsed the filter with the deposited solids was removed and placed in a vertical position in order to dry the sample for 1 min. at the initial vacuum pressure. The cake formed was scraped off completely with a spatula to determine the solid load and the final solid percent. This same procedure was followed at different vacuum pressures and form time for the primary and digested sludges from the BRWTP.

D. Centrifugation

Samples of both primary and digested sludges were centrifuged in 50 ml test tubes using a laboratory centrifuge. The samples were processed at different combinations of speed

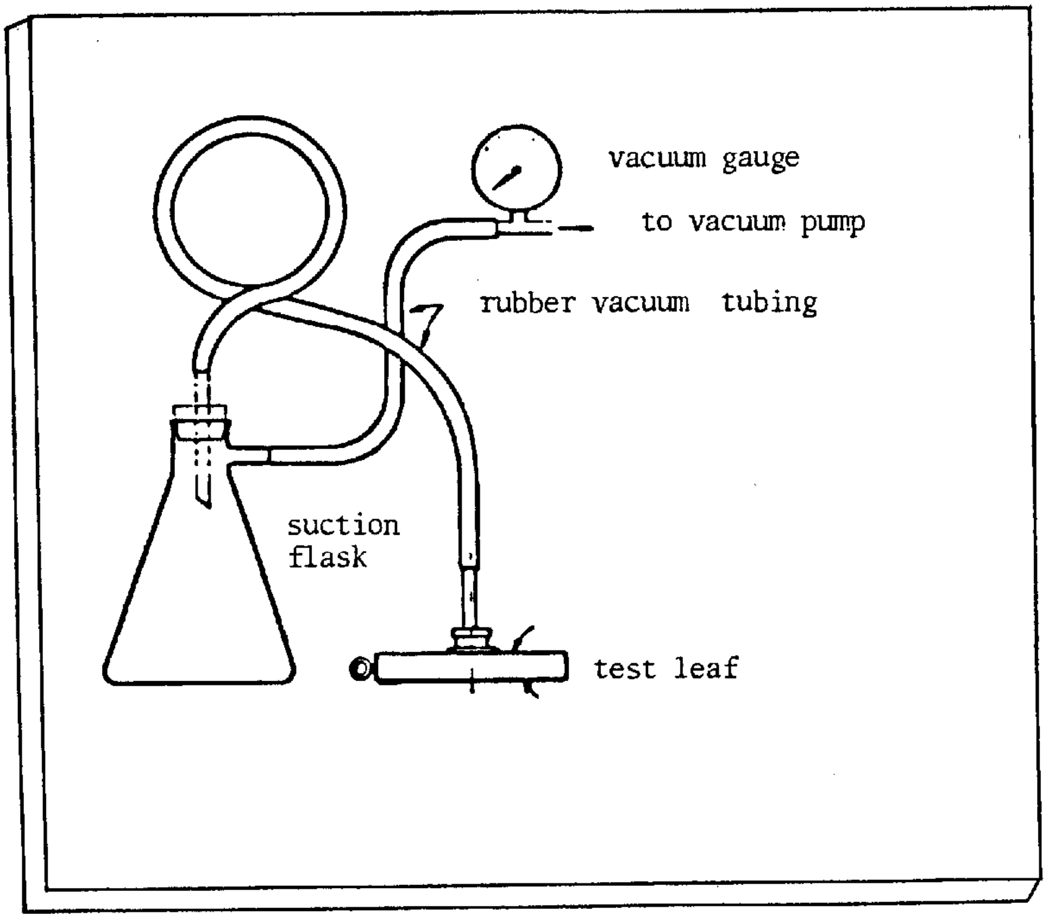

Figure 4 Leaf Test Assembly

(RPM's) and retention time. At the end, the volume of supernatant was recorded, and a sample of it analyzed for total solids content.

EXPERIMENTAL RESULTS

I. Evaluation of the sludge conditioner (polymers)

By means of the modified jar test, 18 polymers were tested. The polymers to be selected had to flocculate both primary and digested sludges. The quality of the flocculation could be appraised easily in a visual way and 6 polymers were selected. The results are shown in tables A.1 and A.2. Tests on polymer quantity and filtration quality for all 6 polymers were effected by means of the modified jar test and the Buchner funnel method. It was established that the CALGON WT-2640 polymer at a dose of 15 ml. per 200 mls of sludge gave the best results for both primary and digested sludges.

II. Specific resistance of primary and digested sludge.

For each run of the Buchner funnel test, duplicate data was collected of volume of filtrate and time for the primary and digested sludges. These data are found in tables B.1 and B.2 respectively.

With the data from each table a plot of t/v vs. v was plotted; an example is shown in Figure 5 for the primary sludge and in Figure 6 for the digested sludge.

The slope of the line is equal to the term b of equation 8. For the primary sludge in the example the value of b was found to be 0.00147 with a correlation coefficient of 0.983321.

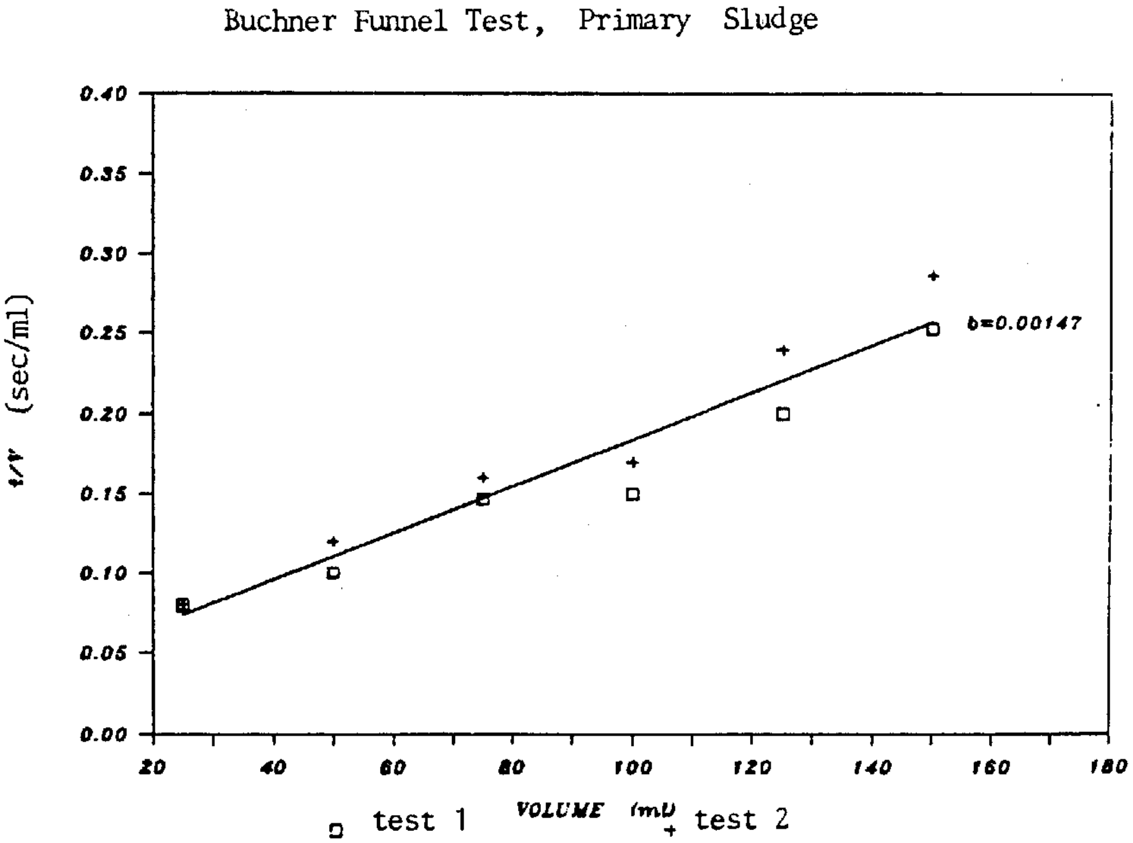

Figure 5 Determination of Slope b

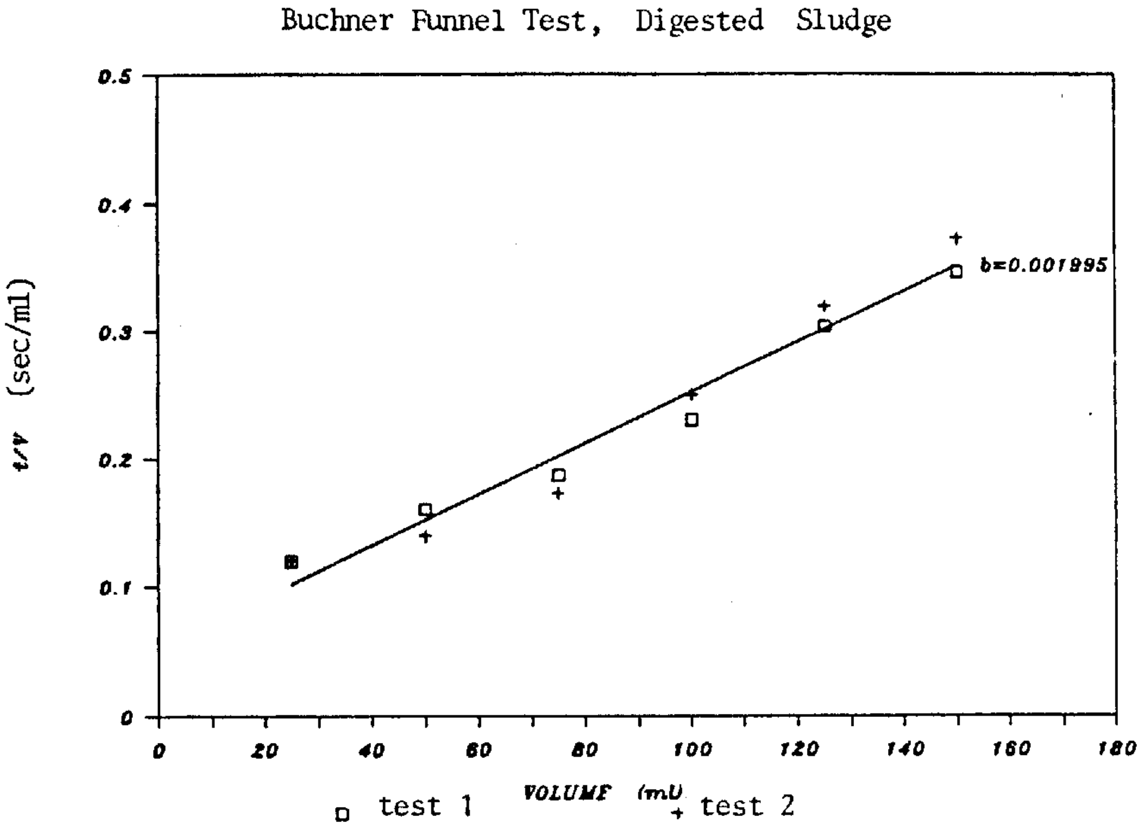

Figure 6 Determination of Slope b

The specific conditions for the tests were:

filter paper area = 104.6 cm^2
applied vacuum = 15 in. Hg
initial and final concentrations, Ci and Cf, for the primary and digested sludges are found in Table B.3.

Equation 14 was used to determine the amount of solid deposited per unit volume of filtrate, c . In the example this value was found to be 5.2332%

Finally, the specific resistances, r, for the primary and digested sludges were determined. These are found in Tables B.4 and B.5, respectively.

III. Filter Leaf Tests:

A. Experimental results for solid loading

1. The data obtained by the filter leaf method are tabulated in tables C.1 and C.2 for the primary and digested sludges, respectively.

2. With the results for the primary and digested sludges the following grafical correlations were made:

   a. Solid load vs. form time (Figures 7 and 8). The slope of this line gives the value of the coefficient n of equation 15. For the case shown in Figure 7, n = 0.66.

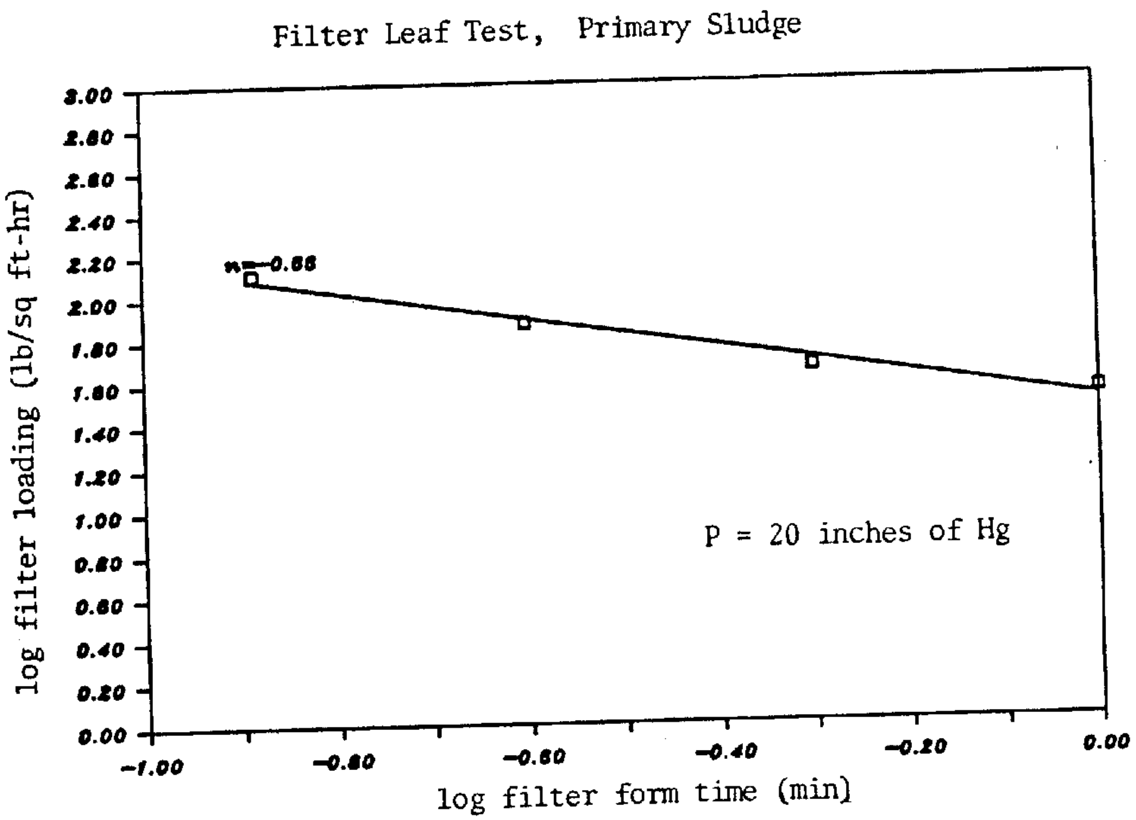

Figure 7 Determination of Coefficient n

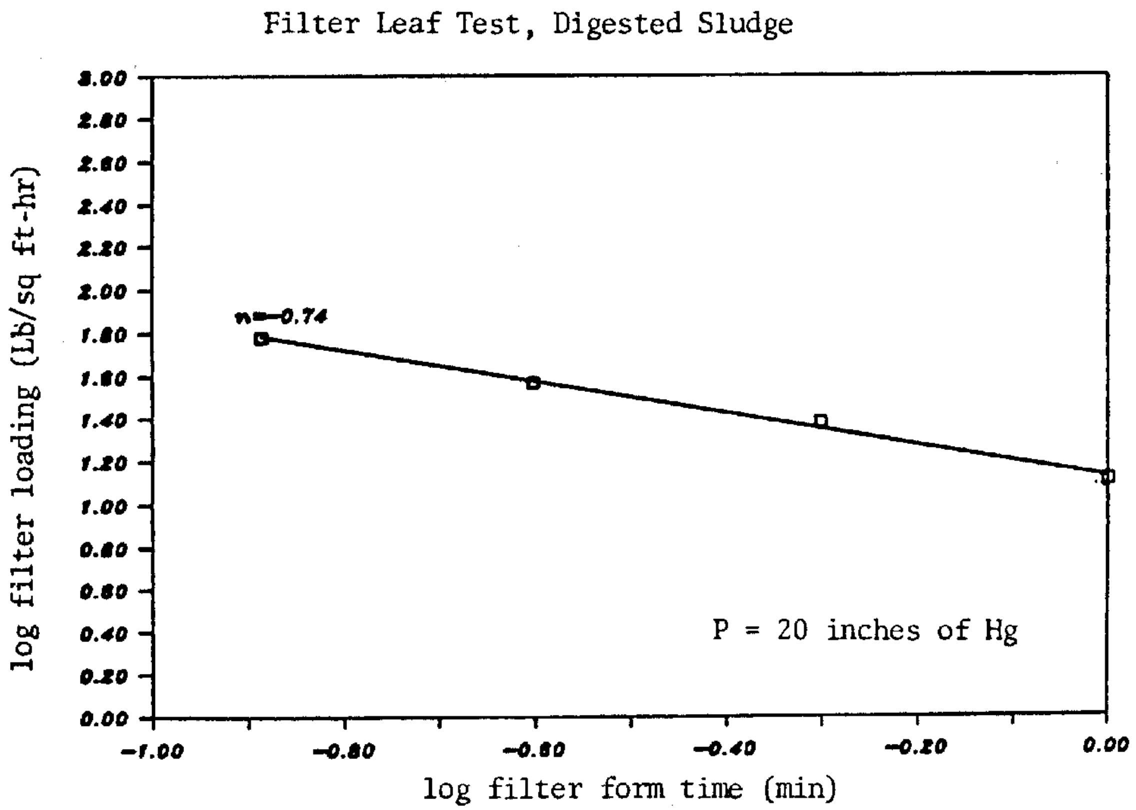

Figure 8 Determination of Coefficient n

b. Solid load vs. initial concentration of solid (Figures 9 & 10). The slope of this line gives the value of m. of equation 15. For the case in Figure 9, $m = 1.5$.

c. Solid load vs. vacuum pressure (figures 11 & 12). In this case the slope of the line gives the value of the term $(1-s)/2$ of equation 15. For the case in Figure 11 this was found to be 0.66.

3. These values served as a basis for the non-linear regression.

B. Non-linear regression results for solid load.

The equation used to relate the solid load with the applied pressure, form time and the amount of solid deposited per unit volume of filtrate was derived in the theory, equation 15. A non-linear regression program (LS) developed in the University of Wisconsin(25) was used in order to evaluate the parameters for the proposed model which is of the form

$$L = PAR(1) \times P^{PAR(2)} \times C^{PAR(3)}/t_f^{PAR(4)}$$

where:

$PAR(1) = K/(\mu R_o)^{1/2}$

$PAR(2) = (1 - s)/2$

Filter Leaf Test, Primary Sludge

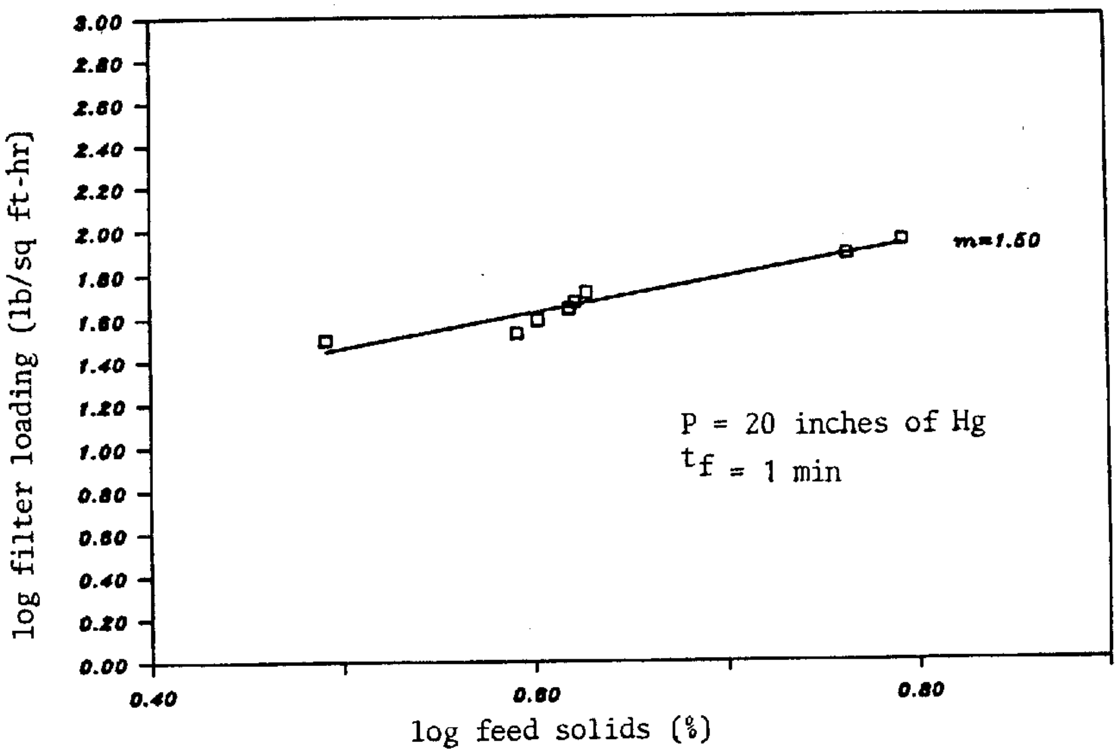

Figure 9 Determination of Coefficient m

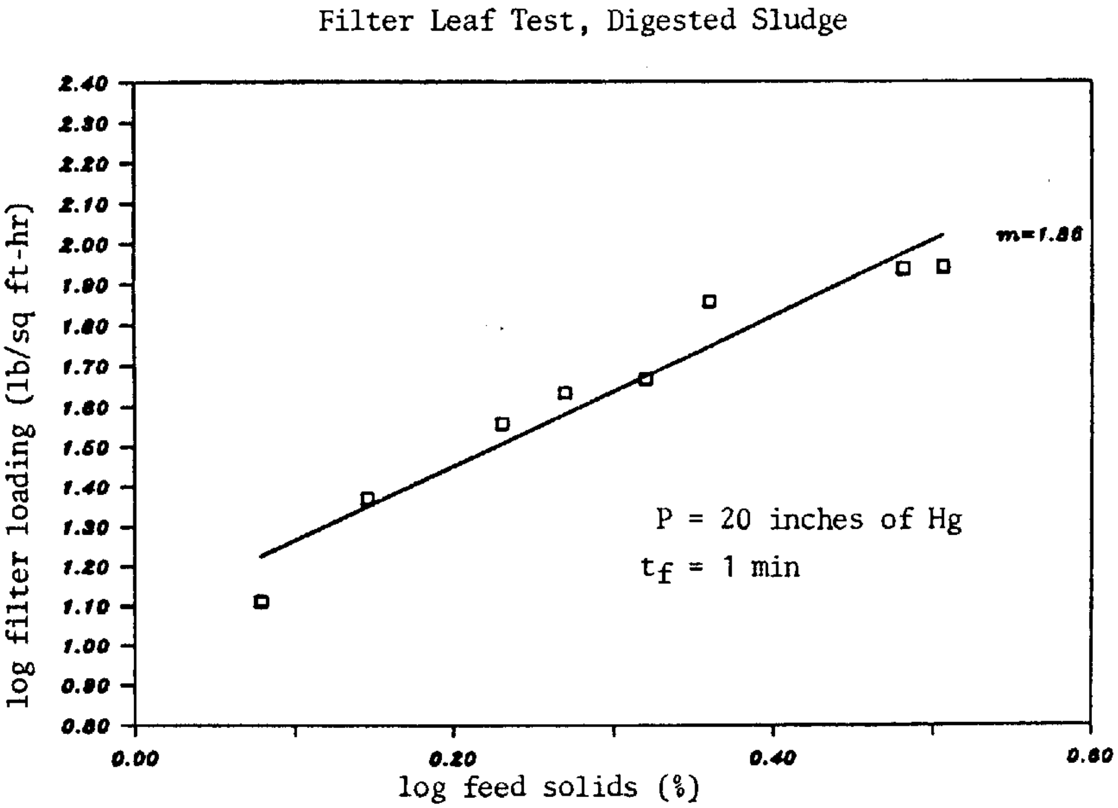

Figure 10 Determination of Coefficient m

Filter Leaf Test, Primary Sludge

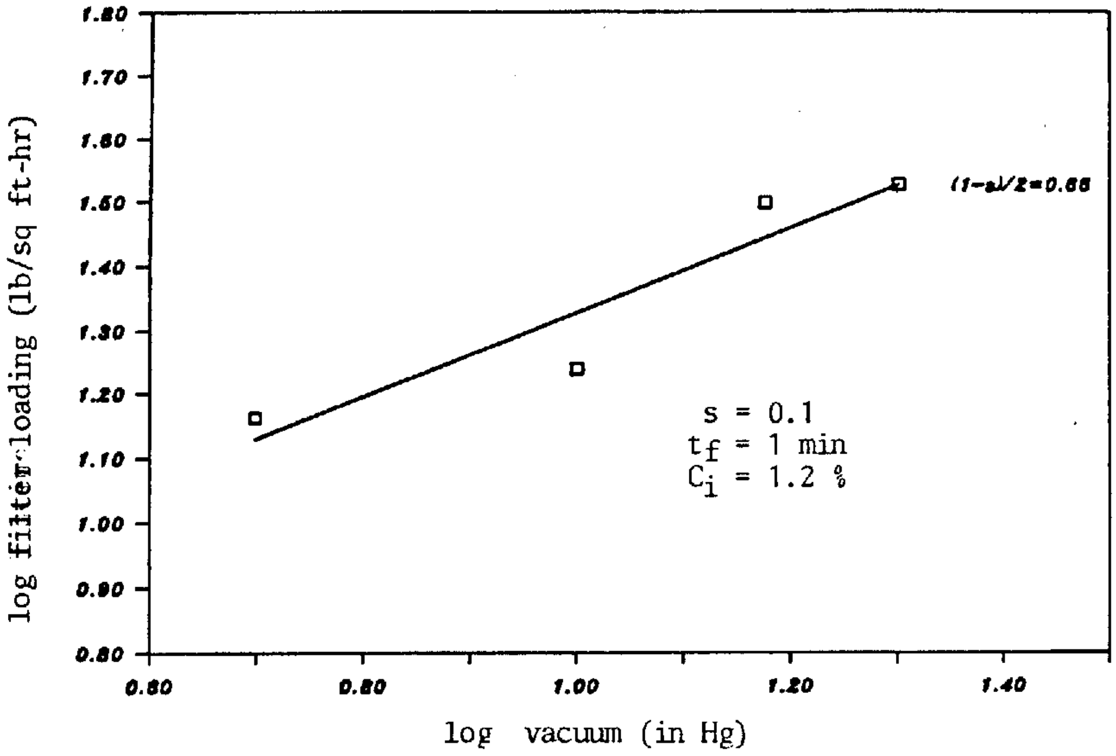

Figure 11 Determination of Coefficient s

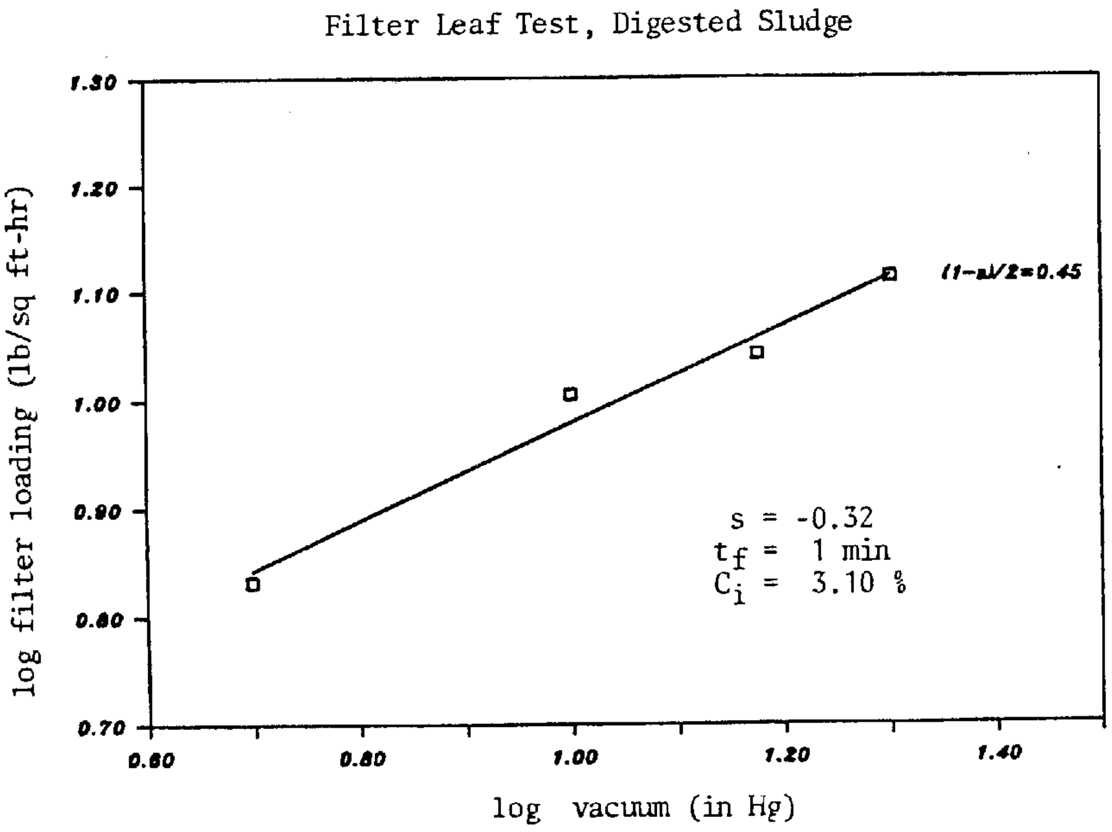

Figure 12 Determination of Coefficient s

PAR(3) = m

PAR(4) = n

To obtain a preliminary estimate of the parameters required for the program, the experimental data were plotted as mentioned in part A.

The results obtained for the best estimates of the parameters by minimizing the sum of squares of errors are shown in Table 2.

IV. Centrifugation

The data obtained for centrifugation of the primary and digested sludges are tabulated in Table G.1.

TABLE 2. RESULTS OF NON LINEAR REGRESSION FOR THE FILTER LOADING

Parameters	Primary Sludge	Digested Sludge
Parameter(1) =	3.340432	3.033051
Ro =	2.050000	2.480000
Parameter(2) =	0.498913	0.675866
s =	0.002173	0.351733
Parameter(3) =	0.825267	0.981168
m =	0.825267	0.981168
Parameter(4) =	0.340485	0.417281
n =	0.340485	0.417281

Where: Parameter(1) = $K\sqrt{(\mu Ro)}$, Parameter(2) = $(1-s)/2$

DISCUSSION OF RESULTS

As the polymers were evaluated is was noticed that the behavior of the primary and digested sludges varies considerably for each type of polymer, although the majority of the polymers used were cationic, the flocculation of the samples did not show the same agglomeration. The polymer CALGON WT-2640 was the polymer which best flocculated the samples of both primary and digested sludges in a quantity of approximately 15 ml. per 200 ml. of sludge. It is important to point out that flocculation was not always achieved with the suggested dose since the samples varied, even from one day to another. A polymer excess, in some cases, degraded the flocculation.

Filtration results using the Buchner Funnel method suggest that the digested sludge is more difficult to filter than the primary sludge. The average specific resistance for the digested sludges (10.352×10^{10} cm/g, see Table D.1) seems to be significantly larger than that corresponding to the primary sludges (2.547×10^{10} cm/g). Due to the fact that a significant variation was observed in the experimental results and analysis of variance was done (see Table D.2 to corroborate). In this analysis it was shown, with 99% certainty, that the difference is significant. This is probably due to the presence of colloidal particles in the digested sludges.

The filter leaf method reproduces satisfactorily the operating conditions of a full scale vacuum filter. It offers the advantage of allowing variations of the operational parameters such as vacuum pressure, form time and drying time. This enables the experimenter to evaluate, with a good number of data, the coefficients that characterize the particles present in the sludge. With the data obtained for the primary and digested sludges, a non-linear regression was done. Afterwards, in order to corroborate the results obtained in the non-linear regression, a statistical analysis was done. From this analysis it was concluded that the results are satisfactory with 95% reliability. Thus, the equations for the solid load of the primary and digested sludges are reliable. The statistical analysis is shown in appendix D and the results for the non-linear regression in appendixes E and F. The EIMCO POPR-859 mesh gave good results with the formation of a uniform and easy to manage cake.

Our preliminary results on dewatering the sludge by centrifugation seem to indicate that this is a very effective method, particularly for the primary sludge. However, it should be pointed out that the experiment was poorly designed for this part of the project. The retention times used were much higher than one would expect in a continuous full-scale dewatering facility.

CONCLUSIONS

From the results obtained in this investigation the following conclusions may be reached:

1. The Buchner Funnel test constitutes a simple and effective method for the determination of the required coagulant dose for a particular sludge. It may also be used to compare the relative difficulties in filtering different sludges.
2. The specific resistance observed for the primary sludges of the BRTP are significantly greater than those of the digested sludges. This may be due to the difference in particle size distribution of both sludges.
3. The solid load equation derived for the filter leaf represents satisfactorily the experimental results obtained. To reach this expression, the contribution to resistance of the filtering medium to the total specific resistance was neglected. Thus, the results in these tests may be used for the design of full scale vacuum filters even if the filtering medium to be used is different from the one used during this experiment.
4. The use of a non-linear multiple regression program for the simultaneous estimate of the empirical parameters of the solid load equation produces better results than the classical method of linearizing the model and plotting the

data in a way that estimates are obtained one by one. The sum of erros squared was reduced by a factor of 10^7 when the multiple regression method was used.

5. For the primary sludge it was observed that the compressibility factor, s, is practically zero. Even though, it was oberved that for both primary and digested sludges the correlation coefficient between parameter 1 (which involves the specific resistance of the sludge at 1 in Hg) and parameter 2 (that involves the compressibility factor) is very significant (see appendix D). This suggests that the relation between the specific resistance of the sludge and the applied pressure is possibly more complex than the relation presumed during the development of the model. It is important to point out that for liquids and gases the compressibility factor is a function of pressure. This may also explain why the compressibility factor for the digested sludges seems to be negative as shown in the results obtained in the regression
6. The equation developed to relate the applied solid load to a vacuum filter with the pressure applied, mass of solids deposited and form time may be a very useful instrument in the design of a full scale system for the dewatering of the sludges of Barceloneta, eliminating the need to, depend of empiricism in an engineering project of such magnitude.

7. Dewatering by centrifugation should be studied further using experimental conditions representative of continuous, full-scale operation.

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Appendix A

Results of Polymers Tests

TABLE A.1 RESULT OF JAR TEST FOR POLYMERS

Polymer	Primary Sludge V (ml)	Primary Sludge Quality	Digested sludge V (ml)	Digested sludge Quality
WT-264-5	18	Poor	10	Regular
WT-2640	15	Good	15	Good
CAT-FLOC-T	35	Poor	20	Good
CAT-FLOC-TL	18	Poor	18	Regular
CAT-FLOC-L	18	Poor	18	Regular
CAT-FLOC-CL	18	Poor	18	Regular
CAT-FLOC-K-10	18	Poor	18	Regular
CAT-FLOC-LS	18	Poor	18	Regular
CA-250	13	Poor	13	Poor
WT-2736	15	Regular	20	Poor
WT-2466	14	Good	12	Good
L-681-E	14	Poor	18	Poor
K-250	16	Good	14	Good
K-200	16	Good	12	Good
CA-25	34	Poor	34	Poor
CA-35	20	Poor	20	Poor
MAFLOC 900	16	Good	24	Poor
BESST FLOC 851P	26	Regular	12	Good

TABLE A.2 RESULTS OF BUCHNER FUNNEL TEST FOR POLYMERS

Polymer	Primary Sludge I.D.(ml)	Primary Sludge S.D.(ml)	Digested Sludge I.D.(ml)	Digested Sludge S.D.(ml)
WT-2640*	15	15	10	15 **
WT-2466	14	21	21	12
K-250	16	24	24	24
K-200	16	16	16	18
BESST FLOC 851P	26	26	26	18
WT-264-5	18	10	10	10

* Good quality polymer
**15 ml of a 1 ppm polymer solution
I.D. = initial dosage
S.D. = suggested dosage

Appendix B

Results of Buchner Funnel Tests

TABLE B.1 BUCHNER FUNNEL TEST PRIMARY SLUDGE

Sample #	Time (sec)	Volume (ml)	t/V (sec/ml)
1P1	3	25	0.12
	7	50	0.12
	9	75	0.12
	12	100	0.12
	21	125	0.17
	30	150	0.20
1P2	3	25	0.12
	8	50	0.16
	14	75	0.19
	20	100	0.20
	28	125	0.22
	35	150	0.23
2P1	2	25	0.08
	5	50	0.10
	11	75	0.15
	15	100	0.15
	25	125	0.20
	38	150	0.25
2P2	2	25	0.08
	6	50	0.12
	12	75	0.16
	17	100	0.17
	30	125	0.24
	43	150	0.29
3P1	3	25	0.12
	8	50	0.16
	20	75	0.27
	30	100	0.30
	46	125	0.37
	75	150	0.50
3P2	3	25	0.12
	9	50	0.18
	24	75	0.32
	32	100	0.32
	52	125	0.42
	80	150	0.53

TABLE B.1 BUCHNER FUNNEL TEST PRIMARY SLUDGE (Cont.)

Sample #	Time (sec)	Volume (ml)	t/V (sec/ml)
4P1	2	25	0.08
	5	50	0.10
	8	75	0.11
	12	100	0.12
	18	125	0.14
	33	150	0.22
4P2	2	25	0.08
	6	50	0.12
	10	75	0.13
	14	100	0.14
	19	125	0.15
	47	150	0.31
5P1	1	25	0.04
	3	50	0.06
	6	75	0.08
	9	100	0.09
	16	125	0.13
	37	150	0.25
5P2	2	25	0.08
	4	50	0.08
	6	75	0.08
	9	100	0.09
	20	125	0.16
	39	150	0.26
6P1	5	25	0.20
	11	50	0.22
	17	75	0.23
	23	100	0.23
	31	125	0.25
	42	150	0.28
6P2	5	25	0.20
	12	50	0.24
	19	75	0.25
	27	100	0.27
	38	125	0.30
	52	150	0.35

TABLE B.1 BUCHNER FUNNEL TEST PRIMARY SLUDGE (Cont.)

Sample #	Time (sec)	Volume (ml)	t/V (sec/ml)
7P1	3	25	0.12
	6	50	0.12
	10	75	0.13
	13	100	0.13
	20	125	0.16
	33	150	0.22
7P2	3	25	0.12
	8	50	0.16
	13	75	0.17
	20	100	0.20
	47	125	0.38
	60	150	0.40
8P1	3	25	0.12
	8	50	0.16
	15	75	0.20
	35	100	0.35
	47	125	0.38
	80	150	0.53
8P2	3	25	0.12
	9	50	0.18
	23	75	0.31
	36	100	0.36
	50	125	0.40
	70	150	0.47

TABLE B.2 BUCHNER FUNNEL TEST DIGESTED SLUDGE

Sample #	Time (sec)	Volume (ml)	t/V (sec/ml)
1S1	8	25	0.32
	17	50	0.34
	28	75	0.37
	46	100	0.46
	72	125	0.58
	102	150	0.68
1S2	7	25	0.28
	15	50	0.30
	27	75	0.36
	41	100	0.41
	60	125	0.48
	84	150	0.56
2S1	3	25	0.12
	8	50	0.16
	14	75	0.19
	23	100	0.23
	38	125	0.30
	52	150	0.35
2S2	3	25	0.12
	7	50	0.14
	13	75	0.17
	25	100	0.25
	40	125	0.32
	56	150	0.37
3S1	3	25	0.12
	8	50	0.16
	15	75	0.20
	27	100	0.27
	41	125	0.33
	57	150	0.38
3S2	2	25	0.08
	6	50	0.12
	12	75	0.16
	21	100	0.21
	32	125	0.26
	50	150	0.33

TABLE B.2 BUCHNER FUNNEL TEST DIGESTED SLUDGE (Cont.)

Sample #	Time (sec)	Volume (ml)	t/V (sec/ml)
4S1	3	25	0.12
	7	50	0.14
	14	75	0.19
	26	100	0.26
	38	125	0.30
	50	150	0.33
4S2	3	25	0.12
	8	50	0.16
	16	75	0.21
	25	100	0.25
	34	125	0.27
	48	150	0.32
5S1	8	25	0.32
	19	50	0.38
	34	75	0.45
	61	100	0.61
	75	125	0.60
	111	150	0.74
5S2	7	25	0.28
	18	50	0.36
	36	75	0.48
	58	100	0.58
	72	125	0.58
	115	150	0.77
6S1	9	25	0.36
	20	50	0.40
	38	75	0.51
	63	100	0.63
	88	125	0.70
	135	150	0.90
6S2	9	25	0.36
	22	50	0.44
	42	75	0.56
	61	100	0.61
	85	125	0.68
	125	150	0.83

TABLE B.2 BUCHNER FUNNEL TEST DIGESTED SLUDGE (Cont.)

Sample #	Time (sec)	Volume (ml)	t/V (sec/ml)
7S1	6	25	0.24
	13	50	0.26
	26	75	0.35
	44	100	0.44
	78	125	0.62
	112	150	0.75
7S2	5	25	0.20
	12	50	0.24
	24	75	0.32
	42	100	0.42
	75	125	0.60
	116	150	0.77
8S1	2	25	0.08
	4	50	0.08
	12	75	0.16
	22	100	0.22
	30	125	0.24
	50	150	0.33
8S2	2	25	0.08
	6	50	0.12
	15	75	0.20
	28	100	0.28
	42	125	0.34
	58	150	0.39

TABLA B.3 INITIALS AND FINALS SOLIDS CONCENTRATIONS (%), BUCHNER FUNNEL TEST

Primary			Digested		
Sample #	Ci (%)	Cf (%)	Sample #	Ci (%)	Cf (%)
P1	3.10	19.17	S1	1.20	14.36
P2	4.15	20.05	S2	3.03	16.34
P3	5.80	23.77	S3	1.70	14.77
P4	4.18	21.02	S4	1.40	14.45
P5	6.20	23.82	S5	2.09	16.11
P6	3.90	19.93	S6	2.29	16.27
P7	4.24	21.57	S7	3.21	16.73
P8	4.00	19.99	S8	1.86	15.53

TABLA B.4 RESULTS OF BUCHNER FUNNEL TEST PRIMARY SLUDGES
(Vacuum = 15 in. Hg, Area = 104.6 cm^2)

Sample #	b (sec/cm6)	R (Cortn.)	Ci (%)	Cf (%)	c (%)	u (poise)	r x 10^{-10} (cm/g)
1P	0.000752	0.980102	3.10	19.17	3.6980	0.0134	1.69
2P	0.001470	0.983321	4.15	20.05	5.2332	0.0126	2.48
3P	0.003046	0.984061	5.80	23.77	7.6720	0.0130	3.39
4P	0.001208	0.870570	4.18	21.02	5.2176	0.0126	2.04
5P	0.001379	0.872686	6.20	23.82	8.3816	0.0126	1.45
6P	0.000820	0.976112	3.90	19.93	4.8488	0.0126	1.49
7P	0.001537	0.931568	4.24	21.57	5.2774	0.0126	2.57
8P	0.003035	0.992809	4.00	19.99	5.0006	0.0128	5.27

TABLA B.5 RESULTS OF BUCHNER FUNNEL TEST DIGESTED SLUDGE
(Vacuum = 15 in. Hg, Area = 104.6 cm^2)

Sample #	b (sec/cm6)	R (Cortn.)	Ci (%)	Cf (%)	c (%)	u (poise)	$r \times 10^{-11}$ (sec^2/g)
1P	0.002619	0.976517	1.20	14.36	1.3095	0.0130	1.71
2P	0.001995	0.987386	3.03	16.34	3.7200	0.0128	0.47
3P	0.002056	0.995821	1.70	14.77	1.9212	0.0127	0.94
4P	0.001716	0.996615	1.40	14.45	1.5502	0.0144	0.85
5P	0.003484	0.980751	2.09	16.06	2.4027	0.0132	1.22
6P	0.003926	0.988872	2.29	16.27	2.6651	0.0137	1.19
7P	0.004437	0.972391	3.21	16.73	3.9724	0.0131	0.95
8P	0.002324	0.991801	1.86	15.53	2.1132	0.0128	0.96

Appendix C

Results of Filter Leaf Tests

TABLE C.1 RESULTS OF FILTER LEAF TESTS OF PRIMARY SLUDGE
FOR THE EVALUATION OF PARAMETERS m, n, s, Ro

Sample #	Form Time (min)	Dry Time (min)	Vacuum (in. Hg)	C (%)	Ci (%)	Cf (%)	Filter Loading (lb/ft^2-h)
1P	1.00	1	20	3.74	3.10	18.00	33.64
	1.00	1	15	3.77	3.10	17.50	31.49
	1.00	1	10	3.83	3.10	16.25	17.35
	1.00	1	5	3.92	3.10	14.76	14.51
	0.50	1	20	3.78	3.10	17.13	44.98
	0.25	1	20	3.82	3.10	16.50	74.34
	0.13	1	20	3.91	3.10	15.00	128.67
2P	1.00	1	20	5.31	4.15	19.00	45.02
	1.00	1	15	5.33	4.15	18.80	43.65
	1.00	1	10	5.38	4.15	18.20	43.40
	1.00	1	5	5.39	4.15	18.00	26.88
	0.50	1	20	5.35	4.15	18.50	68.38
	0.25	1	20	5.38	4.15	18.20	103.05
	0.13	1	20	5.38	4.15	18.10	119.17
3P	1.00	1	20	7.81	5.80	22.50	80.08
	1.00	1	15	7.96	5.80	21.40	76.28
	1.00	1	10	8.07	5.80	20.60	51.47
	1.00	1	5	8.24	5.80	19.60	49.28
	0.50	1	20	8.10	5.80	20.40	108.69
	0.25	1	20	8.26	5.80	19.50	139.82
	0.13	1	20	8.56	5.80	18.00	152.15
4P	1.00	1	20	5.26	4.18	20.40	46.38
	1.00	1	15	5.26	4.18	20.30	46.59
	1.00	1	10	5.27	4.18	20.18	45.88
	1.00	1	5	5.32	4.18	19.46	35.45
	0.50	1	20	5.30	4.18	19.80	71.87
	0.25	1	20	5.33	4.18	19.40	108.41
	0.13	1	20	5.36	4.18	19.00	128.55

TABLE C.1 RESULTS OF FILTER LEAF TESTS OF PRIMARY SLUDGE
FOR THE EVALUATION OF PARAMETERS m, n, s, Ro Cont.

Sample #	Form Time (min)	Dry Time (min)	Vacuum (in. Hg)	C (%)	Ci (%)	Cf (%)	Filter Loading (lb/ft^2-h)
5P	1.00	1	20	8.47	6.20	23.10	90.39
	1.00	1	15	8.49	6.20	23.00	87.58
	1.00	1	10	8.52	6.20	22.80	78.53
	1.00	1	5	8.71	6.20	21.50	51.60
	0.50	1	20	8.47	6.20	23.10	144.90
	0.25	1	20	8.63	6.20	22.00	169.21
	0.13	1	20	9.11	6.20	19.43	161.59
6P	1.00	1	20	4.96	3.90	18.30	39.34
	1.00	1	15	4.98	3.90	18.00	33.67
	1.00	1	10	4.99	3.90	17.80	24.18
	1.00	1	5	5.04	3.90	17.20	16.80
	0.50	1	20	4.95	3.90	18.34	58.58
	0.25	1	20	4.98	3.90	18.00	86.74
	0.13	1	20	5.16	3.90	16.00	97.31
7P	1.00	1	20	5.30	4.24	21.18	56.92
	1.00	1	15	5.31	4.24	21.00	51.50
	1.00	1	10	5.40	4.24	19.80	49.02
	1.00	1	5	5.44	4.24	19.20	46.68
	0.50	1	20	5.25	4.24	22.00	103.78
	0.25	1	20	5.49	4.24	18.60	119.14
	0.13	1	20	5.60	4.24	17.50	128.55
8P	1.00	1	20	5.10	4.00	18.60	44.99
	1.00	1	15	5.11	4.00	18.40	38.47
	1.00	1	10	5.21	4.00	17.20	25.39
	1.00	1	5	5.23	4.00	17.00	25.10
	0.50	1	20	5.10	4.00	18.60	61.77
	0.25	1	20	5.20	4.00	17.30	88.31
	0.13	1	20	5.25	4.00	16.80	103.26

TABLE C.2 RESULTS OF FILTER LEAF TESTS OF DIGESTED SLUDGE FOR THE EVALUATION OF PARAMETERS m, n, s, Ro

Sample #	Form Time (min)	Dry Time (min)	Vacuum (in. Hg)	C (%)	Ci (%)	Cf (%)	Filter Loading (lb/ft^2-h)
1S	1.00	1	20	1.30	1.20	15.78	12.97
	1.00	1	15	1.30	1.20	15.49	11.01
	1.00	1	10	1.30	1.20	15.36	10.10
	1.00	1	5	1.31	1.20	14.00	6.80
	0.50	1	20	1.30	1.20	15.90	24.23
	0.25	1	20	1.30	1.20	15.18	37.09
	0.13	1	20	1.31	1.20	14.49	60.50
2S	1.00	1	20	3.64	3.03	18.00	86.65
	1.00	1	15	3.65	3.03	17.86	57.05
	1.00	1	10	3.65	3.03	17.82	49.36
	1.00	1	5	3.70	3.03	16.64	33.63
	0.50	1	20	3.64	3.03	18.15	151.79
	0.25	1	20	3.67	3.03	17.43	136.37
	0.13	1	20	3.67	3.03	17.35	208.66
3S	1.00	1	20	1.89	1.70	16.54	36.03
	1.00	1	15	1.90	1.70	16.50	35.94
	1.00	1	10	1.90	1.70	16.21	28.74
	1.00	1	5	1.90	1.70	16.17	18.13
	0.50	1	20	1.89	1.70	16.68	74.46
	0.25	1	20	1.89	1.70	16.53	139.66
	0.13	1	20	1.91	1.70	15.70	92.11
4S	1.00	1	20	1.53	1.40	16.36	23.42
	1.00	1	15	1.53	1.40	16.33	23.24
	1.00	1	10	1.53	1.40	16.20	19.68
	1.00	1	5	1.53	1.40	16.08	12.20
	0.50	1	20	1.53	1.40	16.41	43.46
	0.25	1	20	1.53	1.40	16.24	80.33
	0.13	1	20	1.54	1.40	15.61	86.93

TABLE C.2 RESULTS OF FILTER LEAF TESTS OF DIGESTED SLUDGE FOR THE EVALUATION OF PARAMETERS m, n, s, Ro Cont.

Sample #	Form Time (min)	Dry Time (min)	Vacuum (in. Hg)	C (%)	Ci (%)	Cf (%)	Filter Loading (lb/ft^2-h)
5S	1.00	1	20	2.37	2.09	17.46	46.58
	1.00	1	15	2.38	2.09	17.40	39.47
	1.00	1	10	2.38	2.09	17.35	34.14
	1.00	1	5	2.39	2.09	16.85	24.59
	0.50	1	20	2.38	2.09	16.94	41.07
	0.25	1	20	2.39	2.09	16.80	67.17
	0.13	1	20	2.39	2.09	16.72	116.83
6S	1.00	1	20	2.63	2.29	17.92	71.96
	1.00	1	15	2.63	2.29	17.90	61.77
	1.00	1	10	2.63	2.29	17.60	38.11
	1.00	1	5	2.64	2.29	17.14	31.92
	0.50	1	20	2.63	2.29	17.68	64.33
	0.25	1	20	2.63	2.29	17.52	98.28
	0.13	1	20	2.64	2.29	17.40	179.28
7S	1.00	1	20	3.90	3.21	18.23	87.74
	1.00	1	15	3.91	3.21	18.00	73.33
	1.00	1	10	3.93	3.21	17.56	48.74
	1.00	1	5	3.95	3.21	17.20	36.82
	0.50	1	20	3.90	3.21	18.21	140.82
	0.25	1	20	3.93	3.21	17.60	150.53
	0.13	1	20	3.97	3.21	16.75	170.38
8S	1.00	1	20	2.09	1.86	17.23	43.14
	1.00	1	15	2.09	1.86	17.21	40.80
	1.00	1	10	2.09	1.86	16.59	30.25
	1.00	1	5	2.11	1.86	15.83	23.50
	0.50	1	20	2.09	1.86	17.20	43.23
	0.25	1	20	2.09	1.86	17.18	82.86
	0.13	1	20	2.09	1.86	17.12	131.13

Appendix D

Statistical Analysis of the Results

Statistical Analysis of the Results

I. Buchner Funnel Tests

Comparison of the results for specific resistance of primary and digested sludges.

Tabla D.1 Specific Resistance, cm/g x 10 $^{-10}$

	Primary	Digested
	1.686	17.092
	2.477	4.655
	3.393	9.362
	2.041	8.540
	1.451	12.204
	1.490	11.946
	2.568	9.473
	5.269	9.547
Average	2.547	10.352

Analysis of Variance

H_0: $M_1 = M_2$
H_1: $M_1 \neq M_2$

SST = 344.68 (total sum of squares)
SSA = 243.66 (treatments sum of squares)
SSE = 101.02 (error sum of squares)

Table D.2 ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Calculated F
Treatment	243.66	1	243.66	33.8
Error	101.02	14	7.22	
Total	344.68	15	22.98	

Critical value of F = 8.86[26] (V^1 = 1, V^2 = 14, α = 0.01)

Therefore, it can be concluded with 99% confidence that the specific resistance of the digested sludge is significantly higher than for the primary sludge.

II. Non-Linear Regression Results

A. Primary Sludge

1. Confidence intervals for the regression results (27)

$b_i - t_{1\alpha/2}\, s_i < \beta_i < b_i + t_{1-\alpha/2}\, s_i$

where: s_i = standard error of the estimate of parameter b_i

$t_{1-\alpha/2}$ = percentage point in the t distribution with (n - k) degrees of freedom

n = total number of observations

k = number of parameters estimated

For this case, n = 56, k = 4

For 95% confidence ($\alpha = 0.05$)

$t_{1-\alpha/2} = t_{0.975}$ (52 degrees of freedom) = 2.0(25)

The results are obtained directly from the computer program output (see Ap. E)

Parameter	Standard Error	95% Interval
3.3400	1.2225	0.8960-5.7850
0.4989	0.1105	0.2780-0.7199
0.8253	0.0902	0.6449-1.0056
0.3405	0.0358	0.2689-0.4121

2. Show the parameters are significantly different from cero.

Null hypothesis:

$H_0: \beta_i = 0$

$H_i: \beta_i \neq 0$

Calculate $t = b_i/s_i$ and compare with $t_{critical}$

$t_{critical} = t_{1-\alpha/2}$ with (n - k) degrees of freedom

For 95% confidence and 52 degrees of freedom:

$t_{critical} = 2.0$

Parameter	t	Significantly different from cero?
3.340	2.73	yes
0.4989	4.51	yes
0.8253	9.15	yes
0.3405	9.51	yes

3. Covariance analysis

The correlation coefficients matrix is obtained from the computer program output (see AP. E)

```
 1.000
-0.873     1.000
-0.493     0.026     1.000
 0.375    -0.498    -0.084     1.000
```

A significant correlation is observed between the estimates of parameters 1 and 2.

4. Test for random distribution of the residuals

The randomness of the residuals is tested through the runs test. The number of sign changes in the residuals sequence is called r. Brownlee (29) showed this variable can be approximated by a normal distribution with avarage and standard deviation given by:

$$\bar{r} = \frac{2n_1 n_2}{n_1 + n_2} + 1$$

$$T = \frac{2n_1 n_2(2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}$$

where: n_1 = number of positive residuals
n_2 = number of negative residuals

The standardized form of the variable is:

$$z = \frac{r - \bar{r}}{\sigma}$$

For this case (see Ap. E):

$r = 34$
$n^1 = 32$
$n_2 = 24$

$\bar{r} = 28.4$, $= 3.63$

$$z = \frac{34-28.4}{3.63} = 1.54$$

$z_{critical} = 1.96$ for 95% conficence

Therefore, it can be concluded the residuals are randomly distributed and the model is appropriate.

B. Digested Sludge

1. 95% confidence intervals for the parameter estimates

Parameter	Standard Error	95% Interval
3.033	1.513	0.068-6.059
0.676	0.160	0.355-0.996
0.981	0.090	0.802-1.160
0.417	0.044	0.331-0.504

2. Show the parameters are significantly different from cero.

Parameter	t	Significantly different from cero
3.033	2.005	yes
0.676	4.225	yes
0.981	10.900	yes
0.417	9.477	yes

3. Covariance analysis

Correlation matrix

$$\begin{array}{rrrr} 1.000 & & & \\ -0.970 & 1.000 & & \\ -0.212 & 0.006 & 1.000 & \\ 0.363 & -0.472 & -0.014 & 1.000 \end{array}$$

Significant correlation between the estimates of parameters 1 and 2.

4. Runs test

$r = 27$

$n_1 = 30$

$n_2 = 26$

$\bar{r} = 28.86$

$\sigma = 3.69$

$z = -0.504$

$z_{critical} = -1.96$

It can be concluded, with 95% confidence, the residual are distributed randomly.

Appendix E

Computer Program Output for the Primary Sludge

```
FORTRAN IV G1  RELEASE 2.0               MAIN               DATE = WED JUL 01, 1987

                    C      PROGRAM REGRES
 0001                      DIMENSION PAR(4),Y(56)
 0002                      COMMON X1(56),X2(56),X3(56)
 0003                      NPAR=4
 0004                      NOB=56
 0005                      PAR(1)=969.
 0006                      PAR(2)=0.4
 0007                      PAR(3)=1.5
 0008                      PAR(4)=0.44
 0009                      READ(5,10)  (Y(I),X1(I),X2(I),X3(I),I=1,NOB,1)
 0010                 10   FORMAT(4(F9.3))
 0011                      CALL LS(NOB,Y,NPAR,PAR,0)
 0012                      STOP
 0013                      END

  *OPTIONS IN EFFECT*  NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
  *OPTIONS IN EFFECT*  NAME = MAIN    , LINECNT =        56
  *STATISTICS*    SOURCE STATEMENTS =       13,PROGRAM SIZE = 0002DC
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
FORTRAN IV G1  RELEASE 2.0                MODEL               DATE = WED JUL 01, 1987

 0001                     SUBROUTINE MODEL(PAR,F,NOB,NPAR)
 0002                     DIMENSION PAR(NPAR),F(NOB)
 0003                     COMMON X1(50),X2(50),X3(50)
 0004                     DO 1 I=1,NOB
 0005                     F(I)=PAR(1)*(X1(I)**PAR(2))*(X2(I)**PAR(3))*(1./(X3(I)**PAR(4)))
 0006                   1 CONTINUE
 0007                     RETURN
 0008                     END

  *OPTIONS IN EFFECT*  NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
  *OPTIONS IN EFFECT*  NAME = MODEL   , LINECNT =        56
  *STATISTICS*    SOURCE STATEMENTS =        8,PROGRAM SIZE = 000292
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
FORTRAN IV G1  RELEASE 2.0                DIF                 DATE = WED JUL 01, 1987

 0001                   SUBROUTINE DIF(PAR,Z,FO,MOB,NOB,NPAR,DEL)
 0002                   DIMENSION PAR(NPAR),Z(MOB,NPAR),FO(MOB),DEL(NPAR)
 0003                   RETURN
 0004                   END

  *OPTIONS IN EFFECT*  NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
  *OPTIONS IN EFFECT*  NAME = DIF      , LINECNT =        56
  *STATISTICS*    SOURCE STATEMENTS =        4,PROGRAM SIZE = 0001DC
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE LS (NOB, OBS, NPAR, PAR, IRED)
     DOUBLE PRECISIONPIVOT,CMULT,REF,DENOM,TFACT,FACTO,SSRED,TRED,DRIVRED,DRIVE
      DOUBLE PRECISION A(   31,   31), PARB(  30), X, FLR
      DIMENSION OBS(NOB), PAR(NPAR), DEL(  30), CHMAX(  30)
      DIMENSION Z(150,   31), FO(150), F(150), FUP(150), FLW(150)       )
      DIMENSION LEIU(  30), BNDLW(  30), BNDUP(  30)
      DIMENSION SS(3), FL(3), FD(4), SD(4), LSTP(  30), SPDA(  30)      0)
      COMMON/BLOK1/DEL,CHMAX,BNDLW,BNDUP,REDA,RSSTOL,ITMAX,LISTS,IDIF  ,IDIF
      COMMON/BLOK2/IDER
      LOGICAL LG, LG1
      DATAIII/0/
      MOB = 200
      MPAR = 30
      FINF = 1.D30
      IF(IRED.GE.1) GO TO 100
      DO 101 I = 1,NPAR
      LG = PAR(I).EQ.0.
      IF(LG) WRITE(6,25) I
      IF(LG) STOP
 25   FORMAT('0PARAMETER PAR(',I2,') IS EQUAL TO ZERO')
      DEL(I) = -0.01
      CHMAX(I) = 0.2*ABS(PAR(I))
      BNDLW(I) = -FINF
 101  BNDUP(I) = FINF
      REDA = 1.E-4
      RSSTOL = 1.E-3
      ITMAX = 10
      LISTS = 3
      IDIF = 1
      IF(IRED.LE.-1) RETURN
 100  CONTINUE
      III = III + 1
      ZERO = 0
      WRITE(6,14) III, NOB, NPAR
 14   FORMAT('1START OF PROBLEM NO.',I5,' WITH',I5,' OBSERVATIONS AND',S AND',I
     A I5,' PARAMETERS'/'0VERSION 4 OF LS,     AUGUST 1971')
      M = MOB
      IF(NOB.GT.MOB) WRITE(6,15) NOB, M
 15   FORMAT('0INCREASE THE VALUE OF MOB TO',I5,' (',I5,' WAS USED)')  ED)')
      M = MPAR
      IF(NPAR.GT.MPAR) WRITE(6,16)  NPAR, M
 16   FORMAT('0INCREASE THE VALUE OF MPAR TO',I5,' (',I5,' WAS USED)') SED)')
      IF(NOB.GT.MOB.OR.NPAR.GT.MPAR) STOP
      WRITE(6,7)  (BNDUP(I), I = 1,NPAR)
 7    FORMAT(' BNDUP(I)=',10E12.5)
      WRITE(6,8)  (PAR(I), I = 1,NPAR)
 8    FORMAT(' PAR(I)  =',10E12.5)
      WRITE(6,6) (BNDLW(I), I =1,NPAR)
 6    FORMAT(' BNDLW(I)=',10E12.5)
      WRITE(6,651)
      WRITE(6,11)  (DEL(I), I = 1,NPAR)
 11   FORMAT(' DEL(I)  =',10E12.5)
      WRITE(6,13)  (CHMAX(I), I = 1,NPAR)
 13   FORMAT(' CHMAX(I)=',10E12.5)
```

```
      WRITE(6,17) REDA, RSSTOL, ITMAX, LISTS, IDIF
17    FORMAT('0REDA =',E12.4,' RSSTOL =',E12.4,' ITMAX =',I4,'   LISTS'STS'
   A   ,' =',I3,'  IDIF =',I3)
      FMIN = 1.
      DO 4 I = 1,NPAR
      FMIN = AMIN1(FMIN,ABS(DEL(I)))
      LG=PAR(I).LT.BNDLW(I).OR.PAR(I).GT.BNDUP(I)
      IF(LG) WRITE(6,18) I
4     IF(LG) STOP
18    FORMAT('0PARAMETER PAR(',I2,') IS OUTSIDE ITS BOUNDS')
      IF(FMIN.GE.1.E-35) WRITE(6,26)
26    FORMAT('0SUBROUTINE DIF IS NOT USED')
      IF(FMIN.LT.1.E-35) WRITE(6,27)
27    FORMAT('0SUBROUTINE DIF IS USED')
      ITNO = 1
      NPAR1 = NPAR + 1
      NFUNC = 0
1     WRITE(6,3) ITNO, NFUNC
3     FORMAT('0START ITERATION NO.',I3,' NO. OF FUNCTION CALLS',I4)     4)
      IDER = 0
      CALL MODEL(PAR,F0,NOB,NPAR)
      NFUNC = NFUNC + 1
      WRITE(6,2) (PAR(I), I = 1,NPAR)
2     FORMAT(' PAR(I) =',8E14.7)
      ITNO = ITNO + 1
      IF(FMIN.LT.1.E-35) CALL DIF(PAR,Z,F0,MOB,NOB,NPAR,DEL)
      DO 5 IOB = 1,NOB
5     Z(IOB,NPAR1) = -F0(IOB) + OBS(IOB)
      DO 490 IPAR = 1,NPAR
      IDER = IPAR
      IF(ABS(DEL(IPAR)).LT.1./FINF) GO TO 490
      IF (CHMAX(IPAR).NE.0..AND.BNDUP(IPAR) - BNDLW(IPAR).GE.1./FINF)   FINF)
   A  GO TO 410
      DO 400 IOB = 1,NOB
400   Z(IOB,IPAR) = 0.
      GO TO 490
410   LG = DEL(IPAR).GT.0.
      LG1= .NOT.LG.AND.ABS(PAR(IPAR)*DEL(IPAR)).LE.1.E-20
      IF(LG1) WRITE(6,60) IPAR
60    FORMAT('0THE VALUE OF PAR(',I3,') IF TOO SMALL FOR DETERMINING'  NING'
   A     ,' THE DERIVATIVE')
      IF(LG1) STOP
      PARD = PAR(IPAR)
      IF(LG) DPAR = DEL (IPAR)
      IF(.NOT.LG) DPAR = ABS(PAR(IPAR)*DEL(IPAR))
      JDIF =IDIF
      S1 = BNDUP(IPAR) - PARD - DPAR
      S2 = PARD - DPAR - BNDLW(IPAR)
      IF(S1.LT.0..AND.S2.GT.S1.AND.IDIF.GT.0) JDIF = -1
      IF(S2.LT.0..AND.S1.GT.S2.AND.IDIF.LT.0) JDIF = 1
      IF(JDIF.LT.0) GO TO 420
      PAR(IPAR) = AMIN1(PARD + DPAR,BNDUP(IPAR))
      DENOM = PAR(IPAR)
      CALL MODEL(PAR,FUP,NOB,NPAR)
```

```
          WRITE(6,17) REDA, RSSTOL, ITMAX, LISTS, IDIF
  17      FORMAT('0REDA =',E12.4,' RSSTOL =',E12.4,' ITMAX =',I4,'   LISTS'STS'
      A     ,' =',I3,'  IDIF =',I3)
          FMIN = 1.
          DO 4 I = 1,NPAR
          FMIN = AMIN1(FMIN,ABS(DEL(I)))
          LG=PAR(I).LT.BNDLW(I).OR.PAR(I).GT.BNDUP(I)
          IF(LG) WRITE(6,18) I
  4       IF(LG) STOP
  18      FORMAT('0PARAMETER PAR(',I2,') IS OUTSIDE ITS BOUNDS')
          IF(FMIN.GE.1.E-35) WRITE(6,26)
  26      FORMAT('0SUBROUTINE DIF IS NOT USED')
          IF(FMIN.LT.1.E-35) WRITE(6,27)
  27      FORMAT('0SUBROUTINE DIF IS USED')
          ITNO = 1
          NPAR1 = NPAR + 1
          NFUNC = 0
  1       WRITE(6,3) ITNO, NFUNC
  3       FORMAT('0START ITERATION NO.',I3,' NO. OF FUNCTION CALLS',I4)     4)
          IDER = 0
          CALL MODEL(PAR,F0,NOB,NPAR)
          NFUNC = NFUNC + 1
          WRITE(6,2)  (PAR(I), I = 1,NPAR)
  2       FORMAT(' PAR(I) =',8E14.7)
          ITNO = ITNO + 1
          IF(FMIN.LT.1.E-35) CALL DIF(PAR,Z,F0,NOB,NOB,NPAR,DEL)
          DO 5 IOB = 1,NOB
  5       Z(IOB,NPAR1) = -F0(IOB) + OBS(IOB)
          DO 490 IPAR = 1,NPAR
          IDER = IPAR
          IF(ABS(DEL(IPAR)).LT.1./FINP) GO TO 490
          IF  (CHMAX(IPAR).NE.0..AND.BNDUP(IPAR) - BNDLW(IPAR).GT.1./FINF)   FINP)
      A    GO TO 410
          DO 400 IOB = 1,NOB
  400     Z(IOB,IPAR) = 0.
          GO TO 490
  410     LG = DEL(IPAR).GT.0.
          LG1= .NOT.LG.AND.ABS(PAR(IPAR)*DEL(IPAR)).LE.1.E-20
          IF(LG1) WRITE(6,60) IPAR
  60      FORMAT('0THE VALUE OF PAR(',I3,') IF TOO SMALL FOR DETERMINING'  NING'
      A       ,' THE DERIVATIVE')
          IF(LG1) STOP
          PARD = PAR(IPAR)
          IF(LG) DPAR = DEL  (IPAR)
          IF(.NOT.LG) DPAR = ABS(PAR(IPAR)*DEL(IPAR))
          JDIF =IDIF
          S1 = BNDUP(IPAR) - PARD - DPAR
          S2 = PARD - DPAR - BNDLW(IPAR)
          IF(S1.LT.0..AND.S2.GT.S1.AND.IDIF.GT.0) JDIF = -1
          IF(S2.LT.0..AND.S1.GT.S2.AND.IDIF.LT.0) JDIF = 1
          IF(JDIF.LT.0) GO TO 420
          PAR(IPAR) = AMIN1(PARD + DPAR,BNDUP(IPAR))
          DENOM = PAR(IPAR)
          CALL MODEL(PAR,FUP,NOB,NPAR)
```

```
      NFUNC = NFUNC + 1
      GO TO 440
420   DENOM = PARD
      DO 430 IOB = 1,NOB
430   FUP(IOB) = FO(IOB)
440   IF(JDIF.GT.0) GO TO 450
      PAR(IPAR) = AMAX1(PARD - DPAR,BNDLW(IPAR))
      DENOM = DENOM - PAR(IPAR)
      CALL MODEL(PAR,FLW,NOB,NPAR)
      NFUNC = NFUNC + 1
      GO TO 470
450   DENOM = DENOM - PARD
      DO 460 IOB = 1,NOB
460   FLW(IOB) = FO(IOB)
470   PAR(IPAR) = PARD
      DO480 IOB = 1,NOB
480   Z(IOB,IPAR) = (FUP(IOB) - FLW(IOB))/DENOM
490   CONTINUE
      DO 20 IPAR = 1,NPAR1
      DO 20 JPAR = 1,IPAR
      X = 0
      DO 19 IOB = 1,NOB
19    X = X + Z(IOB,IPAR)*Z(IOB,JPAR)
      A(IPAR,JPAR) = X
20    A(JPAR,IPAR) = X
      IF(ITNO.EQ.2) WRITE(6,12) A(NPAR1,NPAR1)
12    FORMAT('0INITIAL SUM OF SQUARES =',D12.4)
21    IF(LISTS.LE.0) GO TO 501
      WRITE(6,22)
22    FORMAT(' MATRIX OF NORMAL EQUATIONS ')
      DO 49 I = 1,NPAR1
49    WRITE(6,50) (A(I,J), J = 1,NPAR1)
50    FORMAT(1X,10D12.4)
501   NES = 0
      NTRANS = 0
      SSB = A(NPAR1,NPAR1)
      DO 502 I = 1,NPAR
      LSTP(I) = 0
      LBIU(I) = 0
      PARB(I) = PAR(I)
502   SPDA(I) = REDA*A(I,I)
503   SSRED = 0
      JBIU = 0
      NPIV = 0
      DO 510 I = 1,NPAR
      IF(LSTP(I).NE.0.OR.A(I,I).LE.SPDA(I).OR.ABS(CHMAX(I)).LT.1./FINF)./FINF)
    A       GO TO 510
      TRED = A(I,NPAR1)**2/A(I,I)
      IF(TRED.LT.SSRED) GO TO 510
      JB = 0
      FACTO = FINF
      DO 508 J = 1,NPAR
      IF(J.NE.I) GO TO 504
      REF = PAR(I)
```

```
      NFUNC = NFUNC + 1
      GO TO 440
420   DENOM = PARD
      DO 430 IOB = 1,NOB
430   FUP(IOB) = FO(IOB)
440   IF(JDIF.GT.0) GO TO 450
      PAR(IPAR) = AMAX1(PARD - DPAR,BNDLW(IPAR))
      DENOM = DENOM - PAR(IPAR)
      CALL MODEL(PAR,FLW,NOB,NPAR)
      NFUNC = NFUNC + 1
      GO TO 470
450   DENOM = DENOM - PARD
      DO 460 IOB = 1,NOB
460   FLW(IOB) = FO(IOB)
470   PAR(IPAR) = PARD
      DO480 IOB = 1,NOB
480   Z(IOB,IPAR) = (FUP(IOB) - FLW(IOB))/DENOM
490   CONTINUE
      DO 20 IPAR = 1,NPAR1
      DO 20 JPAR = 1,IPAR
      X = 0
      DO 19 IOB = 1,NOB
19    X = X + Z(IOB,IPAR)*Z(IOB,JPAR)
      A(IPAR,JPAR) = X
20    A(JPAR,IPAR) = X
      IF(ITNO.EQ.2) WRITE(6,12) A(NPAR1,NPAR1)
12    FORMAT('0INITIAL SUM OF SQUARES =',D12.4)
21    IF(LISTS.LE.0) GO TO 501
      WRITE(6,22)
22    FORMAT(' MATRIX OF NORMAL EQUATIONS ')
      DO 49 I = 1,NPAR1
49    WRITE(6,50) (A(I,J), J = 1,NPAR1)
50    FORMAT(1X,10D12.4)
501   NES = 0
      NTRANS = 0
      SSB = A(NPAR1,NPAR1)
      DO 502 I = 1,NPAR
      LSTP(I) = 0
      LBIU(I) = 0
      PARB(I) = PAR(I)
502   SPDA(I) = REDA*A(I,I)
503   SSRED = 0
      JBIU = 0
      NPIV = 0
      DO 510 I = 1,NPAR
      IF(LSTP(I).NE.0.OR.A(I,I).LE.SPDA(I).OR.ABS(CHMAX(I)).LT.1./FINF)./FINF)
     A        GO TO 510
      TRED = A(I,NPAR1)**2/A(I,I)
      IF(TRED.LT.SSRED) GO TO 510
      JB = 0
      FACTO = FINF
      DO 508 J = 1,NPAR
      IF(J.NE.I) GO TO 504
      REF = PAR(I)
```

```
          DENOM = A(I,NPAR1)/A(I,I)
          GO TO 505
  504     IF(LSTP(J).EQ.0) GO TO 508
          REF = PAR(J) + A(J,NPAR1)
          DENOM = -A(J,I)*A(I,NPAR1)/A(I,I)
  505     IF(DENOM.GT.1./FINF) GO TO 506
          IF(DENOM.GT.-1./FINF) GO TO 508
          IF(BNDLW(J).LE.-FINF) GO TO 508
         TFACT = (BNDLW(J) - REF)/DENOM
         IJ = -J
         GO TO 507
 506     IF(BNDUP(J).GE.FINF) GO TO 508
         TFACT = (BNDUP(J) - REF)/DENOM
         IJ = J
 507     IF(FACTO.LE.TFACT) GO TO 508
         FACTO = TFACT
         JB = IJ
 508     CONTINUE
         IF(FACTO.GT.1.) GO TO 509
         TRED = TRED*FACTO*(2. - FACTO)
 509     IF(TRED.LT.SSRED) GO TO 510
         SSRED = TRED
         FLMAX = FACTO
         JBIU = JB
         NPIV = I
 510     CONTINUE
         IF(NPIV .EQ. 0 .OR. FLMAX .LT. 1./FINF) GO TO 530
         NTRANS = NTRANS + 1
         IF(FLMAX.LE.1.) GO TO 52
         NES = NES + 1
         LSTP(NPIV) = NPIV
         LBIU(NPIV) = 0
         GO TO 57
 52      IRESP = IABS(JBIU)
         IF(IRESP.NE.NPIV) NES = NES -1
         LSTP(IRESP) = 0
         DPIVP = FACTO*A(NPIV,NPAR1)/A(NPIV,NPIV)
         PAR(NPIV) = PAR(NPIV) + DPIVP
         PAR(IRESP) = BNDLW(IRESP)
         IF(JBIU.GT.0) PAR(IRESP) = BNDUP(IRESP)
 57      DO 58 I = 1,NPAR
         IF(PAR(I) - BNDLW(I).LE.1./FINF.AND.LSTP(I).EQ.0) LBIU(I) = -1
 58      IF(BNDUP(I) - PAR(I).LE.1./FINF.AND.LSTP(I).EQ.0) LBIU(I) = 1
         IF(LISTS.GE.5) WRITE(6,59) (LBIU(I), I = 1,NPAR)
 59      FORMAT(1X/(' LBIU =',20I5))
         IF(LISTS.GE.5) WRITE(6,61) (LSTP(I), I = 1,NPAR)
 61      FORMAT(' LSTP =',20I5)
         IF(LISTS.GE.5) WRITE(6,2) (PAR(I), I = 1,NPAR)
         IF(FLMAX.LE.1.) GO TO70
 62      IF(FLMAX.LE.1.) NPIV = IRESP
         PIVOT = A(NPIV,NPIV)
         A(NPIV,NPIV) = 1.D0
 63      DO 512 J = 1,NPAR1
 512     A(NPIV,J) = A(NPIV,J)/PIVOT
```

```
 513    DO 520 I = 1,NPAR1
        IF(I.EQ.NPIV) GO TO 520
        CMULT = A(I,NPIV)
        DO 519 J = 1,NPAR1
 519    IF(J.NE.NPIV) A(I,J) = A(I,J) - CMULT*A(NPIV,J)
        A(I,NPIV) = - A(I,NPIV)/PIVOT
 520    CONTINUE
 521    IF(LISTS.LT.5) GO TO 503
        WRITE(6,30)
 30     FORMAT(' TRANSFORMED MATRIX INCLUDING INVERSE OF EQUATIONS',
       A      'THAT ARE NOW SOLVED')
        DO 522 I = 1,NPAR1
 522    WRITE(6,50) (A(I,J), J = 1,NPAR1)
        GO TO 503
 70     A(NPAR1,NPAR1) = A(NPAR1,NPAR1) - SSRED
        DO 76 I = 1,NPAR
        A(I,NPAR1) = A(I,NPAR1) - DPIVP*A(I,NPIV)
        LG1 = LSTP(I).NE.0
        LG = PAR(I).GT.BNDUP(I)
        IF(LG) PAR(I) = BNDUP(I)
        IF(LG.AND.LG1) A(I,NPAR1) = 0.
        LG = PAR(I).LT.BNDLW(I)
        IF(LG) PAR(I) = BNDLW(I)
        IF(LG.AND.LG1) A(I,NPAR1) = 0.
        IF(LG1) A(NPAR1,I) = -A(I,NPAR1)
 76     IF(.NOT.LG1) A(NPAR1,I) = A(I,NPAR1)
 73     IF(NPIV.EQ.IRESP) GO TO 521
        A(IRESP,NPAR1) = 0.D0
        A(NPAR1,IRESP) = 0.D0
        GO TO 62
 530    IF(NTRANS.GT.0) GO TO 531
 541    WRITE(6,542)
 542    FORMAT('0NO PARAMETER CHANGES PERMITTED. INSPECT BOUNDS AND CHMAX'
       A    ,' ARRAY')
        STOP
 531    SSE1 = A(NPAR1,NPAR1)
 550    DO 552 I = 1,NPAR
        IF(LSTP(I).EQ.0) A(I,NPAR1) = 0.D0
        A(I,NPAR1) = A(I,NPAR1) + PAR(I) - PARB(I)
 552    CONTINUE
        ILAM = 0
        FLAM = 1
        ILMAX = 0
        FLMAX = FINF
        QMAX = FINF
        DO 536 I = 1,NPAR
        ABSA = DABS(A(I,NPAR1))
        IF(ABSA.LT.1./FINF) GO TO 536
        QLAM = ABS(CHMAX(I))
        IF(CHMAX(I).LE.ZERO) QLAM = QLAM*DABS(PARB(I))
        IF(FLAM*ABSA.LE.QLAM) GO TO 534
        ILAM = I
        FLAM= QLAM/ABSA
 534    IF(A(I,NPAR1).GT.ZERO) QMAX = BNDUP(I) - PARB(I)
```

```
          IF(A(I,NPAR1).LT.ZERO) QMAX = PARB(I) - BNDLW(I)
          IF(QMAX.GE.FLMAX*ABSA) GO TO 536
          ILMAX = I
          FLMAX = QMAX/ABSA
  536     CONTINUE
          IF(ILAM.EQ.0) GO TO 547
          WRITE(6,538) ILAM, FLAM
  538     FORMAT(' PARAMETER',I4,' LIMITS THE CORRECTIONS TO ',E12.4,
         A       'TIMES THE GAUSS-NEWTON VALUES.')
  547     IF(FLMAX.LT.1. .AND.ILMAX.NE.ILAM) WRITE(6,538) ILMAX, FLMAX
  548     IF(FLAM.LT.1./FINF) GO TO 541
  560     SBEST = SSB
          FBEST = 0
          FLR = 2*FINF
          SSP = SSE1
          SS(1) = SSB
          FL(1) = 0
          SS(2) = 1.01*FINF
          FL(2) = 1.01*FLMAX
          SS(3) = 1.02*FINF
          FL(3) = 1.02*FLMAX
          FLT = FLAM
          KEY = 0
          LG = .TRUE.
  561     DO 590 IGRID = 1,ITMAX
  558     DO 562 I = 1,NPAR
          PAR(I) = PARB(I) + FLT*A(I,NPAR1)
          IF(PAR(I).GT.BNDUP(I)) PAR(I) = BNDUP(I)
  562     IF(PAR(I).LT.BNDLW(I)) PAR(I) = BNDLW(I)
          IDER = -1
          CALL MODEL(PAR,F,NOB,NPAR)
          NFUNC = NFUNC + 1
          SST = 0.
          DO 563 IOB = 1,NOB
          DF = ABS(F(IOB) - OBS(IOB))
          IF(DF.GT.1.E15) WRITE(6,566) IOB, F(IOB)
  566     FORMAT('DF(',I3,') =',E10.3,' IS TOO LARGE')
          IF(DF.GT.1.E15) STOP
  563     SST = SST + DF**2
          SSR = SST
          LG = LG.AND.SST.GT.SSB
          IF(KEY.EQ.1) GO TO 581
          IF(LISTS.GE.4) WRITE(6,564) FLT, SST, IGRID, FLR, SSP
  564     FORMAT('0FLT =',E13.5,' SST =',E13.5,' IGRID =',I3,' FLR =  ',D11.
         A    5,' SSP =',E13.5)
          IF((ABS(FLT-1.).GT.ESSTOL.AND.ABS(FLT-FLMAX).GT.ESSTOL).OR.ABS(SST
         A    -SSE1).GT.ABS(SSE1)*RSSTOL.OR.LG) GO TO 565
          FLR = FLT
          GO TO 581
  565     INS = 0
          K = 0
          DO 575 I = 1,3
          IF(FL(I).GT.FLT.AND.INS.EQ.0) INS = 1
          IF(INS.GT.0) K = 1
```

```
        IK = I + K
        FD(I K) = FL(I)
 575    SD(I K) = SS(I)
        IF(INS.EQ.0) INS = 4
        FD(INS) = FLT
        SD(INS) = SST
        K = 0
        IF((SD(2).GT.SD(3).OR.INS.EQ.4).AND.IGRID.GT.2) K = 1
        IF(SD(1).LE.SD(2)) K = 0
        DO 576 I = 1,3
        IK = I + K
        FL(I) = FD(I K)
 576    SS(I) = SD(I K)
        IF(LISTS.GE.6) WRITE(6,577) (FD(J), J = 1,4)
        IF(LISTS.GE.6) WRITE(6,568) (SD(J), J = 1,4)
 577    FORMAT(' FD TABLE ',4E13.5)
 568    FORMAT(' SD TABLE ',4E13.5)
        IF(SST.GE.SBEST) GO TO 578
        SBEST = SST
        FBEST = FLT
 578    IF(FL(3).LE.FLMAX) GO TO 583
        IF(SS(1).LE.SS(2)) GO TO 587
        FLT = 0.1*FL(1) + 0.9*FL(2)
        GO TO 590
 583    DENOM = (FL(3)-FL(1))*(SS(2)-SS(1))+ (FL(1)-FL(2))*(SS(3)-SS(1))
        IF(DENOM.LE.-1./FINF.AND.FL(3).LT.FINF) GO TO 584
        SSP = FINF
        IF(SS(1).GT.SS(2)) GO TO 585
 587    FLT = 0.9*FL(1) + 0.1*FL(2)
        GO TO 590
 585    FLT = FLMAX
        IF(FL(3).GE.0.98*FLMAX) FLT = 0.1*FL(2) + 0.9*FL(3)
        IF(FL(3).LT.0.49*FLMAX) FLT= 2.*FL(3)
        GO TO 590
 584    FOLD = FLR
        FLR = ((FL(3)**2 - FL(1)**2)*(SS(2) - SS(1))+ (FL(1)**2 - FL(2)
       A      **2)*(SS(3) - SS(1)))/2./DENOM
        IF(FLR.GE.FLMAX) FLR = FLMAX
        IF(FLR.LE.FL(1)) FLR = FL(1)
        SSR = SS(1) + (SS(2) - SS(1))*(FLR-FL(1))*(FLR-FL(3))/(FL(2)
       A -FL(1))/(FL(2) - FL(3)) + (SS(3)-SS(1))*(FLR-FL(1))*(FLR-FL(2))
       B      /(FL(3)-FL(1))/(FL(3) - FL(2))
        IF(ABS(SSR-SSP).GT.ABS(RSSTOL*SSP).AND.DABS(FOLD-FLR).GT.DABS
       A      (RSSTOL*FLR)) GO TO 580
        IF(SSR.LT.0..OR.FLR.LE.FL(1).OR.FLR.GT.FL(3).OR.LS) GO TO 580
        FLT = FLR
        KEY = 1
        GO TO 558
 581    WRITE(6,579) IGRID, FLR, SSR
 579    FORMAT(' SEARCH CONVERGED AFTER',I3,' CYCLES, WITH LAMBDA=',
       A      D13.5,' AND SSQ =',E13.5)
        GO TO 626
 580    SSP = SSR
 582    FLT = 0.9*FL(1) + 0.1*FL(2)
```

```
      IF(FLR.GT.FLT) FLT = FLR
      FT = 0.1*FL(1) + 0.9*FL(2)
      IF(FLR.GT.FT)FLT = FT
      FT = 0.9*FL(2) + 0.1*FL(3)
      IF(FLR.LT.FT) FLT = FT
      IF(FLR.GE.FT) FLT = FLR
      FT = 0.1*FL(2) + 0.9*FL(3)
      IF(FLR.GT.FT) FLT = FT
      IF(FLR.GT.FL(3)) GO TO 585
 590  CONTINUE
      IGRID = ITMAX+1
      WRITE(6,591) ITMAX, FBEST, SBEST
 591  FORMAT(' SEARCH TOOK THE FULL',I4,' CYCLES. BEST TRIAL POINT,'
     A     ,' LAMBDA =',E13.5,' SSQ =',E13.5)
      FLR = FBEST
      SSR = SBEST
 626  DO 628 I = 1,NPAR
      PAR(I) = PARB(I) + A(I,NPAR1)*FLR
      IF(PAR(I).GT.BNDUP(I)) PAR(I) = BNDUP(I)
      IF(PAR(I).LT.BNDLW(I)) PAR(I) = BNDLW(I)
 628  CONTINUE
      IF(SSB.LT.SSR) WRITE(6,629) SSR, SSB
 629  FORMAT('0CURRENT SUM OF SQUARES',E15.8,' EXCEEDS RESULT',
     A       E15.8,' OF PREVIOUS ITERATION')
      IF(ITNO.LE.ITMAX.AND.ABS((SSR - SSB)/RSSTOL).GT.SSB.AND.IGRID.GT.1
     A     ) GO TO 1
      IF(ABS((SSR - SSB)/RSSTOL).GT.SSB.AND.IGRID.GT.1) WRITE(6,639)
 639  FORMAT('0********** CONVERGENCE CRITERION IS NOT SATISFIED,',
     A       ' MAXIMUM NUMBER OF ITERATIONS WAS REACHED **********')
      NDF = NOB - NES
      SEXT = 0
      IF(NDF.GT.0) SEXT = SQRT(SSR/FLOAT(NDF))
      DO 630 I = 1,NPAR
      IF(LSTP(I).NE.0) A(I,NPAR1) =DSQRT(A(I,I))
 630  IF(LSTP(I).NE.0) PARB(I) = A(I,NPAR1)*SEXT*2.
      WRITE(6,631)
 631  FORMAT('0BEST PARAMETER VALUES AND 2-SIGMA CONFIDENCE LIMITS'
     A     ,' ESTIMATED',/,' BY LINEARIZATION FOR THE INDIVIDUAL'
     A     ,'PARAMETERS ARE AS FOLLOWS.')
      J1 = (NPAR + 7)/8
      DO 650 J2 = 1,J1
      I1= (J2 - 1)*8 + 1
      I2 = MIN0(NPAR,J2*8)
      DO 632 I = I1,I2
      PARB(I) = PAR(I) + PARB(I)
 632  IF(LSTP(I).EQ.0) PARB(I) = FINF
      WRITE(6,633) (PARB(I), I = I1,I2)
 633  FORMAT('0UPR(I) =',8(1X,D13.7))
      WRITE(6,2) (PAR(I), I = I1,I2)
      DO 634 I = I1,I2
      PARB(I) = 2*PAR(I) - PARB(I)
 634  IF(LSTP(I).EQ.0) PARB(I) = - FINF
 650  WRITE(6,635) (PARB(I), I = I1,I2)
 635  FORMAT(' LWR(I) =',8(1X,D13.7))
```

```
          WRITE(6,636) SEXT, NOB, NDF
  636     FORMAT('OSTANDARD ERROR OF WEIGHTED RESIDUALS =',E13.5,
         A' ESTIMATED WITH',I5/' RESIDUALS AND',I5  ,'DEGREES OF FREEDOM.')
          IF(LISTS.LT.2) GO TO 661
          DO 640 I = 1,NPAR
          DO 640 J = 1,I
          IF(LSTP(I)*LSTP(J).NE.0) A(I,J) = A(I,J)/A(I,NPAR1)/A(J,NPAR1)
  640     IF(LSTP(I)*LSTP(J).EQ.0) A(I,J) = 1.E8
          WRITE(6,641)
  641     FORMAT('ONORMALIZED CORRELATION MATRIX')
          J1 = (NPAR + 9)/10
          DO 660 J2 = 1,J1
  652     FORMAT(1X)
          I1 = (J2 - 1)*10 + 1
          I2= MIN0(NPAR,J2*10)
          DO 660 I = I1,NPAR
          II= MIN0(I,I2)
          WRITE(6,652)
  660     WRITE(6,655) (A(I,J), J = I1,II)
  655     FORMAT(1H ,5X,10(F7.5,3X))
  651     FORMAT(1H0)
  661     IF(LISTS.LT.3) GO TO 666
          IDER = -2
          CALL MODEL(PAR,F,NOB,NPAR)
          NFUNC = NFUNC + 1
          X = 0
          DO 680 I = 1,NOB
  680     X = X + (F(I) - OBS(I))**2
          WRITE(6,662)
  662     FORMAT('OFINAL FUNCTION VALUES'/1H )
          WRITE(6,663) (F(I), I = 1,NOB)
  663     FORMAT(1X,8E12.5)
          DO 670 I = 1,NOB
  670     F(I) = F(I) - OBS(I)
          WRITE(6,665)
  665     FORMAT('ORESIDUALS'/1H )
          WRITE(6,663) (F(I), I = 1,NOB)
          WRITE(6,681) X
  681     FORMAT('OFINAL SUM OF SQUARES =',D12.4)
  666     WRITE(6,671) III, NFUNC
  671     FORMAT('OEND OF PROBLEM NO.',I4,', NO. OF FUNCTION CALLS =',I4)
  690     RETURN
          END

*OPTIONS IN EFFECT*  NOTERM,ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT*  NAME = LS      , LINECNT =       56
*STATISTICS*    SOURCE STATEMENTS =      453,PROGRAM SIZE = 00835C
*STATISTICS*  NO DIAGNOSTICS GENERATED

*STATISTICS*  NO DIAGNOSTICS THIS STEP
/DATA                                                                  11.65
00FC68 BYTES USED
EXECUTION BEGINS
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```
START OF PROBLEM NO.    1 WITH   56 OBSERVATIONS AND    4 PARAMETERS

VERSION 4 OF LS,     AUGUST 1971
BNDUP(I)= 0.10000E+31 0.10000E+31 0.10000E+31 0.10000E+31
PAR(I)   = 0.96900E+03 0.40000E+00 0.15000E+01 0.44000E+00
BNDLW(I)=-0.10000E+31-0.10000E+31-0.10000E+31-0.10000E+31


DEL(I)   =-0.10000E-01-0.10000E-01-0.10000E-01-0.10000E-01
CHMAX(I)= 0.19380E+03 0.80000E-01 0.30000E+00 0.88000E-01

REDA =  0.1000E-03 RSSTOL =  0.1000E-02 ITMAX =  10   LISTS =  3  IDIF =  1

SUBROUTINE DIF IS NOT USED

START ITERATION NO.  1 NO. OF FUNCTION CALLS   0
PAR(I) = 0.9690000E+03 0.4000000E+00 0.1500000E+01 0.4400000E+00

INITIAL SUM OF SQUARES =  0.2931D+12
MATRIX OF NORMAL EQUATIONS
  0.3129D+06  0.8909D+09  0.6021D+09  0.4124D+09 -0.3028D+09
  0.8909D+09  0.2555D+13  0.1715D+13  0.1204D+13 -0.8623D+12
  0.6021D+09  0.1715D+13  0.1179D+13  0.8007D+12 -0.5828D+12
  0.4124D+09  0.1204D+13  0.8007D+12  0.7212D+12 -0.3992D+12
 -0.3028D+09 -0.8623D+12 -0.5828D+12 -0.3992D+12  0.2931D+12
PARAMETER   1 LIMITS THE CORRECTIONS TO   0.2005E+00TIMES THE GAUSS-NEWTON VALUES.
SEARCH CONVERGED AFTER  6 CYCLES, WITH LAMBDA=  0.10015D+01 AND SSQ =  0.34003E+05

START ITERATION NO.  2 NO. OF FUNCTION CALLS  12
PAR(I) = 0.9093600E+00 0.4001111E+00 0.1499253E+01 0.4398559E+00
MATRIX OF NORMAL EQUATIONS
  0.3121D+06  0.8339D+06  0.5635D+06  0.3860D+06  0.4784D+05
  0.8339D+06  0.2245D+07  0.1506D+07  0.1058D+07  0.1257D+06
  0.5635D+06  0.1506D+07  0.1035D+07  0.7031D+06  0.7142D+05
  0.3860D+06  0.1058D+07  0.7031D+06  0.6334D+06  0.3391D+05
  0.4784D+05  0.1257D+06  0.7142D+05  0.3391D+05  0.3407D+05
PARAMETER   3 LIMITS THE CORRECTIONS TO   0.3783E+00TIMES THE GAUSS-NEWTON VALUES.
SEARCH CONVERGED AFTER  3 CYCLES, WITH LAMBDA=  0.25652D+00 AND SSQ =  0.28341E+05

START ITERATION NO.  3 NO. OF FUNCTION CALLS  21
PAR(I) = 0.1268617E+01 0.4361997E+00 0.1295828E+01 0.4002339E+00
MATRIX OF NORMAL EQUATIONS
  0.1573D+06  0.5860D+06  0.3896D+06  0.2613D+06  0.3846D+05
  0.5860D+06  0.2199D+07  0.1452D+07  0.9996D+06  0.1419D+06
  0.3896D+06  0.1452D+07  0.9833D+06  0.6530D+06  0.8460D+05
  0.2613D+06  0.9996D+06  0.6530D+06  0.5924D+06  0.4762D+05
  0.3846D+05  0.1419D+06  0.8460D+05  0.4762D+05  0.2834D+05
PARAMETER   3 LIMITS THE CORRECTIONS TO   0.5443E+00TIMES THE GAUSS-NEWTON VALUES.
SEARCH CONVERGED AFTER  3 CYCLES, WITH LAMBDA=  0.34695D+00 AND SSQ =  0.23108E+05

START ITERATION NO.  4 NO. OF FUNCTION CALLS  30
PAR(I) = 0.1767276E+01 0.4672974E+00 0.1104606E+01 0.3664550E+00
MATRIX OF NORMAL EQUATIONS
  0.8321D+05  0.4314D+06  0.2823D+06  0.1859D+06  0.2720D+05
  0.4314D+06  0.2254D+07  0.1464D+07  0.9912D+06  0.1404D+06
  0.2823D+06  0.1464D+07  0.9770D+06  0.6363D+06  0.8584D+05
  0.1859D+06  0.9912D+06  0.6363D+06  0.5819D+06  0.5257D+05
  0.2720D+05  0.1404D+06  0.8584D+05  0.5257D+05  0.2311D+05
PARAMETER   3 LIMITS THE CORRECTIONS TO   0.9432E+00TIMES THE GAUSS-NEWTON VALUES.
SEARCH CONVERGED AFTER  4 CYCLES, WITH LAMBDA=  0.63668D+00 AND SSQ =  0.17407E+05

START ITERATION NO.  5 NO. OF FUNCTION CALLS  40
PAR(I) = 0.2540170E+01 0.4957768E+00 0.9020909E+00 0.3371863E+00
MATRIX OF NORMAL EQUATIONS
  0.4259D+05  0.3222D+06  0.2071D+06  0.1345D+06  0.1515D+05
```

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  0.1222D+06  0.2454D+07  0.1567D+07  0.1047D+07  0.1147D+06
  0.2071D+06  0.1567D+07  0.1029D+07  0.6594D+06  0.7195D+05
  0.1345D+06  0.1047D+07  0.6596D+06  0.6094D+06  0.4744D+05
  0.1515D+05  0.1147D+06  0.7195D+05  0.4744D+05  0.1741D+05
SEARCH CONVERGED AFTER  5 CYCLES, WITH LAMBDA=  0.12706D+01 AND SSQ =  0.11869E+05

START ITERATION NO.  6 NO. OF FUNCTION CALLS  51
PAR(I) = 0.34796631E+01 0.50222982E+00 0.80064465E+00 0.33425352E+00
MATRIX OF NORMAL EQUATIONS
  0.3025D+05  0.3084D+06  0.1965D+06  0.1284D+06  0.1474D+03
  0.3084D+06  0.3174D+07  0.2006D+07  0.1349D+07  0.1594D+04
  0.1965D+06  0.2006D+07  0.1305D+07  0.8413D+06  0.1681D+04
  0.1284D+06  0.1349D+07  0.8413D+06  0.7841D+06  0.1673D+04
  0.1474D+03  0.1596D+04  0.1681D+04  0.1673D+04  0.1139D+05
SEARCH CONVERGED AFTER  1 CYCLES, WITH LAMBDA=  0.10000D+01 AND SSQ =  0.11868E+05

BEST PARAMETER VALUES AND 2-SIGMA CONFIDENCE LIMITS ESTIMATED
BY LINEARIZATION FOR THE INDIVIDUALPARAMETERS ARE AS FOLLOWS.

UPR(I) = 0.57844640D+01 0.71986110D+00 0.10055850D+01 0.41206940D+00
PAR(I) = 0.33404321E+01 0.49891131E+00 0.82526768E+00 0.34048554E+00
LWR(I) = 0.89564800D+00 0.27796510D+00 0.64494420D+00 0.26490140D+00

STANDARD ERROR OF WEIGHTED RESIDUALS =  0.15107E+02 ESTIMATED WITH   56
RESIDUALS AND   52DEGREES OF FREEDOM.

NORMALIZED CORRELATION MATRIX

     1.00000

     -.87303   1.00000

     -.49323   0.02640   1.00000

     0.37537   -.49832   -.04367   1.00000

FINAL FUNCTION VALUES

 0.44226E+02 0.39546E+02 0.31916E+02 0.23022E+02 0.56492E+02 0.72153E+02 0.75675E+02 0.59061E+02
 0.51323E+02 0.42248E+02 0.29942E+02 0.75246E+02 0.95716E+02 0.11959E+03 0.81204E+02 0.71460E+02
 0.59038E+02 0.42502E+02 0.10596E+03 0.13635E+03 0.17544E+03 0.58602E+02 0.50746E+02 0.41534E+02
 0.29621E+02 0.74664E+02 0.94942E+02 0.11922E+03 0.86427E+02 0.75364E+02 0.61741E+02 0.44493E+02
 0.10994E+03 0.14137E+03 0.18449E+03 0.55829E+02 0.48525E+02 0.39704E+02 0.28328E+02 0.70572E+02
 0.89804E+02 0.11554E+03 0.58969E+02 0.51164E+02 0.42378E+02 0.30171E+02 0.74084E+02 0.97329E+02
 0.12361E+03 0.57127E+02 0.49569E+02 0.41143E+02 0.29207E+02 0.72333E+02 0.93046E+02 0.11723E+03

RESIDUALS

 0.10586E+02 0.70761E+01 0.14564E+02 0.85122E+01 0.11512E+02-0.21871E+01-0.52795E+02 0.14041E+02
 0.76732E+01-0.11520E+01 0.30422E+01 0.84665E+01-0.73336E+01 0.41670E+00 0.11241E+01-0.44199E+01
 0.75676E+01-0.67777E+01-0.27299E+01-0.34717E+01 0.23292E+02 0.12222E+02 0.41762E+01-0.43462E+01
-0.58291E+01 0.27956E+01-0.13428E+02-0.93303E+01-0.35630E+01-0.12216E+02-0.16789E+02-0.71066E+01
-0.34961E+02-0.27841E+02 0.23104E+02 0.16489E+02 0.14835E+02 0.15524E+02 0.11520E+02 0.11992E+02
 0.30641E+01 0.18226E+02 0.20491E+01-0.33575E+00-0.66425E+01-0.16509E+02-0.24696E+02-0.21611E+02
-0.49418E+01 0.12137E+02 0.11049E+02 0.15753E+02 0.41064E+01 0.10563E+02 0.47557E+01 0.13337E+02

FINAL SUM OF SQUARES =  0.11870D+05

END OF PROBLEM NO.   1, NO. OF FUNCTION CALLS =  58
STOP        0
```

Appendix F

Computer Program Output for the Digested Sludge

```
START OF PROBLEM NO.    1 WITH   56 OBSERVATIONS AND    4 PARAMETERS

VERSION 4 OF LS,    AUGUST 1971
BNDUP(I)= 0.10000E+31 0.10000E+31 0.10000E+31 0.10000E+31
PAR(I)   = 0.96500E+03 0.40000E+00 0.15000E+01 0.44000E+00
BNDLW(I)=-0.10000E+31-0.10000E+31-0.10000E+31-0.10000E+31


DEL(I)   =-0.10000E-01-0.10000E-01-0.10000E-01-0.10000E-01
CHMAX(I)= 0.19300E+03 0.80000E-01 0.30000E+00 0.88000E-01

EPDA =  0.1000E-03 ASSTOL =  0.1000E-02 ITMAX =  10   LISTS =  3  IDIF =  1

SUBROUTINE DIF IS NOT USED

START ITERATION NO.  1 NO. OF FUNCTIONCALLS   0
PAR(I) = 0.9650000E+03 0.4000000E+00 0.1500000E+01 0.4400000E+00

INITIAL SUM OF SQUARES =  0.2342D+11
MATRIX OF NORMAL EQUATIONS
  0.2513D+05  0.7148D+08  0.2855D+08  0.3221D+08 -0.2426D+08
  0.7148D+08  0.2045D+12  0.8120D+11  0.9405D+11 -0.6900D+11
  0.2855D+08  0.8120D+11  0.3431D+11  0.3667D+11 -0.2756D+11
  0.3221D+08  0.9405D+11  0.3667D+11  0.5585D+11 -0.3109D+11
 -0.2426D+08 -0.6900D+11 -0.2756D+11 -0.3109D+11  0.2342D+11
PARAMETER   1 LIMITS THE CORRECTIONS TO   0.2007E+00TIMES THE GAUSS-NEWTON VALUES.
SEARCH CONVERGED AFTER  5 CYCLES, WITH LAMBDA=  0.99991D+00 AND SSQ =  0.25487E+05

START ITERATION NO.  2 NO. OF FUNCTIONCALLS  11
PAR(I) = 0.3686024E+01 0.4005666E+00 0.1497781E+01 0.4398674E+00
MATRIX OF NORMAL EQUATIONS
  0.2514D+05  0.2720D+06  0.1086D+06  0.1225D+06 -0.6316D+03
  0.2720D+06  0.2965D+07  0.1175D+07  0.1361D+07 -0.2350D+04
  0.1086D+06  0.1175D+07  0.4963D+06  0.5305D+06 -0.1852D+05
  0.1225D+06  0.1361D+07  0.5305D+06  0.6082D+06 -0.2036D+04
 -0.6316D+03 -0.2350D+04 -0.1852D+05 -0.2036D+04  0.2549D+05
PARAMETER   2 LIMITS THE CORRECTIONS TO   0.3139E+00TIMES THE GAUSS-NEWTON VALUES.
SEARCH CONVERGED AFTER  6 CYCLES, WITH LAMBDA=  0.88334D+00 AND SSQ =  0.15743E+05

START ITERATION NO.  3 NO. OF FUNCTIONCALLS  23
PAR(I) = 0.3582910E+01 0.6261120E+00 0.9868796E+00 0.4094415E+00
MATRIX OF NORMAL EQUATIONS
  0.2770D+05  0.2942D+06  0.1066D+06  0.1312D+06 -0.1259D+04
  0.2942D+06  0.3141D+07  0.1132D+07  0.1422D+07 -0.1239D+05
  0.1066D+06  0.1132D+07  0.4482D+06  0.5059D+06 -0.5029D+04
  0.1312D+06  0.1422D+07  0.5059D+06  0.8355D+06 -0.2984D+04
 -0.1259D+04 -0.1239D+05 -0.5029D+04 -0.2984D+04  0.1574D+05
SEARCH CONVERGED AFTER  2 CYCLES, WITH LAMBDA=  0.56470D+00 AND SSQ =  0.15630E+05

START ITERATION NO.  4 NO. OF FUNCTIONCALLS  31
PAR(I) = 0.3268908E+01 0.6523224E+00 0.9840094E+00 0.4138754E+00
MATRIX OF NORMAL EQUATIONS
  0.3250D+05  0.3154D+06  0.1140D+06  0.1414D+06 -0.8281D+02
  0.3154D+06  0.3075D+07  0.1106D+07  0.1399D+07 -0.3598D+03
  0.1140D+06  0.1106D+07  0.4372D+06  0.4972D+06 -0.3927D+03
  0.1414D+06  0.1399D+07  0.4972D+06  0.8223D+06  0.9601D+03
 -0.8281D+02 -0.3598D+03 -0.3927D+03  0.9601D+03  0.1563D+05
SEARCH CONVERGED AFTER  1 CYCLES, WITH LAMBDA=  0.10000D+01 AND SSQ =  0.15619E+05

BEST PARAMETER VALALUES AND 2-SIGMA CONFIDENCE LIMITS ESTIMATED
BY LINEARIZATION FOR THE INDIVIDUALPARAMETERS ARE AS FOLLOWS.

UPR(I) = 0.6059258D+01 0.9963474D+00 0.1160397D+01 0.5037277D+00
PAR(I) = 0.3033051E+01 0.6758665E+00 0.9811689E+00 0.4172816E+00
```

```
LWR(I) = 0.6844887D-02 0.3553849D+00 0.8019402D+00 0.3308356D+00

STANDARD ERROR OF WEIGHTED RESIDUALS =  0.17331E+02 ESTIMATED WITH   56
RESIDUALS AND   52DEGREES OF FREEDOM.

NORMALIZED CORRELATION MATRIX

     1.00000

     -.97025   1.00000

     -.21245   0.00632   1.00000

     0.36323   -.47160   -.01415   1.00000

FINAL FUNCTION VALUES

 0.29717E+02 0.24466E+02 0.18601E+02 0.11731E+02 0.39684E+02 0.52994E+02 0.70145E+02 0.81609E+02
 0.67370E+02 0.51221E+02 0.32493E+02 0.10898E+03 0.14671E+03 0.19274E+03 0.42900E+02 0.35503E+02
 0.26993E+02 0.16896E+02 0.57289E+02 0.76504E+02 0.10155E+03 0.34867E+02 0.28706E+02 0.21825E+02
 0.13662E+02 0.46562E+02 0.62179E+02 0.82210E+02 0.53566E+02 0.44284E+02 0.33669E+02 0.21162E+02
 0.71829E+02 0.96317E+02 0.12653E+03 0.59327E+02 0.48844E+02 0.37136E+02 0.23332E+02 0.79225E+02
 0.10580E+03 0.13951E+03 0.87324E+02 0.72075E+02 0.55074E+02 0.34646E+02 0.11661E+03 0.15690E+03
 0.20019E+03 0.47350E+02 0.38983E+02 0.29639E+02 0.18727E+02 0.63231E+02 0.84440E+02 0.11093E+03

RESIDUALS

 0.16747E+02 0.13456E+02 0.85013E+01 0.49315E+01 0.15454E+02 0.15904E+02 0.96455E+01-0.50412E+01
 0.10320E+02 0.18612E+01-0.11369E+01-0.42809E+02 0.10341E+02-0.15921E+02 0.68700E+01-0.43709E+00
-0.17471E+01-0.12336E+01-0.17171E+02-0.63156E+02 0.94395E+01 0.11447E+02 0.54660E+01 0.21452E+01
 0.14617E+01 0.31017E+01-0.18151E+02-0.47197E+01 0.69865E+01 0.48139E+01-0.47092E+00-0.34277E+01
 0.30759E+02 0.29147E+02 0.97045E+01-0.12633E+02-0.12926E+02-0.97415E+00-0.85879E+01 0.14895E+02
 0.75181E+01-0.39771E+02-0.41556E+00-0.12550E+01 0.63338E+01-0.21741E+01-0.24206E+02 0.63724E+01
 0.37806E+02 0.42099E+01-0.18168E+01-0.61102E+00-0.47731E+01 0.20001E+02 0.15799E+01-0.20199E+02

FINAL SUM OF SQUARES =  0.1562D+05

END OF PROBLEM NO.    1, NO. OF FUNCTION CALLS =   38
STOP         0
```

Appendix G

Preliminary Centrifugation Results for Primary and Digested Sludges

Table G.1

Centrifugation Results

Type of Sludge: P = primary
D = Digested

Polymer used: WT - 2640
Dose: 3 mls/40 ml sludge

Total mass of solids in the original sample:

P = 0.1896 g D = 0.0543 g

Sample	Speed RPM	Retention Time Min.	Percent Removal of Solids
P1	355	5	88.76
P2	355	5	88.92
P3	440	5	88.03
P4	440	5	89.08
P5	355	10	86.13
P6	355	10	85.86
P7	440	10	87.50
P8	440	10	86.39
D1	355	5	61.69
D2	355	5	63.17
D3	440	5	65.38
D4	440	5	62.80
D5	355	10	52.49
D6	355	10	53.41
D7	440	10	56.91
D8	440	10	51.20