A PLANNING MODEL FOR THE CONTROL AND TREATMENT OF STORMWATER RUNOFF THROUGH DETENTION STORAGE

by Rafael I. Segarra
Department of Civil Engineering
University of Puerto Rico

and

G.V. Loganathan
Department of Civil Engineering
Virginia Polytechnic Institute and State University

Project No. 03
Grant Agreement No. 14-08-0001-G-1041

FINAL TECHNICAL REPORT
TO
U.S. DEPARTMENT OF THE INTERIOR
WASHINGTON, D.C. 20240

"The work on which this report is based was supported in part by funds provided by the United States Department of the Interior under the Water Research and Development Act of (P.L. 98-242)".

January 1989

ACKNOWLEDGEMENTS

I wish to express my appreciation to the members of my graduate committee: Drs. G.V. Loganathan, C.W. Randall, T.A. Dillaha, C.Y. Kuo, and S. Ardekani, for their assistance and review of my dissertation.

I also wish to extend a special note of gratitude to Dr. G.V. Loganathan, my Chairman, for his patience, his guidance, and his friendship, which have made this a truly rewarding experience.

I am also indebted to Dr. O. Yucel for his assistance in the early stages of this work.

I would like to thank the Water Resources Research Institute of the University of Puerto Rico for their support of this research. The support of Dr. Munoz Candelario was of great help in getting the project approved. The patience, skill, and cooperativeness of Miss Aida Feliciano, who typed the dissertation and project report, is gratefully acknowledged.

I would like to dedicate this work to my parents and my wife, to whom I will forever be indebted for all they have given me.

The early stage of this project was conducted at the Virginia Polytechnic Institute and State University. The latter stage was conducted, and supported by, the Water Resources Research Institute of the University of Puerto

Rico, under a grant from the Department of the Interior, project no. G-1041-03.

TABLE OF CONTENTS

Table of Contents (continued)

Tables of Contents (continued)

Table of contents (continued)

Page

LIST OF TABLES

List of Tables (continued)

LIST OF FIGURES

List of Figures (continued)

List of Figures (continued)

List of Figures (continued)

Page

LIST OF SYMBOLS

a = rate of water withdrawal from the storage unit (inches/hours).

a_2 = time factor in runoff duration equation.

b = storage device design capacity; the maximum volume available for treatment (inches).

c = runoff trapping capacity of the inlet system (inches/hour).

d = decrease in interevent time due to rainfall duration transformation (hours).

f = the fraction of the runoff that arrives at the storage unit, and is captured by the device.

f_t = the fraction of the total runoff volume that is captured by the storage unit.

g = $\frac{\alpha c + \beta}{a - c}$; a coefficient (inches^{-1}).

H = $\frac{\beta\gamma}{(\alpha a + \beta)(\gamma - ag)} \exp(\beta x_c)$; a coefficient (hours^{-1}).

H^* = $\frac{\beta\gamma}{(\alpha a + \beta)(\alpha a + \gamma)} \exp(-\alpha k_2)$; a coefficient (hours^{-1}).

i = uniform runoff intensity arriving at the detention unit (inches/hour).

i_{av} = the ratio of the mean runoff volume and the mean runoff duration (inches/hour).

i_m = uniform total runoff intensity (inches/hour).

i_o = a specified value of the total runoff intensity (inches/hour).

i_r = rate at which the detention unit fills up during an event (inches/hour).

$I(a)$ = conditional expectation of the total runoff intensity (inches/hour).

K = first order pollutant washoff decay rate (inches^{-1}).

k_1 = $(a-c)x_c$; a parameter (inches).

k_2 = ax_c; a parameter (inches).

L = runoff event pollutant load captured by the detention unit (pounds).

L_t = total pollutant load obtained from an event (pounds).

$L_{t,x}$ = total pollutant load in terms of total runoff volume (pounds).

$L_{t,z}$ = total pollutant load in terms of runoff volume arriving at the storage unit (pounds).

m_I = mean value of the runoff volume exceeding the inlet capacity (inches).

m_Z = mean value of the runoff volume available to the detention unit (inches).

M_x = $\rho_{\ell x} E[L_{t,x}]$; product of the trap efficiency and the mean value of the total load in terms of the total event runoff volume, giving the pollutant load trapped by the storage unit (pounds).

M_z = $\rho_{\ell z} E[L_{t,z}]$; product of the trap efficiency and the mean value of the total load in terms of the runoff volume arriving at the storage unit, giving the pollutant load trapped by the unit (pounds).

n = the n^{th} runoff event arriving at the detention unit.

p = $\dfrac{i_r}{i_r - a}$; a term.

P_o = available pollutant mass on the ground surface at the start of the runoff event, (pounds).

q_1 = $\dfrac{r_1(\alpha + K)}{\gamma/ap + \alpha + K}$; a coefficient.

q_2 = $\dfrac{r_1 \gamma}{\gamma + (\alpha + K)ap}$; a coefficient (pounds).

q_3 = $\dfrac{r_3 \gamma}{\gamma + (\beta/c + \alpha + K)ap}$; a coefficient (pounds).

$q_4 \quad = \dfrac{r_3(\beta/c + \alpha + K)}{\beta/c + \alpha + K + \gamma/ap}$; a coefficient (pounds).

$Q \quad = \dfrac{\alpha a \gamma}{(\alpha a + \beta)(\gamma - \beta)} \exp(\beta x_c)$; a coefficient (hours^{-1}).

$r_1 \quad = \dfrac{KP_o}{\alpha + K}$; a coefficient (pounds).

$r_2 \quad = \dfrac{KP_o\beta}{(\alpha + K)[\beta + (\alpha + K)c]} \exp[-(\alpha + K)cx_c]$; a coefficient (pounds).

$r_3 \quad = \dfrac{KP_o}{\beta/c + \alpha + K}$; a coefficient (pounds).

s = available storage variable (inches).

s_o = the value of the available storage capacity at the end of the previous runoff event (inches).

S_b = a storage index that classifies the storage distributions (inches).

$S(n-1)$ = available storage space at the end of the $(n-1)^{th}$ event (inches).

$S(n)$ = available storage (empty space) at the end of the n^{th} runoff event (inches).

T = $\mathrm{Min}[S(n-1) + aX_3, b]$, the storage space available at the start of the n^{th} event (inches).

t_o = the time required to fill the detention unit during an event (hours).

U = pT, a variable in the pollutant load formulation (inches).

W = $aX_2 - Z$, a variable representing the difference between the runoff volume treated during an event at rate a and the incoming runoff volume (inches).

$X_1^{(n)}$ = event runoff volume (inches).

$X_2^{(n)}$ = runoff duration of the n^{th} event (hours).

$X_2^{\prime(n)}$ = effective rainfall duration of the n^{th} event (hours).

$X_3^{(n)}$ = time between the end of the $(n-1)^{th}$ event and the start of the n^{th} event (hours).

x_c = a time base parameter used to obtain the duration of the runoff event, estimated by the time of concentration (hours).

y = overflow volume variable (inches).

$Y(n)$ = the volume of storge overflow of the n^{th} event (inches).

$Y_I(n)$ = runoff volume not captured by inlets (inches).

Z = Min (X_1, cX_2), amount of runoff that is available to the detention unit (inches).

α = the inverse of the mean runoff volume (inches^{-1}).

β = the inverse of the mean runoff event duration (hours^{-1}).

β_2 = the inverse of the mean effective rainfall duration (hours^{-1}).

γ = the inverse of the mean interevent time (hours^{-1}).

δ = the fraction of the storage capacity available at the end of the previous event.

ε = exceedance probability, or the probability of having an overflow.

ε_I = exceedance probability of the runoff trapping system.

ρ = runoff capture efficiency of the detention unit in terms of the runoff volume arriving at the unit.

ρ_ℓ = pollutant load trap efficiency.

ρ_t = runoff capture efficiency of the detention unit in terms of the total runoff volume.

$\rho_{\ell,x}$ = polluant load trap efficiency in terms of the total runoff volume.

$\rho_{\ell,z}$ = pollutant load trap efficiency in terms of the runoff volume arriving at the detention unit.

A STATISTICAL APPROACH TO URBAN STORMWATER DETENTION

by

Rafael I. Segarra

(ABSTRACT)

A statistical model has been developed to study the long-term behaviour of a stormwater detention unit. This unit stores a portion of the incoming runoff, corresponding to the empty space available in the unit, from which runoff is pumped to a treatment plant. The objective is to avoid, as much as possible, the discharge of untreated runoff to receiving bodies of water.

The model was developed by considering the arrival of independent runoff events at the urban catchment. The process variables of event depth, duration, and interevent time were treated as independent, identically distributed random variables. A storage equation was formulated from which the probability of detention unit overflow was obtained. With this distribution it was possible to define the trap efficiency of the unit in terms of the long-term fraction of the runoff volume trapped by the storage unit.

The trap efficiency expressions define storage/treatment isoquants, which represent the combinations of storage capacity, treatment rate, and the sewer system runoff trapping capacity, which provide a fixed level of runoff control.

A pollutant load model was also formulated, based on a first-order washoff model. This model was used to define pollutant control isoquants.

Optimal values of the required storage capacity and treatment rate were obtained by treating the isoquants as production functions. Applying the results of production function theory, a cost minimization problem was solved for the value of the storage capacity and treatment rate, for prescribed runoff and pollutant trap efficiency levels.

The results obtained with the statistical model compared well with results obtained from major simulation models.

The statistical approach offers an advantage in that no simulation is required to obtain the isoquants, as the expressions are analytical, thus greatly simplifying the optimization process. Also, the evaluation of the storage unit pollutant trap efficiency can be easily evaluated for any type of pollutant whose washoff rate is known.

CHAPTER I.

1.1 Introduction

Urban Stormwater Management has, in recent years, been recognized as an independent field within the water resources environment. This has been brought about by the realization of the relative uniqueness of the urban runoff process, both in terms of quantity and quality. The major feature of the urban catchment condition is its high degree of imperviousness. A large percentage of the rainfall volume becomes available as runoff, as opposed to natural watersheds. The runoff is routed through man-made channels to a point of discharge. In its flow over the urban catchment the runoff becomes the transport mode of a wide variety of substances made available on the surface through human activity. These substances, through physical/chemical transport processes, are borne in the runoff and discharged at the stormwater outlets. A schematic representation of the process is shown in Figure 1.1.

In the past Urban Stormwater Management received little attention beyond that associated with stormsewer design. With the increase in size of the urban locations and the advancement of industry and technology the environmental problems associated with heavily developed and industrialized catchments have become more numerous and of a greatly increased complexity. The major impacts associated with urban development are increased runoff volumes, increased

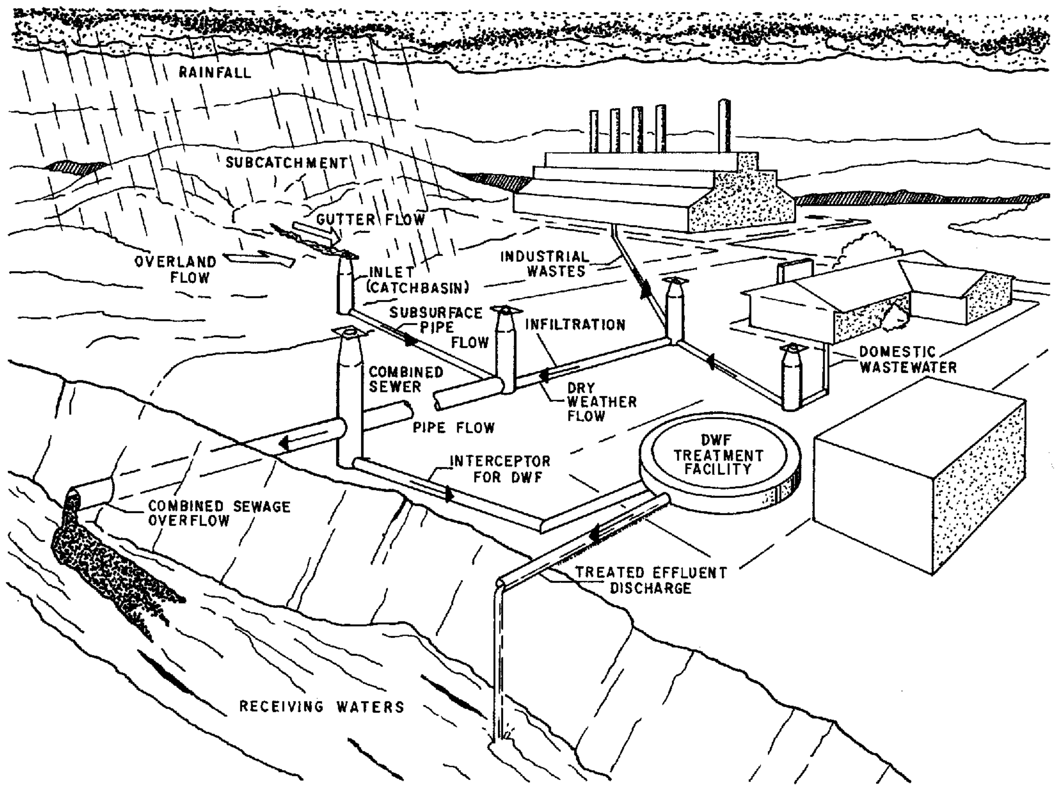


Figure 1.1 Typical urban drainage system [Roesner,1982a].

runoff rates, and deterioration of the water quality.

The water-borne substances that are transported through the gutter/stormsewer system are eventually discharged to a receiving body of water, be it a lake, river, or ocean. These discharges inflict what is called a "receiving water impact". The receiving water impact may be negligible if the assimilation capacity of the body of water is large with respect to the nature of the pollutant load, or it may be disastrous if the stream cannot properly assimilate the pollutant load it receives. Assimilation in this context refers to the capacity of the receiving water to reduce, through natural processes, the concentration of a given substance to a certain level. This level is referred to as a water quality standard. Public law requires standards of acceptable water quality for natural bodies of water. In the past these standards were concerned mainly with controlling point discharges. Point discharges are those occurring from specific source locations such as treatment plants. With the development of environmental legislation, such as PL 92-500, attention has been also focused on nonpoint pollution. Nonpoint, or Areawide, pollution emanates from source areas and is borne by wind and water. The urban water quality problem falls in the nonpoint pollution category.

With the development of urban nonpoint pollution assessment projects, most notably the so called 208 Assessment Projects, and the execution of water quality surveys, awareness ensued of the enormity of the water quality problem associated with urban centers throughout the nation. It has been found that in some locations the impact from nonpoint urban pollution exceeds that of municipal sewage. As an illustration Tables 1.1 and 1.2 compare the water quality from a combined sewer and stormwater for a given urban location. It can be seen that the potential for a greater impact, especially for solids, is there.

Accompanying the realization of the existence of the problem came the need to adequately address it. The practice of using combined sewer systems was the simplest approach. The approach is found to be lacking when considered for modern urban development. Larger cities provide larger runoff volumes to handle. These, when routed through combined sewers to a treatment plant will produce larger overflow volumes with a large pollutant load into the receiving body of water. The use of large treatment plants is also a possibility. However, these are very costly and it is not economical to design them for zero overflow. Another factor working against the use of simple combined systems is the necessity of avoiding untreated municipal discharges to receiving waters, even in diluted form. One of the best alternatives developed thus far is the use of a separate stormsewer system combined with a storage/release system.

In separate stormsewer systems the conduits will only convey stormflow. The stormflow is conveyed to a storage

TABLE 1.1 Characteristics of Combined Sewer Overflows

Characteristic	Range of Values
BOD_5 (mg/l)	30-600
TSS (mg/l)	20-1,700
TS (mg/)	150-2,300
Volatile TS (mg/l)	15-820
pH	4.9-8.7
Settleable solids (ml/l)	2-1,550
Organic N (mg/l)	1.5-33.1
NH_3N (mg/l)	01.12.5
Soluble PO_4 (mg/l)	0.1-6.2
Total Coliforms (number/100 ml)	10,000-90 x 10^6
Fecal coliforms (number/100 ml)	20,000-17 x 10^6
Fecal streptococci (number/100 ml)	20,000-2 x 10^6

From Roesner (1982)

TABLE 1.2 Characteristics of Urban Stormwater

Characteristic	Range of Values
BOD_5 (mg/l)	1-700
COD (mg/l)	5-3,100
TSS (mg/l)	2-11,300
TS (mg/l)	450-14,600
Volatile TS (mg/l)	12-1,600
Setteable solids (ml/l)	0.5-5,400
Organic N (mg/l)	0.1-16
NH_3N (mg/l)	0.1-10
Soluble PO_4 (mg/l)	0.1-125
Chlorides (mg/l)	2-25,000[a]
Oils (mg/l)	0-110
Phenols (mg/l)	0-0.2
Lead (mg/l)	0-1.9
Total coliforms (number/100 ml)	200-146 x 10^6
Fecal coliforms (number/100 ml)	55-112 x 10^6
Fecal streptococci (number/100 ml)	200-1.2 x 10^6

[a]With highway deicing.

From Roesner (1982)

device where it fills available storage capacity. From the device water is withdrawn to a treatment plant for treatment along with the municipal discharge. If the storage capacity is exceeded by the incoming volume then a storage overflow will occur, the overflow going to a receiving body of water. In areas with already existing combined sewers it is usually not economical to construct a new separate system. In these cases storage capacity is made available "in line", or within the sewer itself.

Different combinations of storage/treatment devices can be constructed. In some situations the storage device will also function as a treatment device, either being the main source of runoff treatment, or providing preliminary treatment before further processing at the main treatment facility. The particular setup at a given site will be a function of engineering and economic considerations.

The major costs associated with the implementation of a stormwater management program are those related to the construction of facilities such as the sewer system, the storage units, and the treatment plant. Additional related costs involve the cost of treatment and operation and maintenance costs. These costs are very high. Cost estimates for upgrading the water quality using best available technology run into the billions (Labadie, 1976). It is therefore imperative that the design of water management systems be undertaken in an efficient manner. While sounding like an understatement this observation has great implications for the planning process. It compels the planning process to consider techniques which yield "optimal" solutions to management problems. Optimality is a condition in which the system is designed to fulfill its required function at the least possible cost, or the greatest possible benefit.

There are two major general outlooks on the stormwater management problem. In one the system is viewed from the point of view of a deterministic formulation. In the other the system is viewed in terms of statistical/stochastic processes. The former approach assumes perfect information. The random behaviour of involved variables is not considered. The latter considers all possible realizations of events, each with a given probability of occurrence.

This work will concentrate on the development of a statistical model for urban stormwater management. The main objectives of the work are the following:

a) to develop a statistical model in order to assess long-term bahaviour of an urban storage/release system, accounting for the randomness associated with the occurrences, durations, and volumes of rainfall/runoff events;

b) to obtain a probability distribution of the overflows obtained from stormwater storage devices;

c) to obtain, in an optimal fashion, the storage capacity and the treatment/release rate necessary to control pollution from stormwater;

d) and to obtain within this formulation, a closed-form solution in order to gain flexibility for the planning process, and to facilitate the incorporation of these techniques into larger and more comprehensive land use planning frameworks.

The remainder of this chapter will present a review of related techniques that address the urban stormwater management problem within the categorizations of this section.

1.2 Deterministic Models for Storage/Release Systems

A relatively large number of models and techniques exist for designing stormsewer systems. These are to be differentiated from those models specifically designed for the planning of storage/release systems. Representative of modern design models are those developed at the University of Illinois [see Terstriep and Stall (1974), Yen et al. (1976), and Yen (1986], which involve physically-based formulations and include optimization as a design tool. A class of detention storage design models are those that conceptualize the system in a relatively simple fashion while still maintaining various essential features of the process. These are not generally concerned with an accurate hydraulic description of catchment flow but are preoccupied with hydrologic aspects, such as the rainfall-runoff transformation, abstractions, and travel times. Representative of the latter approach is the work of Ordon (1974), and Smith and Bedient (1980).

the planning process. It compels the planning process to consider techniques which yield "optimal" solutions to management problems. Optimality is a condition in which the system is designed to fulfill its required function at the least possible cost, or the greatest possible benefit.

There are two major general outlooks on the stormwater management problem. In one the system is viewed from the point of view of a deterministic formulation. In the other the system is viewed in terms of statistical/stochastic processes. The former approach assumes perfect information. The random behaviour of involved variables is not considered. The latter considers all possible realizations of events, each with a given probability of occurrence.

This work will concentrate on the development of a statistical model for urban stormwater management. The main objectives of the work are the following:

a) to develop a statistical model in order to assess long-term bahaviour of an urban storage/release system, accounting for the randomness associated with the occurrences, durations, and volumes of rainfall/runoff events;

b) to obtain a probability distribution of the overflows obtained from stormwater storage devices;

c) to obtain, in an optimal fashion, the storage capacity and the treatment/release rate necessary to control pollution from stormwater;

An extensive analytical study of urban stormwater storage/release systems has been conducted by Medina et al. (1981). Medina's model describes the transient response of storage/treatment systems to variable forcing functions of flow and concentration, for completely mixed systems of constant and variable volumes, and for one-dimensional advective-dispersive systems. The model is based on a solution to the conservation of mass equation, accounting for the movement, decay, storage, and treatment of stormwater pollutants and dry weather flows through natural and engineering transport systems. A limitation of the model is that it was developed using a linear relationship between outflow and basin volume. Also, the pollutant and runoff input functions are assumed to be completely defined by the user.

Next, the two most popular, and widely used planning models-the Storage, Treatment, Overflow, Runoff Model (STORM), and the Stormwater Management Model (SWWM) will be discussed.

These models have explicit storage/release formulations. They have the capability of working with continuous rainfall records and thus can simulate a large number of events. This is a very useful capability because it allows for the statistical analysis of the simulation results, which in turn can be compared with the results obtained from statistical models.

The present study will compare the results obtained to those obtained from the statistical analysis of simulation results.

1.2.1 The U.S. Army Corps STORM Model

The STORM model (U.S. Army Corps of Engineers, 1977) was designed to generate long-term runoff and pollutant load records with a continuous long-term hourly rainfall record. The hourly runoff is, in turn, routed to a storage/treatment system. The interaction between storage, treatment, and release used in STORM is depicted in Figure 1.2.

To generate runoff the STORM model can use the Coefficient method or the Soil Conservation Service Curve Number technique. The most popular use of the model is with the Coefficient method. With the Coefficient method the hourly rate of runoff is computed by multiplying the difference between the actual precipitation and depression storage by a composite runoff coefficient.

The generated hourly runoff sequence is routed to the appropriate device. Runoff exceeding the maximum treatment rate is stored for release at a later time. If the storage capacity is exceeded, the excess overflows directly into the receiving water. When runoff abates to the point where the treatment rate is no longer exceeded, the storage unit is drained at a rate equivalent to the difference between the maximum treatment rate and the runoff rate.

In STORM an overflow event is defined as that starting

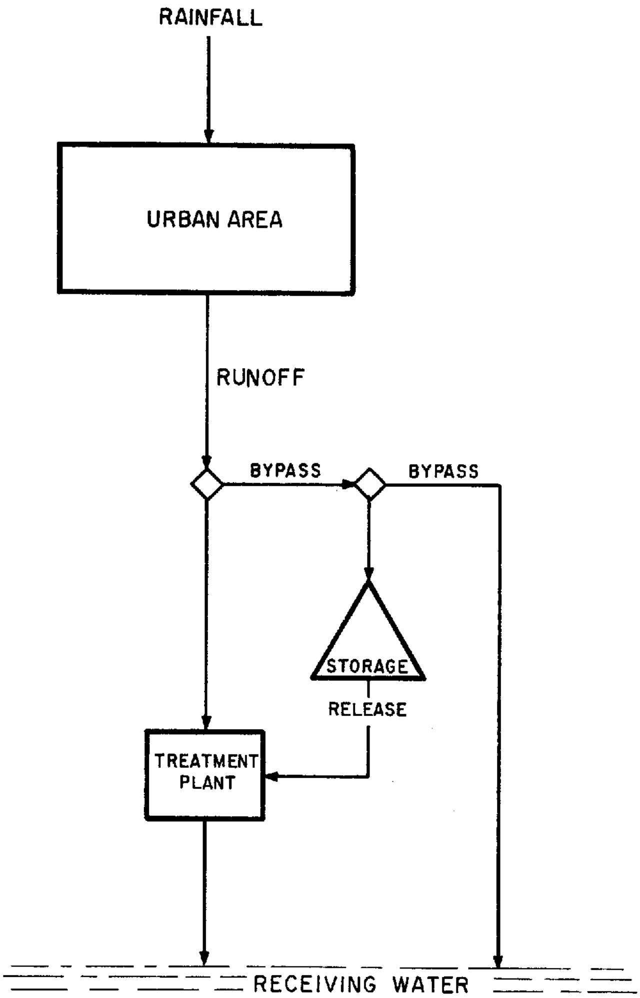


Figure 1.2 Storage/Release System, STORM (U.S. Army Corps of Engineers, 1977).

when overflow occurs and ending when the system can again handle the runoff. It is implicitly assumed that the storage capacity can be filled in one hour, and that the drainage system has the capacity to convey even the largest runoff produced.

The quality aspects are handled by assuming that the pollutant washoff is proportional to the remaining quantity. This corresponds to a linear first-order model. The manner in which STORM handles the runoff quality considerations is in general use. It is also utilized by the SWWM model, to be described next. STORM has no provision for simulating pollutant removal mechanisms, as does SWWM. It does keep yearly statistics of the qualities and quantities treated, stored and/or overflowed.

Modifications to STORM have appeared over the years. A notable modification is the one introduced by Dendrou et al. (1978). It was motivated by the fact that most simulation models do not account for street flooding. Since some of the flood volume may be lost to the sewer system as a result of flooding (via infiltration or diversion), this volume cannot be assumed to be available to the system. In the words of Dendrou et al.: "...the frequency of occurrence and the volume of street flooding should be determined by considerations such as the acceptable level of reliability of performance of the system (return period of the local flood) and the economics of the corresponding street flooding damage". To define a local flooding event Dendrou et al. assumed that such an event would occur only after the treatment facility had been operating to capacity, the storage facility filled to capacity, and the quantity to be overflowed constrained by a maximum allowable overflow rate. In this case, then, local flooding events corresponds to ficticious STORM overflowing events. A ficticious computational treatment rate is defined by the actual treatment rate augmented by the maximum allowable overflow rate.

The difference between the largest runoff rate and the ficticious treatment rate provides an estimate of the level of local street flooding.

The use of a ficticious treatment rate assumes that any backup of runoff that results in street flooding results from the exceedance of capacity of the treatment and storage units. However, pipe capacity may be exceeded, resulting in surcharge, before the treatment/storage capacity is exceeded. Additionally, inlets to the sewer system, because they exercise runoff control through their limited capacity, can cause local flooding of a nature that perhaps cannot be assessed properly by the use of ficticious treatment rates. The statistical model developed through this work will propose a physically-based formulation to account for the loss of runoff volume to the storage/treatment system.

The model STORM is found to be useful for the development of storage/treatment isoquants. Storage-treatment isoquants are obtained from a frequency analysis of simulation data. A point on a storage-treatment isoquant is obtained by determining the relative number of times that the storage device overflows for a given storage capacity and treatment rate. Needless to say this requires a large number of simulation runs. In practice the isoquants are defined for the fraction of the runoff that is captured and for the fraction of the pollutant load that is trapped by the storage unit. The locus of storage/treatment rate values for a given fraction defines the isoquant. This capability of the model STORM was employed in a national assessment of combined sewer overflows (Heany et al., 1977). Additional illustrations can be found in Melville and Bell, 1979, and Padmanabhan and Delleur, 1978. The proposed statistical method will estimate the shape of these isoquants without the need for simulation.

1.2.2 The Stormwater Management Model, Version III (SWWM III)

The SWWM III model (Huber et al., 1984) is a comprehensive, and widely used, urban stormwater management model. It has undergone a number of revisions since its formulation, most notably the development of a storage/treatment block and a hydraulic routing block named EXTRAN. These developments place SWWM a big leap ahead of STORM, at a corresponding increase in complexity. The program is divided into four major blocks, each corresponding to a different process. These are given as follows (Huber et al., 1984):

1) The input sources:

   The Runoff Block generates surface runoff based on arbitrary rainfall hyetograhs, antecedent conditions, land use, and topography. Dry weather flow and infiltration into the sewer may be optionally generated using the Transport Block.

2) The central core:

   The Transport and Extended Transport Blocks carry and combine the inputs through the sewer system.

3) The correctional devices:

   The Storage/Treatment Block characterizes the effects of control devices upon flow and quality. Elementary cost computations are also made.

4) The effect (receiving waters):

   The Receiving Block routes hydrographs and pollutographs through the receiving waters, which may consist of a stream, river, lake, estuary, or bay.

For runoff generation the catchment is divided into sub-basins, with hydrographs generated on each of these and then routed through the sewer system. Each subcatchment is treated as a lumped non-linear reservoir. The outflow is computed with Manning's equation.

Figure 1.3 illustrates the conceptualization of the

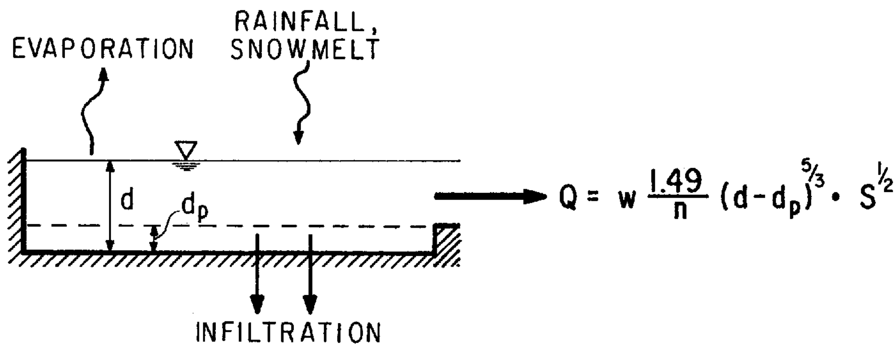


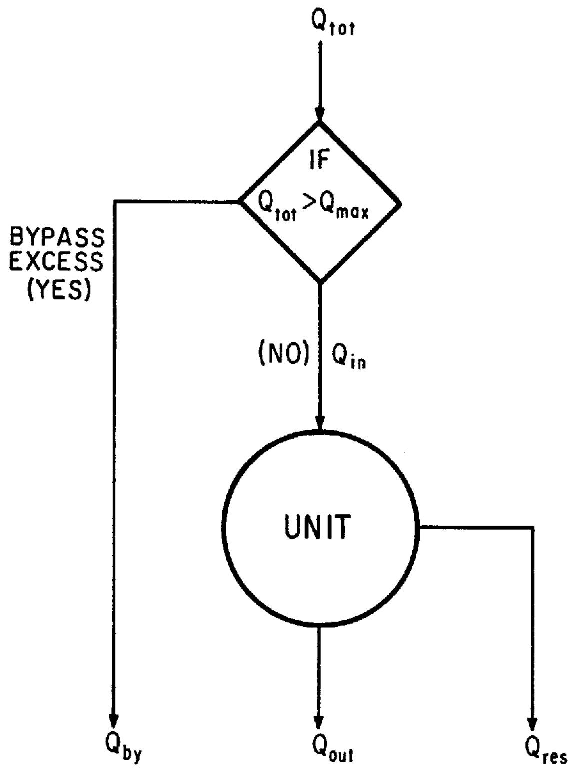
Figure 1.3 Non-linear Reservoir Model of Subcatchment, SWWM III.
(Huber et al., 1984)

subcatchments in SWWM.

The runoff is the transport mode by which pollutants are transported to the storage or treatment device. The generation of pollutants on the surface and their subsequent washoff are handled in a manner similar to that of STORM, with some additional features, such as availability factors to account for the fact that not all of the pollutant will be available for washoff during any event. Another feature included is the consideration of catchbasins and their effect on the pollutant load.

The pollutant transported through the sewer system is conveyed to the storage/treatment device. The Storage/Treatment Block accepts a flowrate and pollutant concentrations to be routed through a network of as many as five storage and/or treatment units. The flow interactions in the storage/treatment units is shown in Figure 1.4. Units may or may not have detention capability. A unit may remove pollutants through a generalized removal equation or through particle settling and/or obstruction. Pollutants are characterized by their concentration and, if desired, by particle size and specific gravity distributions or terminal settling velocity distributions.

SWWM is found to be a simulator capable of modeling a wide range of storage/treatment systems. It does require a large body of input data which must be painstakingly gathered for the model to produce meaningful results. While some of the model components have come under criticism, especial-



<u>LEGEND</u>

Q_{tot} = total inflow, ft^3/sec
Q_{max} = maximum allowable inflow, ft^3/sec
Q_{by} = bypassed flow, ft^3/sec
Q_{in} = direct inflow to unit, ft^3/sec
Q_{out} = treated outflow, ft^3/sec
Q_{res} = residual flow, ft^3/sec

Figure 1.4 Storage/Treatment Unit, SWWM III. (Huber et al., 1984) ly the pollutant generation mechanism (see Whipple and Hunter, 1977), the use of SWWM is fairly well established in the Stormwater Management field.

Its shortcomings are those typical of large simulation models. A certain degree of difficulty exists, because of its scope, in obtaining statistical information from the model. This is due to the high cost associated with running the model for the sufficient number of times needed to adequately define statistical quantities. This to an extent has been solved by running the program for a period of time, say a year, with a rainfall record that is representative of the historical record. Also, most of the modeled processes have, in some way or another, been simplified for use in the model, so, the user must be aware of these in order to adequately interpret the simulation results.

SWWM can also be used to generate storage/treatment rate isoquants in a manner analogous to STORM - through succesive runs of the model for given storage capacities and treatment rates. Studies conducted by Nix (1982), and Goforth et al. (1983), using SWWM as a management tool, resulted in the development of storage/treatment rate isoquants for two locations.

From the point of view of statistical formulations simulation models are a complementary tool. Statistical methods require good estimates of the distribution moments. It is the general condition that a good data base for urban stormwater management studies is usually lacking, in the form and to the extent required by statistical methods. As an illustration consider the distribution of runoff for a certain urban area. To obtain the mean and variance of the probability distribution of runoff, good gaging records should be available. These are usually lacking, and those that are available may be seasonal or of relatively short duration. Furthermore, these records may originate from locations different from the ones of interest.

Comprehensive monitoring of storage/release systems is a vast and costly undertaking. Fortunately, simulation models provide a tool with which to assess system behaviour under varying conditions and for extended lengths of time. Statistical results can therefore be compared and verified against simulation results. This procedure has already been employed by Loganathan and Delleur (1982). Such a strategy will be used here to compare the statistically derived storage/treatment isoquants with isoquants obtained from the application of SWWM and STORM.

The following section will review the currently available statistical models for urban stormwater management.

1.3 Statistical Models in Urban Stormwater Management

The statistical approach to urban stormwater management views the rainfall/runoff process as a random process, utilizing probability distribution functions to describe the process of interest. The fundamental motivation for the approach is the underlying randomness characterizing the rainfall/runoff process in general. A major advantage of the statistical techniques is that they allow variables of interest to take any value within their population range, assigning a measure of probability to every realization of the process, and thus providing an estimate of the likelihood of a particular occurrence. Additionally, statistical techniques allow for the assessment of long-term system behaviour, without resorting to costly simulations, providing an estimate of system reliability. Their shortcomings are related to the need to conceptualize the catchment in a relatively simple fashion, and the requirement of adequate runoff data on which to estimate distribution parameters.

The statistical approach has generated new insights into urban stormwater management problems, and provides a powerful tool with which to assess system reliability. A description of the available statistical techniques which are related to the formulation proposed in this work will occupy the remainder of this chapter.

1.3.1 The Hydroscience Statistical Method

The Hydroscience Statistical Method (Hydroscience, 1979; Di Toro and Small, 1979) represents a major achievement in the application of statistical techniques to urban stormwater management problems. It analyzes, statistically, an urban storage/treatment/interception system. The methodology analyzes long-term hourly rainfall records and develops a set of statistics describing the resulting stormwater runoff characteristics of urban areas. The stochastic rainfall or runoff process is segregated into a series of independent, randomly occurring events, as shown in Figure 1.5(a). From the hourly rainfall data available a runoff record is generated through the use of a transformation - a rainfall/runoff coefficient. A statistical analysis, to be discussed in the following chapter, segregates the record into independent runoff events by defining a minimum interevent time for independence. Separated into independent events, contiguous runoff increments are combined into a single event of uniform intensity q, of duration d_γ, runoff volume v_γ, and interevent time as shown in Figure 1.5(b).

Interception of runoff is accounted for as a constant rate Q_I, defined as the available treatment plant capacity. This is shown in Figure 1.5(c). The storage unit is defined in terms of a storage capacity, v_B. As shown in Figure 1.5(d), a portion of the incoming runoff volume will go to fill the storage capacity of the reservoir. The combined effect of interception and storage is shown in Figure 1.5(e). The volume not intercepted or stored will become storage overflow.

The runoff variables - event duration, interevent time, and event flows - are all assigned probability distribution functions. Storm flows are assumed to be gamma distributed.

The event duration is also assumed to be gamma distributed. The interevent time is assumed to be exponentially distributed, to be in accordance with the prescribed Poisson

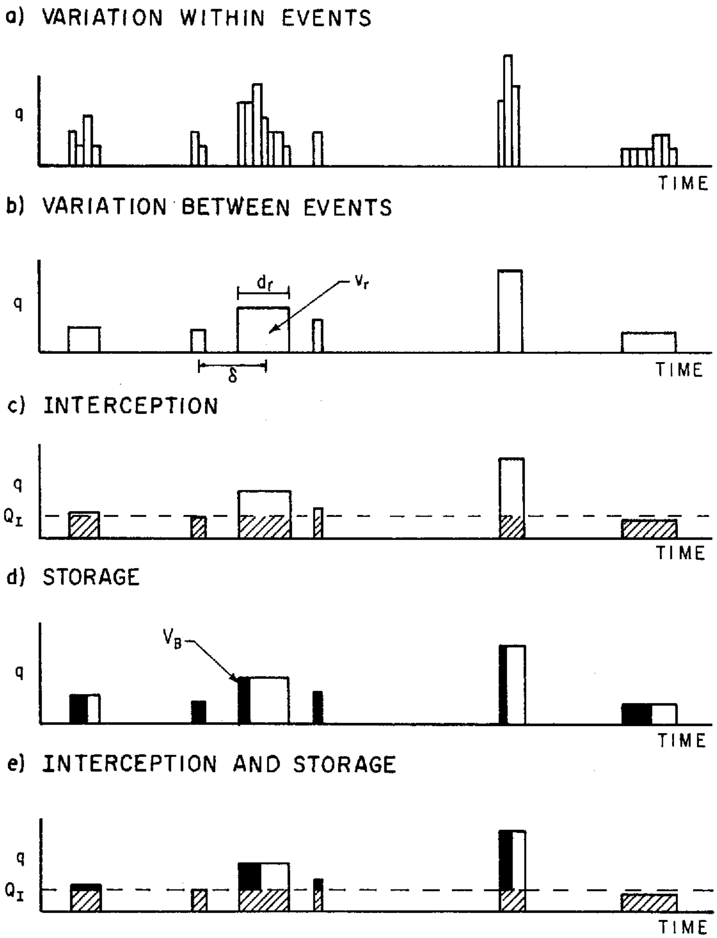
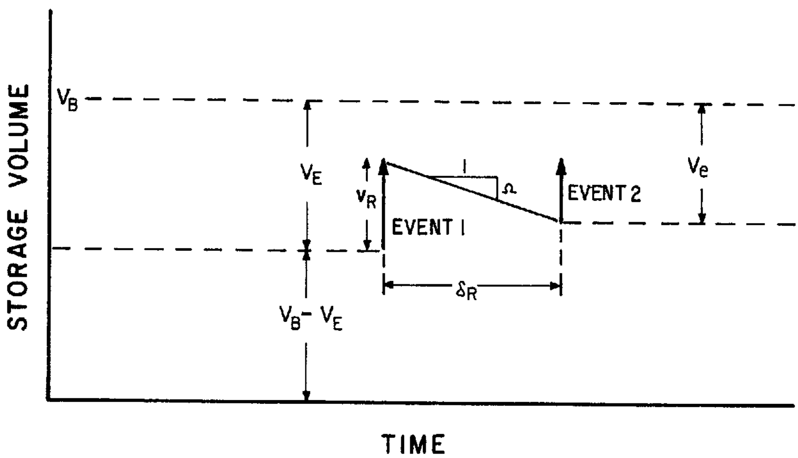


Figure 1.5 Representation of Storm Runoff Process, Interception and Storage in the Hydroscience Method (DiToro and Small, 1979; Hydroscience, Inc., 1979).

arrival of independent random events. The use of these distributions is justified from hydrologic studies (see, for example, Chow, 1976).

The storage basin operates in terms of the available or effective capacity, V_E. The operation of the device is as shown in Figure 1.6. The event with volume v_R arrives at the unit, which has a fixed available empty space, defined by v_E. The runoff reduces the space available by v_R. During the interevent time the space available is increased at rate Ω, which is the treatment rate. The arrival of the next event will reduce the storage available to V_e. The model will always assume that at the end of the previous event the storage available will always be given by V_E. V_E is a mean value.

The effective storage capacity is, essentially, a stochastic process with a memory. At least, the effective storage at the end of an event is dependent on the storage at the end of the previous event, in itself dependent on the storage at the end of the next to the previous event. In stochastic processes theory such processes are called Markov processes of the first order. The use of a fixed, mean available previous storage capacity is unfortunate because this parameter is a very important random component. However, to treat it as random would make the formulation unwieldy, if not impossible to solve. The statistical model developed in this study will overcome this shortcoming by allowing the system to be at any previous storage level,



LEGEND

V_B = maximum storage capacity, L^3
V_E = mean effective storage capacity, L^3
v_R = event 1 runoff volume, L^3
Ω = release rate, $L^3\ T^{-1}$
V_e = effective storage capacity, L^3
δ_R = time between runoff event midpoints, T

Figure 1.6 Determination of Effective Storage Capacity, V_e (DiToro and Small, 1979; Hydroscience, Inc., 1979).

from empty to full, thus allowing consideration of the full range of possibilities.

Using the probability distributions and the value of V_E, the long term fraction of the runoff volume bypassing the storage is calculated as the mean runoff event volume bypassing the storage unit, V', divided by the mean runoff volume, V.

Given that the pollutant concentration in runoff is known, and expressed through its mean value, the fraction of the pollutant load not captured by the storage unit is given by:

$$f_V = C_R V'/C_R V \tag{1.1}$$

where: C_R = mean runoff event pollutant concentration.

Because C_R is treated as a mean value, and constant, the fraction of the runoff, and the fraction of the load not captured will be equivalent. Equation (1.1) is solved numerically and is reproduced here as Figure 1.7.

A similar type of analysis is carried out to assess the long term behaviour of an interceptor, which can be taken to represent a treatment plant. The long-term fraction of the runoff bypassing an interceptor of wet-weather capacity Q_I is defined as f_I and requires numerical evaluation.

When an interceptor and storage basin are operated together the long term fraction of the runoff volume bypassing the interceptor and the storage basin is defined as f_{IV},

Numerical evaluation by DiToro an Small (1979) has

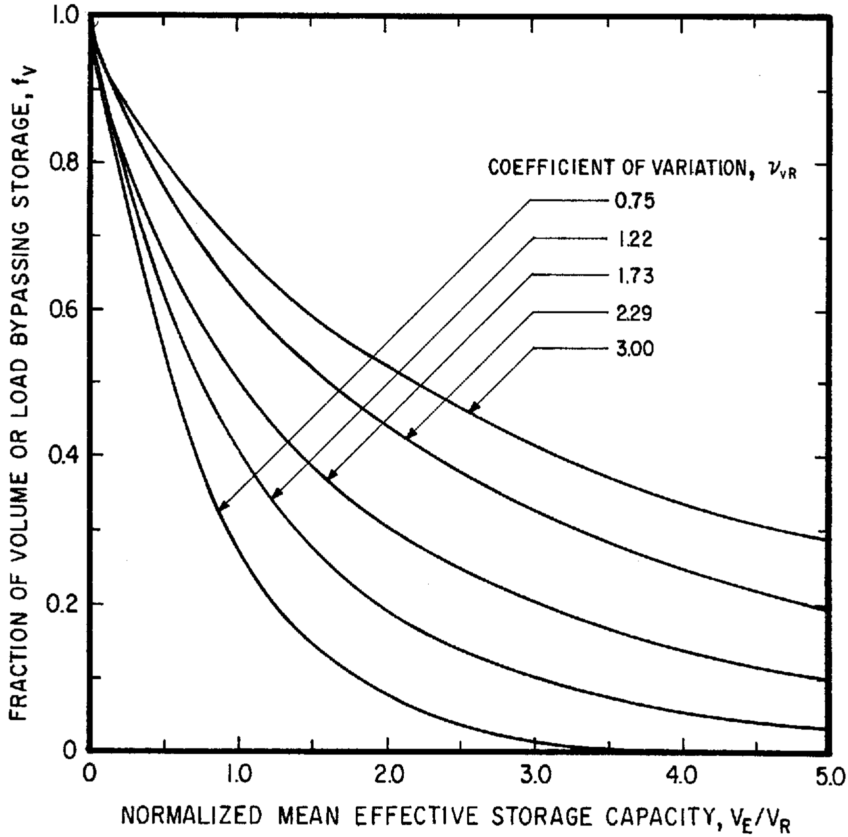


Figure 1.7 Determination of the Long-Term Fraction of the Total Pollutant Load or Runoff Volume Not Captured by the Storage Basin (DiToro and Small, 1979; Hydroscience, Inc., 1979).

shown that f_{IV} can be approximated in terms of the other fractions. The approximation is expressed as:

$$f_{IV} \approx f_I f_V \qquad (1.2)$$

The evaluation of these long-term fractions can be simplified if the coefficient of variation of these distributions are found to be, or assumed, equal to one. In such a case the gamma distribution will become exponential.

From the graphs of the fraction equations values of the storage and interception rate can be obtained in order to construct the storage/treatment isoquants, as previously defined. While the fraction equations have been shown by Hydroscience (1979) to give good results when compared to data from some locations, Nix (1982) has found that they provide only fair results when used to construct storage/treatment isoquants for comparison with simulation results from SWWM. However, the method does yield the range within which the efficient treatment rate/storage capacity are to be found.

A further development of the Hydroscience method considers the effects of a varying pollutant concentration. The concentration is assumed to vary exponentially, in a decreasing fashion, with time. The form of the equation is analogous to Horton's infiltration equation. Further discussion on this topic is postponed until Chapter V.

The Hydroscience method does not consider the effect of pipe storage in the fraction equations. Neither does it consider the inlet system as a factor which may control the amount of runoff available to the storage unit. The proposed statistical method will seek to overcome these shortcomings by formulating a more generalized model. Nevertheless, the Hydrocomp method introduces a new approach to urban stormwater management.

1.3.2 Howard's Statistical Method

Howard's Statistical Method (Howard, 1976; Howard et al., 1981) was developed through the use of derived distribution techniques (see Benjamin and Cornell, 1970, for applications of the theory in Civil Engineering). The method in which the runoff process is treated is similar to that of the Hydrocomp approach - independent events arriving at the system defined by runoff intensity, event duration, and interevent time. Figure 1.8 shows the system configuration used in Howard's Statistical Analytical method.

Runoff is generated by subtracting depression storage and multiplying the hyetograph by a runoff coefficient. It is assumed that the concentration of pollutant in runoff is a constant, independent of all event parameters. The treatment plant rate is a constant, operating so long as there is water in the reservoir. The treatment efficiency of the plant is also taken as a constant.

The other major assumption is that the reservoir is assumed to be full at the end of the previous storm.

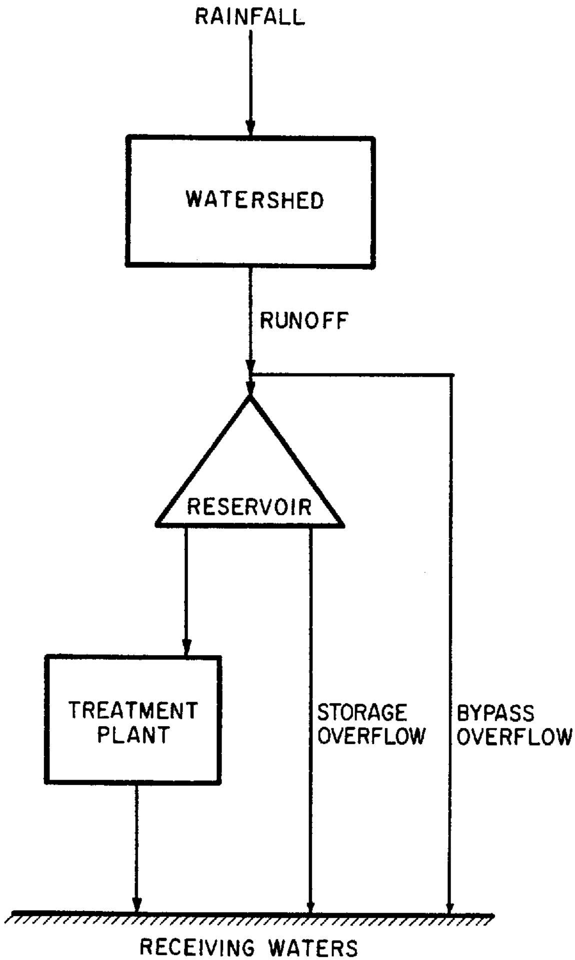


Figure 1.8 Schematic Representation of the System used by Howard's Statistical Analytical Method (Howard et al., 1981).

Because of this simplifying, and conservative, assumption the overflow volume from storage can be expressed as a simple relationship in terms of the storm runoff volume, the interevent time, the treatment rate, and the storage capacity of the tank.

Based on related studies Howard assumes that the event durations, interevent times, and storm intensities are exponentially distributed. The volume of a storm is defined as the product of the average storm intensity and its duration:

$$V_e = T_e I_e \tag{1.3}$$

Using derived distribution techniques the probability density function of V_e is obtained. To simplify the analysis, Howard disregards the Bessel function form of the distribution for V_e and approximates the distribution by an exponential distribution. This procedure is utilized by Howard again and again to simplify the form of the resulting distribution functions. For example, both the distributions of runoff intensity and the storage-utilizing runoff intensity are simplified to exponential distributions.

In a modification of the original procedure (Howard et al., 1981), Howard considered, among other modifications, the use of a varying treatment rate, the use of a time-varying rate of rainfall abstractions, and the consideration of the dynamic routing and storage effects in pipelines.

Utilizing the derived distributions, Howard developed annual statistics for the storage device in terms of the means, or expected values, of the distributions. The expected number of annual overflow events is given by:

$$N_o = N_s \, G_p(0) \qquad (1.4)$$

where: N_o = expected number of overflows,

N_s = average annual number of events which utilize storage;

$G_p(0)$ = probability of having an overflow greater than zero.

The average annual percent runoff control is given as:

$$C_R = 100 \, (1 - P_u/R) \qquad (1.5)$$

where: P_u = average annual volume of overflows,

R = average annual runoff.

The study conducted by Howard (Howard et al., 1981), showed that both the Hydrocomp's and Howard's statistical method produced the same maximum deviation from results when compared with STORM simulations. The maximum percentage deviation in percent runoff control for Howard's method when compared to STORM ranged from 13 to 19, while for the Hydrocomp's method it ranged from -10 to -14.

Howard's method suffers a loss of accuracy from the manner in which the distributions have been simplified to achieve manageable exponential forms. Also, it does not consider carryover storage, assuming that storage will always be previously full. It is also assumed, for simplicity, that the runoff event duration will always be one hour less than the storm duration. A further, and understandable, limitation, is that the treatment efficiency equation is not substantiated by an adequate amount of data. The treatment efficiency when an overflow occurs is not defined well because its real value has not been estimated.

The statistical method proposed herein will overcome some of the limitations of Howard's and Hydrocomp's methods. By using exponential forms it will derive distributions in a closed-form manner, without the need for approximations. Howard does introduce the need to account for storage and routing effects due to the conveyance system. This consideration will be carried over into the present work in a physically meaningful manner.

1.3.3 Other Related Statistical Methods

The two statistical methods presented in the previous sections comprise two of the major approaches to date to the statistical treatment of storage release systems. The other major approach is the work of Loganathan, Delleur, and Segarra (1985). This work extends and generalizes further the theory developed by Loganathan and Delleur (1982), which suffered from limitations similar to the ones already mentioned in relation to Hydrocomp's and Howard's methods.

The original work of Loganathan and Delleur (1982) utilized uniform intensity blocks of runoff arriving at a storage device in an independent manner. From storage, water was continuously withdrawn for treatment. The method employs exponential distributions for the event depth, duration, and time between events. The method assumed that the storage unit was previously filled to capacity, and no consideration was given to pipe storage effects or inlet control. The water quality was handled through the use of a constant concentration.

Such limitations are understandable in view of the purposes of that report. Its objective was to develop a multiobjective land use planning model, of which the storage/treatment system was only a part. However, the fundamental theory allowed further development, and this was encouraged by the good results that the theory yielded when compared to the model STORM (Loganathan and Delleur, 1984). Further work on the theory relaxed the limiting assumption of full storage at the end of the previous event and allowed the proper consideration of carryover storage (Loganathan, Delleur, and Segarra, 1985). However, the theory still did not account for the effects of inlet control, and the consideration of a water quality formulation.

Other statistical approaches to particular aspects of urban stormwater management problems exist but they will not be reviewed here, either because they are not as versatile as the ones already mentioned, or treat other aspects not directly related to storage/release systems. An example of the former is the work of Schwarz and Adams (1981), which is based on the work of Smith (1980). Schwarz and Adams obtained analytical expressions for the probability distributions of spill volumes from two detention storage reservoirs in series. However, the procedure is not easily extended to a higher number of storage devices which would generalize the process. The assumptions as to the fundamental hydrology are those typical of the statistical models discussed here, and no water quality considerations are included.

An example of the latter is the work of Chan and Bras (1979). Based on the Kinematic wave theory, Chan and Bras developed, through the use of the derived distribution technique, the distribution of flood volumes above a given threshold. This allows them to compute the flood volume frequency for a given catchment. The equations obtained through the kinematic wave method could not be integrated analytically to obtain the desired distributions so an approximation was devised. The formulation does not consider storage devices explicitly, and therefore is not directly related to the type of problem considered here.

CHAPTER II

Stormwater Detention Planning Model

2.1 Introduction

The development of the statistical model in the present work requires the conceptualization of the urban drainage system in a manner that effectively accounts for the relevant parameters of the process. The major components are (Roesner, 1982):

1. Surface Runoff
2. Transport through sewers and major drainage facilities.
3. Receiving water.

The runoff generated over the urban catchment travels overland to the inlets of the stormsewer system. Passing through the inlet system the runoff travels to the storage or treatment unit for processing. Hydraulically, the inlets, because of their limited capacity, will tend to create ponding for large storms (Burke, 1978). The efficiency of the inlets in trapping runoff flows depends on geometric and hydraulic factors (Akan, 1973; Yu, 1979). These factors determine the depth of flow obtained over an inlet during a runoff event.

The flooding depth obtained during an event may be of such magnitude that a portion of the runoff volume may be lost to the inlets through overflow into adjacent areas not drained by the inlet system. From the management point of view it may be desirable in some situations to provide some relief to the sewer system by conveying a fraction of the runoff volume to another location. This would reduce the storage requirement for the major storage treatment device. Studies along this line have been conducted by Wisner et al. (1981), and Wisner and Kassem (1982). Wisner defined the concept of dual storage in terms of inlet control of runoff volume. The idea is to utilize the limited capacity of inlets, or to limit this capacity by devices, to divert a portion of the runoff volume to a surface storage unit, such as a park. This diverted volume may never be treated. The objective is to reduce the storage requirements on the sewer system and lessen the impact of local flooding. It is based on the assumption that it is less costly to provide limited surface storage than to provide additional sewer storage capacity.

The typical planning formulations assume that surface runoff passes down to the sewer system without suffering any effect from the inlet devices. The inlet is assumed to posses infinite capacity to pass the flow without modification (U.S. Army Corps of Engineers, 1977). In the statistical approach this may not be a good assumption because extreme events have a finite probability of occurrence, and the resulting flows will certainly be affected by the limited capacity of the inlet system. It is expected that for major flows the limited capacity of the inlets will be exceeded, producing local flooding. This affects the design of the major storage unit because it cannot be assumed that all runoff volumes will be intercepted by the inlet system.

In the sequel formulation and derivation of a stormwater detention planning model that effectively accounts for the processes described above is provided.

2.2 Formulation of the Storage Equation

The representation of the urban stormwater catchment used in the model is shown in Figure 2.1, and the following discussion is referred to that figure. The representation comprises the catchment, the inlet or trapping system, the storage unit, and the treatment plant. The system is described by the following event based parameters and variables:

$S(n)$ = available storage (empty space) at the end of the n^{th} runoff event (inches) (see Fig. 2.2);

$X_1^{(n)}$ = actual runoff volume of the n^{th} event (inches);

$X_2'^{(n)}$ = effective rainfall duration of the n^{th} event (hours);

$X_2^{(n)}$ = runoff duration of the n^{th} event (hours), a function of the effective rainfall duration;

$X_3^{(n)}$ = time between the end of the $(n-1)^{th}$ event

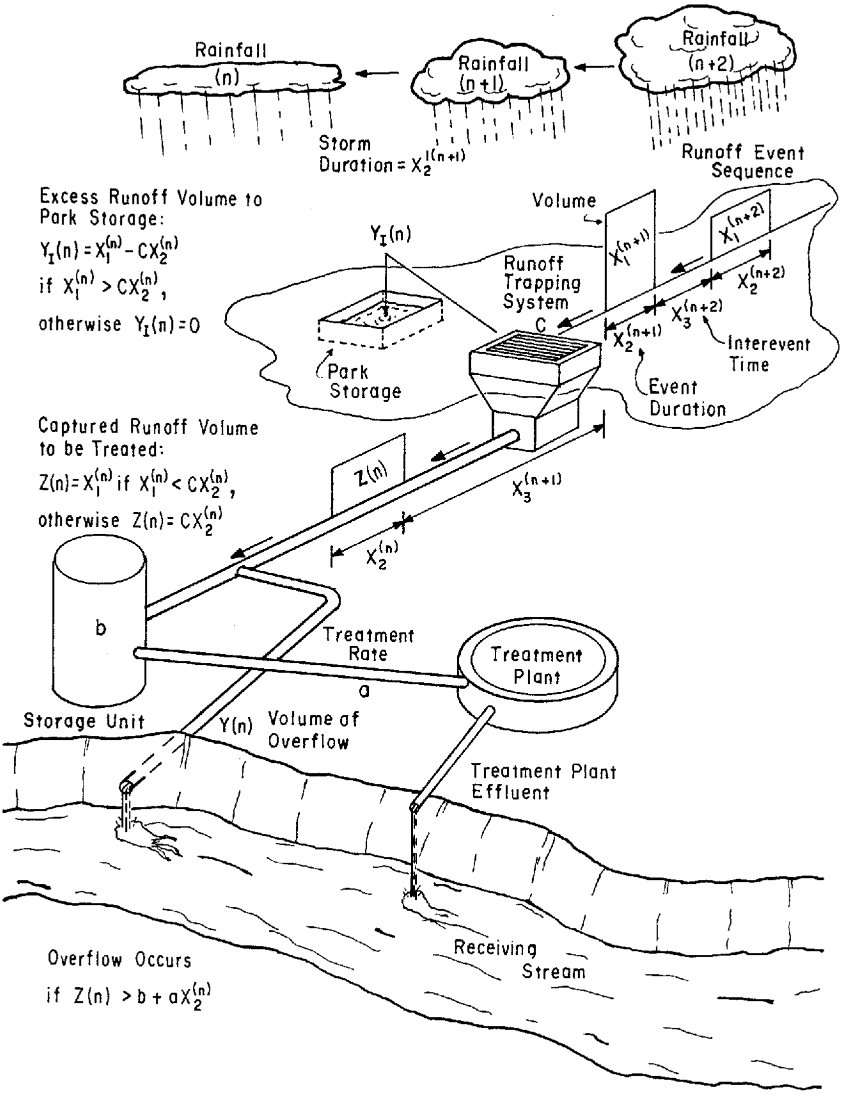


Figure 2.1 The Urban Stormwater System

and the start of the n^{th} event (hours);

$Y(n)$ = volume of storage overflow at the n^{th} event (inches);

$Y_I(n)$ = runoff volume not captured by inlets which contributes to park storage or is diverted elsewhere (inches);

$S(n-1)$ = available storage space at the end of the $(n-1)^{th}$ event (inches);

a = rate of water withdrawal from the storage unit (inches/hour);

b = storage device design capacity, the maximum volume available for treatment (inches); and

c = a measure of the surface runoff trapping capacity of the inlet system, which determines the runoff volume available to park storage.

Runoff events are produced over the urban catchment by rainfall events having a certain depth and duration. Runoff events are separated by time periods called interevent times. Catchment runoff is routed to the trapping system, which comprises the inlets to the stormsewer system. Over the catchment the runoff events are described by an event depth $X_1^{(n)}$, by an event duration $X_2^{(n)}$, and by an interevent time $X_3^{(n)}$. The event depth is the actual runoff volume obtained from the rainfall event. The runoff duration and interevent time are related to the effective rainfall duration and interevent time, respectively. The actual hydrograph is simplified to a block hydrograph of constant, average, runoff rate. The use of such shapes is justified for lumped models where interest lies in the total event volume and duration rather than on the actual runoff rate (Morris and Wiggert, 1972; Hydrocomp, 1979).

A runoff event gets divided into two parts, namely (i) contribution to park storage or detention storage, which is generally not treated, and (ii) contribution to the storm sewer system, which is treated. It is assumed here that water will be diverted to the park storage or other location only when the surface runoff trapping capacity is exceeded. Therefore the amount of water (in volume units) which passes through the sewer system in an event can be expressed as

$$Z(n) = \mathrm{Min}\ (X_1^{(n)},\ cX_2^{(n)}). \tag{2.1}$$

The term $cX_2^{(n)}$ represents the maximum amount of runoff that the inlet system can capture over the event duration. This is defined by the limited system trapping rate c.

The amount of water diverted is given by the difference between the total runoff volume $X^{(n)}$, and the maximum amount captured by the inlet system over the event duration $cX^{(n)}$, if the total runoff volume is the greater amount; otherwise no water is diverted. Letting Y_I represent the diverted volume, it can be expressed as

$$Y_I^{(n)} = \text{Max}\ (X_1^{(n)} - cX_2^{(n)},\ 0) \qquad (2.2)$$

The runoff volumes passing through the stormsewer system will travel to the storage unit. The shape of the hydrographs is assumed to remain the same. The sewer network is treated as a linear channel system. This implies that the event time base is unaltered. The block hydrographs are only shifted in time, which does not affect the volume relationships [see time shift routing in ILLUDAS (Terstriep & Stall, 1974)].

As stated previously the runoff duration is related to the excess rainfall duration. Howard (1981) considered this relationship in his statistical model. To assess catchment and sewer effects on hydrograph durations, Howard assumed a symmetric triangular hydrograph with a peak runoff rate equivalent to the effective precipitation rate. From continuity the hydrograph time base is given as

$$X_2 = 2X_2' \qquad (2.3)$$

Howard also observes that for large catchments with significant pipe storage the peak runoff rate will be less than the effective rainfall intensity. Thus the time base must be larger than twice the effective rainfall duration.

volume, it can be expressed as

$$Y_I^{(n)} = \text{Max}\ (X_1^{(n)} - cX_2^{(n)},\ 0) \tag{2.2}$$

The runoff volumes passing through the stormsewer system will travel to the storage unit. The shape of the hydrographs is assumed to remain the same. The sewer network is treated as a linear channel system. This implies that the event time base is unaltered. The block hydrographs are only shifted in time, which does not affect the volume relationships [see time shift routing in ILLUDAS (Terstriep & Stall, 1974)].

As stated previously the runoff duration is related to the excess rainfall duration. Howard (1981) considered this relationship in his statistical model. To assess catchment and sewer effects on hydrograph durations, Howard assumed a symmetric triangular hydrograph with a peak runoff rate equivalent to the effective precipitation rate. From continuity the hydrograph time base is given as

$$X_2 = 2X_2' \tag{2.3}$$

Howard also observes that for large catchments with significant pipe storage the peak runoff rate will be less than the effective rainfall intensity. Thus the time base must be larger than twice the effective rainfall duration.

event time is much longer than the event duration, by at least an order of magnitude (Nix, 1981; Goforth, 1983). In effect, $E[X_3] >> E[X_2']$. It is expected that, if runoff events last longer than effective rainfall duration, the runoff interevent time should, on the average, be shorter than the effective rainfall interevent time. From the above argument it is seen that the correction required on the mean interevent time should be small, and would be somewhat offset by the time lags introduced by pipe flow.

Without altering the distribution postulated for X_3, a correction factor can be applied to the mean of X_3, the effective rainfall interevent time, to obtain the mean of the runoff event interevent time, and still utilize the original distribution of X_3. A simple correction is suggested by Equation (2.6). The decrease in interevent time because of the transformation of rainfall duration is given by

$$d = X_2 - X_2' \tag{2.7}$$

which yields

$$d = (a_2 - 1)\ X_2' + x_c \tag{2.8}$$

The ratio between the mean runoff interevent time and the mean rainfall excess interevent time, r_3, is given by

$$r_3 = 1 - E[d]/E[X_3] \qquad (2.9)$$

Because $E[X_3] >> E[X_2']$ it is seen that the correction is bound to be small. The other parameters in Equation (2.8) are of the same order of magnitude, or smaller than, X_2'.

The event defining the n^{th} arrival may find that the tank is not empty - some carryover volume of water may be present. The amount present is a function of the past loading history of the storage device. Upon the arrival of the n^{th} event the tank will be empty or partially full. The amount of storage space available upon the arrival of the n^{th} event is given by

$$\text{Min } [S(n-1) + aX^{(n)}, b] \qquad (2.10)$$

The quantity $aX_3^{(n)}$ is the amount of water treated during the interevent time. The minimum reflects the fact that the physical capacity of the system, or empty space available, is limited to b.

At the storage unit, the incoming runoff event of volume Z(n) and duration $X_2^{(n)}$ is stored while being treated continuously at rate a. If the incoming volume is greater than the empty space available the excess volume is dumped to a water body. This excess volume is the quantity of greatest interest here. If the incoming volume is less than the empty space available no overflow occurs and all of the runoff is treated. Using the definition of $Z(n)$, the change in storage during an event is defined as

$$aX_2^{(n)} - \text{Min}\ (X_1^{(n)},\ cX_3^{(n)}) \qquad (2.11)$$

A positive value of Equation (2.11) indicates an increase in available storage (empty space) while a negative one indicates a decrease in available storage. Figure 2.2 illustrates the relationships.

The storage available at the end of the n^{th} event is the sum of the storage available at the start of the n^{th} event and the net change in storage produced during the event. As seen from Equation (2.11) this change can be positive or negative. The storage made available cannot exceed tank capacity, b. However, the process is left unbounded in the other direction because overflows can occur. It will be shown that negative available storage is a measure of the overflow volume.

A the end of the n^{th} event the equation for available storage is given by

$$S(n) = \text{Min}\,\{[\text{Min}(S(n-1)+aX_3,\ b)+aX_2-\text{Min}(X_1,cX_2)],\ b\,\} \qquad (2.12)$$

where;

$$S(n-1) = \begin{cases} S(n-1) \text{ if } S(n-1) > 0 \\ 0 \quad \text{if } S(n-1) \leq 0 \end{cases}$$

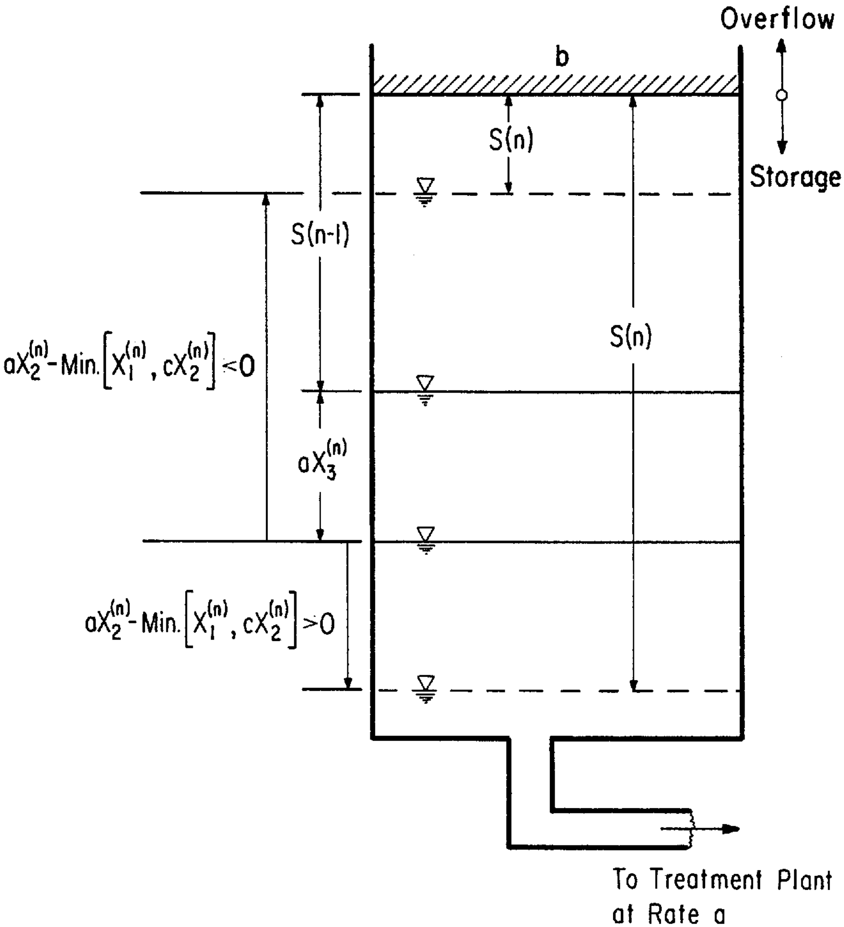


Figure 2.2 Variations in Available Storage
During an Event

The equation is valid for all n. The value of S(n-1) when $S(n-1) \leqslant 0$ indicates an overflow, and thus no available empty storage at the end of the $(n-1)^{st}$ event, because the tank is full.

Negative storage, which is indicative of an overflow, can occur if

$$\text{Min }(X_1^{(n)}, cX_2^{(n)}) > \text{Min }[S(n-1) + aX_3^{(n)}, b] + aX_2^{(n)}$$

In such a case, an overflow, equal to S(n), will occur and the tank will be full at the end of the n^{th} event.

If, on the other hand,

$$\text{Min }(X_1^{(n)}, cX_2^{(n)}) < \text{Min }[S(n-1) + aX_3^{(n)}, b] + aX_2^{(n)}$$

then some storage is available at the end of the n^{th} event.

The process variables described are to be treated as random variables. These are the event duration, depth and interevent time. The probability law of these variables is to be specified, and the probability law of the storage and overflow functions derived from these. The dynamic nature of the process is assessed by specifying a Poisson arrival process for the excess rainfall events. The Poisson process has been shown to describe accurately the arrival of storm events (Todorovic and Yevjevich, 1969; Eagleson, 1978).

An important related question is the nature of the interdependence between event parameters. Studies by Loganathan (1984), and Eagleson (1978), for locations in Indiana and New England, respectively, demonstrated the statistical independence between event depth, event duration, and interevent time.

If the Poisson process is used to describe the arrival of independent storm events it is necessary to define the duration of the interevent time that makes events statistically independent. A convenient procedure for determining this time makes use of the fact that interevent times for Poisson arrivals are exponentially distributed, with a coefficient of variation of unity. By varying the minimum number of dry hours that are expected to separate independent events a coefficient of variation is obtained which approaches unity. This dry period then defines the minimum number of dry hours that separate independent storm events. Events occurring at the time intervals smaller than this minimum interevent time are considered to be part of the same event. It should be noted that this definition prescribes a maximum number of hours for event duration. The Hydrocomp Statistical Model (Hydrocomp, 1979), and Restrepo - Posada and Eagleson (1982) make use of this approach.

Two other approaches to determining the minimum interevent time for event definition have been suggested by Heany et al. (1977). The first is to run an autocorrelation analysis of the hourly rainfall record to determine the time lag at which the autocorrelation coefficient becomes insignificant according to some tolerance level. The lag at which the times become uncorrelated gives the minimum value of the dry period that separates independent events. The second approach makes use of the fact that the number of storm events in a given record depends on the chosen length of the interevent time. By plotting the number of events as a function of the minimum interevent time, the time can be chosen at which the number of events in the period does not change appreciably with an increase in interevent time. This time would represent the minimum interevent time. These techniques are a way of screening the data to fit a particular distribution to observed data. They do not by themselves guarantee the independence of the arrivals for all realizations of the process.

In a similar fashion, for the runoff process, the same type of analysis can be conducted to determine the minimum runoff interevent time. Since rainfall records are more readily available than runoff records, a rainfall/runoff transformation usually precedes the statistical analysis. The most commonly used approach is the runoff coefficient method. (U.S. Army Corps of Engineers, 1977; Chow, 1964). The basic procedure of the method is the determination of the average runoff coefficient which will multiply the increment of rainfall depth for that period to obtain the increment of runoff or effective rainfall depth. The coefficient is, basically, a function of physiographic characteristics. The method can be expanded to account for (in a lumped fashion) antecedent moisture conditions, infiltration losses, and depression storage.

To obtain the appropriate distributions the process will be assumed stationary. This implies that the process is invariant with respect to time, that is, it is the same when viewed at different times. The index n can then be dropped from the variables.

Furthermore, the process random variables are assumed to be exponentially distributed. For Poisson event arrivals the exponential distribution is prescribed for interevent times. The event depth and duration are also assumed to be exponential. The use of the exponential distribution for rainfall and runoff parameters is justified from the relatively good description of actual data obtained with exponential distributions by various researchers (Chow and Yen, 1976; Eagleson, 1978; Delleur, 1983; Loganathan and Delleur, 1984; and Pagán, 1984).

Thus, X_1, X_2', and X_3, the runoff event depth, the effective rainfall duration, and the interevent time, are exponentially distributed with parameters α, β_2, and γ, respectively. The distributions are expressed as:

$$f_{X_1}(x_1) = \alpha \exp(-\alpha x_1); \qquad x_1 \geqslant 0 \qquad (2.13)$$

$$f_{X_2}(x_2) = \beta_2 \exp(-\beta_2 x_2); \qquad x_2 \geqslant 0 \qquad (2.14)$$

$$f_{X_3}(x_3) = \gamma \exp(-\gamma x_3); \qquad x_3 \geqslant 0 \qquad (2.15)$$

These distributions are used to obtain the distributions of the available storage and overflow via the derived distribution approach. The following sections are concerned with the derivation of the distributions.

2.3 Derivation of the Distribution Functions

2.3.1 The distribution of the excess volume not captured by inlets

The excess volume not captured by the inlets, which is defined as inlet overflow or park storage volume, is obtained from equation (2.2). Under the assumed stationarity condition of the process this equation becomes:

$$Y_I = \mathrm{Max}(X_1 - cX_2, 0) \qquad (2.16)$$

The runoff duration X_2, is related to the effective rainfall duration, X_2', via Equation (2.6). The relationship is expressed as

$$X_2 = a_2 X_2' + x_c \tag{2.17}$$

in which a_2 and x_c are parameters.

The distribution of X_2' is specified as exponential [Equation (2.14)]. Because X_2 is a linear function of X_2', the probability density function of X_2 is obtained from that of X_2' via the derived distribution approach. Using Equations (2.17) and (2.14), the probability density function of X_2 is obtained as

$$f_{X_2}(x_2) = \beta \exp[-\beta(x_2 - x_c)]; \quad x_2 \geqslant x_c \qquad (2.18)$$
$$= 0 \text{ otherwise}$$

in which $\beta = \beta_2/a_2$.

The distribution of X_1 is exponential and is obtained from Equation (2.13). The probability distribution function of Y_I is obtained from Equation (2.16). It is noted that the distribution of Y_I will have a point mass probability at $Y_I = 0$. In the range $Y_I > 0$ the cumulative distribution function is obtained from

$$P[0 < Y_I \leqslant y_I] = P[0 < X_1 - cX_2 \leqslant y_I], \tag{2.19}$$

or

$$P[Y_I \leqslant y_I] = P[X_1 \leqslant y_I + cX_2] \tag{2.20}$$

where P[ ] represents the probability of the given argument.

Because X_1 and X_2 are independent random variables the cumulative distribution is given by:

$$P[Y_I \leqslant y_I] = \int_{x_c}^{\infty} P[X_1 \leqslant y_I + cx_2] f_{X_2}(x_2) dx_2 \qquad (2.21)$$

Substitution of the appropriate expressions in Equation (2.21) yields the equation:

$$P[Y_I \leqslant y_I] = \beta \int_{x_c}^{\infty} \{1-\exp[-\alpha(y_I + cx_2)]\} \exp[-\beta(x_2-x_c)] dx_2 \qquad (2.22)$$

Integration finally obtains:

$$P[Y_I \leqslant y_I] = 1 - \frac{\beta}{\alpha c + \beta} \exp[-\alpha(y_I + cx_c)]; \quad Y_I > 0 \qquad (2.23)$$

The point mass probability is obtained by setting $Y_I=0$ in Equation (2.23):

$$P[Y_I = 0] = 1-\frac{\beta}{\alpha c + \beta} \exp[-\alpha c\ x_c] \qquad (2.24)$$

The probability density function of Y_I is obtained by differentiating Equation (2.23):

$$f_{Y_I}(y_I) = \begin{cases} \frac{\beta}{\alpha c + \beta} \exp[-\alpha(y_I + cx_c)]; & y_I > 0 \quad (a) \\ 1-\frac{\beta}{\alpha c + \beta} \exp(-\alpha c\ x_c); & y_I = 0 \quad (b) \end{cases} \qquad (2.25)$$

The distribution of the excess runoff volume Y_I has a "spike" at the origin corresponding to $P[Y_I=0]$. Thus, there always exists the possibility of having no excess volume from the inlet system.

From the cumulative distribution we can specify a probability of exceedance of overflow as:

$$P[Y_I \geq y_I] \leq \varepsilon_I \tag{2.26}$$

where

$$P[Y_I \geq y_I] = \frac{\beta}{\alpha c + \beta} \exp[-\alpha(y_I + cx_c)] \tag{2.27}$$

Here ε_I represents the exceedance probability, which is a measure of the system reliability. From a design point of view Equation (2.26) can be solved to yield values of the overall inlet capacity for specified levels of exceedance probability. Consider the probability of an overflow, defined as $P[Y_I>0]$, being less than some value ε_I. This is given by the expression

$$\frac{\beta}{\alpha c + \beta} \exp(-\alpha c \ x_c) \leq \varepsilon_I \tag{2.28}$$

Equation (2.28) can be solved to yield values of the inlet capacity c in terms of the exceedance probability and the mean runoff rate. The mean runoff rate is obtained from the term β/α. It is known from the properties of the exponential distribution that $\alpha = E[X_1]^{-1}$, and $\beta = a_2\ E[X_2']^{-1}$ - the parameter of the distribution is the inverse of the expected value of the variable. The ratio β/α is seen to give the mean runoff intensity. Equation (2.28), now expressed in equality form, is expressed as

$$\frac{i_{av}}{c + i_{av}} \exp(-\alpha c\ x_c) = \varepsilon_I \tag{2.29}$$

where $i_{av} = \beta/\alpha$.

The value of c obtained from Equation (2.29) is distributed among a fixed number of inlets to obtain the design discharge for specific inlets.

The values of system capacity and the average runoff intensity are related to the probability of exceedance or the probability of having an overflow from the inlet system during any event. The inlet capacity c is a positive or zero variable. It is thus seen that, as $\varepsilon_I \longrightarrow 1$, indicating that all runoff will go off as surface flow, the capacity c must go to zero. This is the situation wherein no inlet system exists and all flow is overland flow. Conversely, as $\varepsilon_I \longrightarrow 0$, indicating that no inlet overflow will occur, the capacity c must be infinite, indicating that the inlets will intercept all runoff.

The tradeoff between ε_I and the probability of storage overflow is readily observed. For a small value of ε_I, c will be large, allowing more runoff entrance to the storm-sewer and thus to the storage unit, increasing the chance of having an overflow. For a large value of ε_I, c is reduced, allowing less runoff to enter the sewer, thus decreasing the chance of an overflow into the receiving stream from the storage unit, but it also drastically increases local flooding. Essentially the area containing the inlet becomes a storage unit because of local flooding.

Equation (2.29) is derived from a general conceptualization of the urban runoff system, with the extensive simplification inherent to that approach. Nevertheless the expression derived provides for an assessment of the overall efficiency of the runoff collection system - one based on runoff event parameters.

2.3.2 Distribution of the storage S(n)

The storage equation already formulated is used to derive the probability function for storage levels. The storage equation is given as:

$$S(n) = \text{Min}\{[\text{Min}(S(n-1) + aX_3, b) + aX_2 - \text{Min}(X_1, cX_2)], b\} \quad (2.30)$$

The distributions of X_1, X_2, and X_3 are used to derive the corresponding distribution for S(n). The process represented by the storage equation has been assumed stationary. The distributions of the random variables X_1, X_2, and X_3 are assumed stationary in the sense that their distributions are invariant with respect to time. The storage level at the end of any event is a function only of the storage level at the end of the previous event and the random hydrologic occurrences in the intervening period. The process is thus classified a first order Markov process.

Because of its intricacy the derivation of the distribution function will proceed in stages. First, the distribution of the following variables is obtained:

$$Z = \mathrm{Min}(X_1, cX_2), \tag{2.31}$$

$$W = aX_2 - Z, \tag{2.32}$$

and

$$T = \mathrm{Min}[S(n-1) + aX_3, b] \tag{2.33}$$

The storage equation is then expressed as:

$$S(n) = \mathrm{Min}\ (T + W, b) \tag{2.34}$$

The derivation of the distributions for Z, W, and T now proceeds.

2.3.2.1 The distribution of $Z = \mathrm{Min}\ (X_1, cX_2)$

To obtain the distribution of Z it is first noted that X_1 and X_2 are independent random variables. The distribution of X_1 is given by Equation (2.13), and that of X_2 by Equation (2.18). The most convenient form of obtaining the distribution of Z is through the expression:

$$P[Z \geqslant z] = P[X_1 \geqslant z]\ P[X_2 \geqslant z/c] \qquad (2.35)$$

If $0<z \leqslant cx_c$, then $P[X_2 \geqslant {}^{z}/c] = 1$, because the distribution of X_2 is defined for $X_2 \geqslant x_c$. In this case Equation (2.35) becomes:

$$P[Z \geqslant z] = \exp(-\alpha z);\ 0 < z < cx_c \qquad (2.36)$$

For $z > cx_c$, both distributions are used in Equation (2.35) to yield:

$$P[Z \leqslant z] = \exp(\beta x_c)\ \exp[-(\alpha+\beta/c)z];\ z > cx_c \qquad (2.37)$$

The probability density function of Z, obtained from Equations (2.36) and (2.37) is given by

$$f_Z(z) = \begin{cases} \alpha \exp(-\alpha z);\ 0 \leqslant z \leqslant cx_c & \text{(a)} \\ (\alpha+\beta/c)\exp(\beta x_c)\exp[-(\alpha+\beta/c)z];\ z> cx_c & \text{(b)} \\ 0 \qquad \text{elsewhere} \end{cases} \qquad (2.38)$$

Equation (2.38a) indicates that for the range $z \leqslant cx_c$ the distribution of the runoff volume entering the stormwater system is given by the total runoff distribution, implying that all of it is trapped.

It is of interest to find the mean value of Z. This is done by taking the expectation of Z:

$$E[Z] = \int_0^\infty z\, f_Z(z)\, dz \qquad (2.39)$$

For the particular pdf of Z it is obtained

$$E[Z]= \alpha \int_0^{cx_c} z \exp(-\alpha z)dz + (\alpha+\beta/c)\exp(\beta x_c)\int_{cx_c}^\infty z \exp[-(\alpha+\beta/c)z]dz \qquad (2.40)$$

Integration by parts yields

$$E[Z] = \frac{1}{\alpha} - \frac{\beta}{\alpha(\alpha c+\beta)} \exp(-\alpha c x_c) \qquad (2.41)$$

E[Z] represents the mean value of the runoff volume which enters the stormwater system. It is a function of runoff duration and depth parameters, and of the inlet parameter c. It is to be noted that, as c increases, E[Z] will approach $1/\alpha$, which is the mean runoff volume, $E[X_1]$. As c becomes small, eventually becoming zero, E[Z] will be reduced to zero also because no runoff will be entering the system.

It is also of interest to compare Equation (2.41) with $E[Y_I]$, the mean value of the runoff that is not trapped by the system, and contributes to park storage. Using Equation

(2.25):

$$E[Y_I] = 0\ P[Y_I=0] + \frac{\alpha\beta}{\alpha c + \beta} \exp(-\alpha c x_c) \int_{0+}^{\infty} y_I \exp(-\alpha y_I) dy_I \qquad (2.42)$$

which yields:

$$E[Y_I] = \frac{\beta}{\alpha(\alpha c + \beta)} \exp(-\alpha c x_c) \qquad (2.43)$$

For $E[Y_I]$, as c increases, it will go to zero, indicating that all flow is trapped by the inlets. As c becomes small $E(Y_I)$ will tend to become $1/\alpha$, indicating that all the runoff remains as overland flow.

From Equations (2.41) and (2.43) it is seen that

$$E[Z] + E[Y_I] = E[X_1] \qquad (2.44)$$

2.3.2.2 The Distribution of W = a - Z

Because $Z = \mathrm{Min}\ (X_1, cX_2)$ this term will not be independent from aX_2, and appropriate regions of integration must be defined. To achieve this W is split into the following ranges:

$$W = \begin{cases} aX_2 - X_1 & \text{if} \quad X_1 < cX_2 \\ (a-c)\ X_2 & \text{if} \quad X_1 \geqslant cX_2 \end{cases} \qquad (2.45)$$

It will be assumed from here on that c>a, that is, the inlet capacity must always be greater than the treatment rate, otherwise a storage unit would not be required and there would be no overflow, thus invalidating the problem.

For any $w \in R$, the cdf is defined by

$$P[W \leqslant w] = P[(W \leqslant w \cap X_1 < cX_2) \cup (W \leqslant w \cap X_1 \geqslant cX_2)] \qquad (2.46)$$

Because the events are mutually exclusive:

$$P[W \leqslant w] = P[W \leqslant w \cap X_1 < cX_2] + P[W \leqslant w \cap X_1 \geqslant cX_2] \qquad (2.47)$$

Substituting from Equation (2.45):

$$P[W \leqslant w] = P[(X_1 \geqslant aX_2 - w) \cap (X_1 < cX_2)] + P[(X_2 \geqslant \frac{w}{a-c}) \cap (X_1 \geqslant cX_2)] \qquad (2.48)$$

Because X_2 is a bounded variable the region of integration for the cdf will be related to x_c, the lower bound of X_2.

From Equation (2.45) it is evident that W can be a negative variate, so that positive and negative ranges for the cdf are introduced. The derivation will be carried out by specifying the range and the relative location of x_c in Equation (2.48).

<u>Range $w>0$, $w/a> x_c$</u>:

This situation, for Equation (2.48), is shown in Figure 2.3. For the bounds of the regions in the figure:

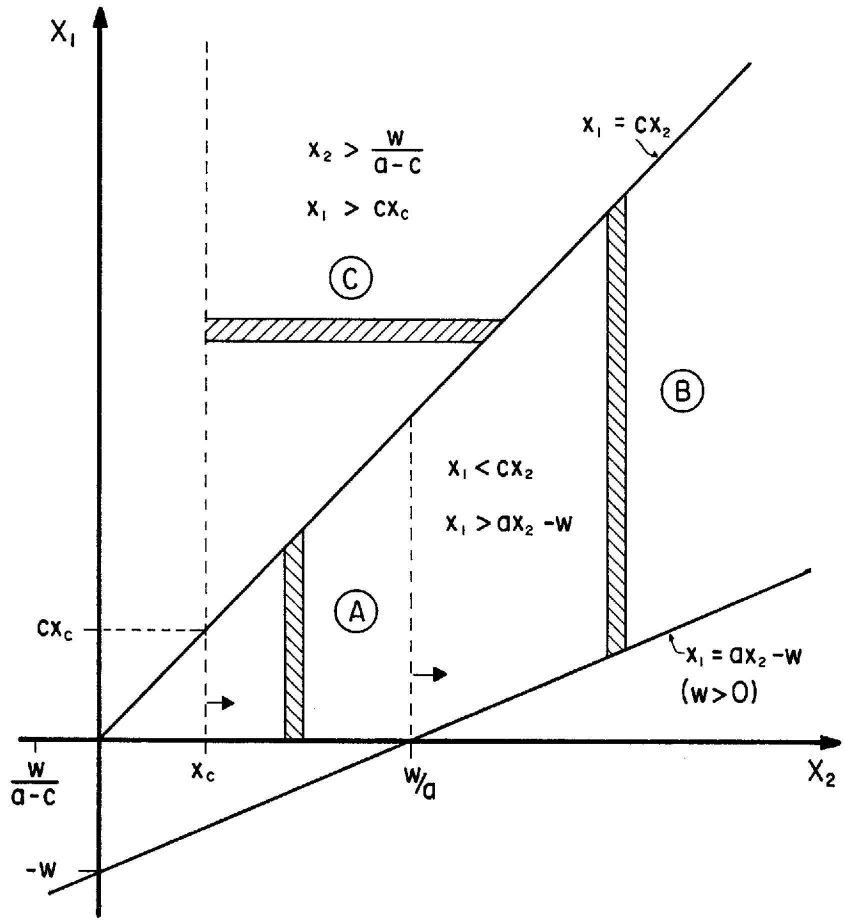


Figure 2.3 Integration Regions for the Cumulative Density Function of W, Region W>0, $\frac{W}{a} > x_c$

$$P[W \leqslant w] = \int_{x_2=x_c}^{w/a} \int_{x_1=0}^{cx_2} f_{X_1}(x_1)\, f_{X_2}(x_2)dx_1\, dx_2 \text{ [Region (A)]}$$

$$+ \int_{x_2=\frac{w}{a}}^{\infty} \int_{x_1=ax_2-w}^{cx_c} f_{X_1}(x_1)\, f_{X_2}(x_2)\, dx_1\, dx_2 \text{ [Region (B)]} \quad (2.49)$$

$$+ \int_{x_1=cx_c}^{\infty} \int_{x_2=x_c}^{x_1/c} f_{X_1}(x_1)\, f_{X_2}(x_2)dx_1\, dx_2 \quad \text{[Region (C)]}$$

Substitution of the distribution of X_1 and X_2 in Equation (2.49) yields the following intermediate results for each region:

$$\text{Region (A)} = \beta \int_{x_c}^{w/a} [1 - \exp(-\alpha\, cx_2)] \exp[-\beta(x_2-x_c)]dx_2$$

$$\text{Region (B)} = \beta \int_{w/a}^{\infty} \{\exp[-\alpha(ax_2-w)]-\exp[-\alpha cx_2)]\}\exp[-\beta(x_2-x_c)]\, dx_1$$

$$\text{Region (C)} = \alpha \int_{cx_c}^{\infty} \{ 1-\exp[-\beta(\frac{x_1}{c} - x_c)]\} \exp(-\alpha x_1)\, dx_1$$

Carrying out the integration for each region and adding the results yields:

$$P[W \leqslant w] = 1 - \frac{\alpha\beta}{\alpha a+\beta} \exp[-\beta(w/a - x_c)];\; w \geqslant ax_c \quad (2.50)$$

<u>Range $w>o$, $w/a \leqslant x_c$</u>

For this range the appropriate regions are depicted in Figure 2.4. The cdf is now given by:

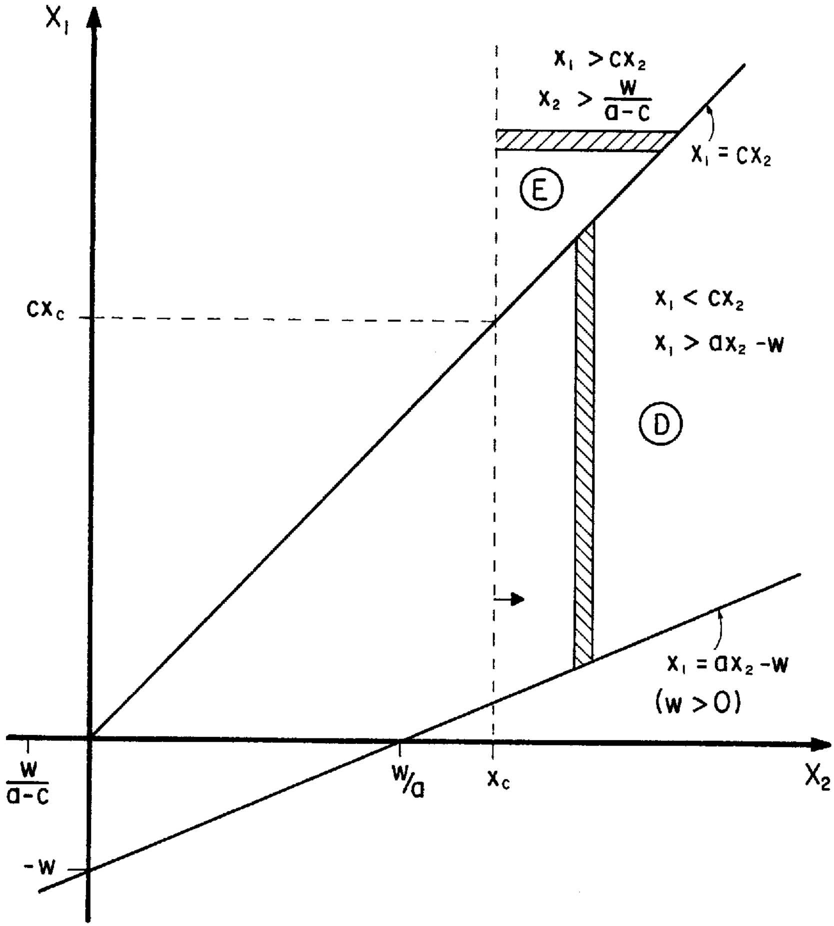


Figure 2.4 Integration Regions for the Cumulative Density Function of W, Region W>0, $\frac{W}{a} < x_c$

$$P[W \leqslant w] = \beta \int_{x_c}^{\infty} P[ax_2 - w \leqslant X_1 \leqslant cx_2]\ f_{X_2}(x_2)\ dx_2 \quad \text{[Region (D)]}$$

$$+ \alpha \int_{cx_c}^{\infty} P[X_2 \leqslant \frac{x_1}{c}]\ f_{X_1}(x_1)\ dx_1 \quad \text{[Region (E)]} \tag{2.51}$$

Integration and summation yields

$$P[W \leqslant w] = \frac{\beta}{\alpha a+\beta} \exp[-\alpha(ax_c - w)]; \quad w \leqslant ax_c \tag{2.52}$$

Now, the negative ranges are considered.

<u>Range $w \leqslant 0,\ w/(a-c) > x_c$</u>

Figure 2.5 illustrates this range. The cdf is given by

$$P[W \leqslant w) = \int_{\frac{w}{a-c}}^{\infty} P[ax_2 - w \leqslant X_1 \leqslant \frac{x_2}{c}]\ f_{X_2}(x_2)\ dx_2 \quad \text{[Region (F)]}$$

$$+ \int_{\frac{w}{a-c}}^{\infty} P[\frac{w}{a-c} \leqslant X_2 \leqslant \frac{x_1}{c}]\ f_{X_1}(x_1)\ dx_1 \quad \text{[Region (G)]} \tag{2.53}$$

Integration and summation of the two terms yields:

$$P[W \leqslant w] = \frac{\beta}{\alpha a + \beta} \exp(\beta x_c) \exp(-\frac{\alpha c+\beta}{a-c} w); \quad w \leqslant (a-c)x_c \tag{2.54}$$

The inequality of the range in the above equation changes sense because it was assumed that $c > a$.

<u>Range $w \leqslant 0,\ w/(a-c) \leqslant x_c$</u>

Figure 2.6 shows this range. The cdf is given by

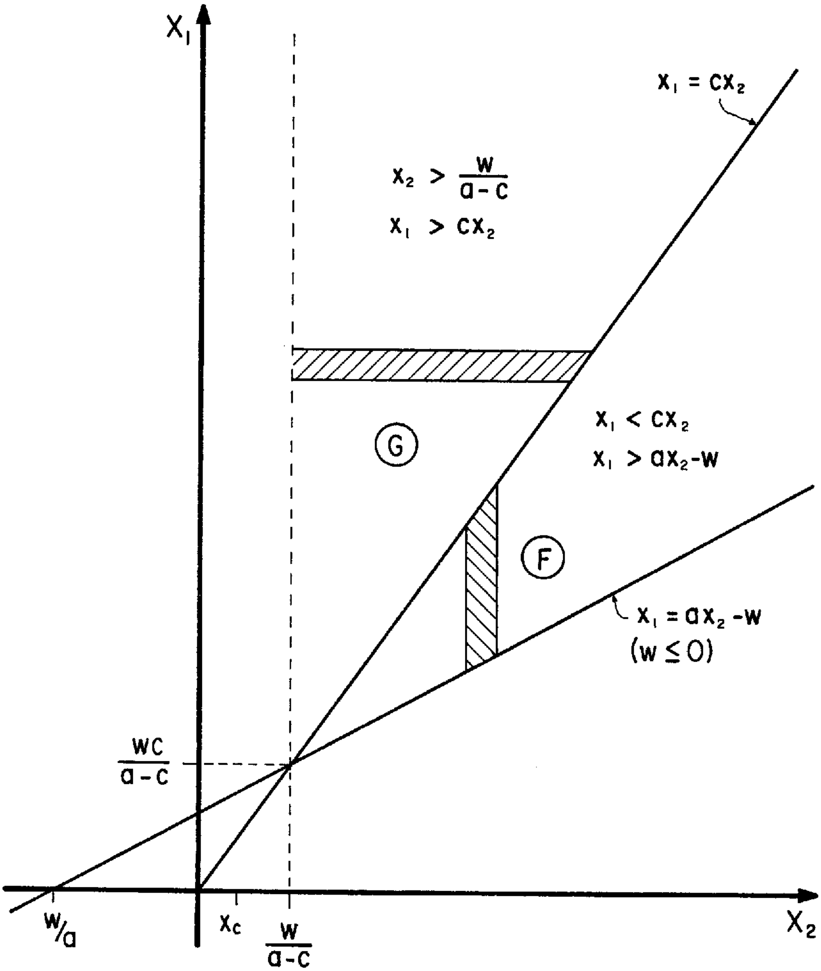


Figure 2.5 Integration Regions for the Cumulative Density Function of W, Range W≤0, $\frac{W}{a-c} > x_c$

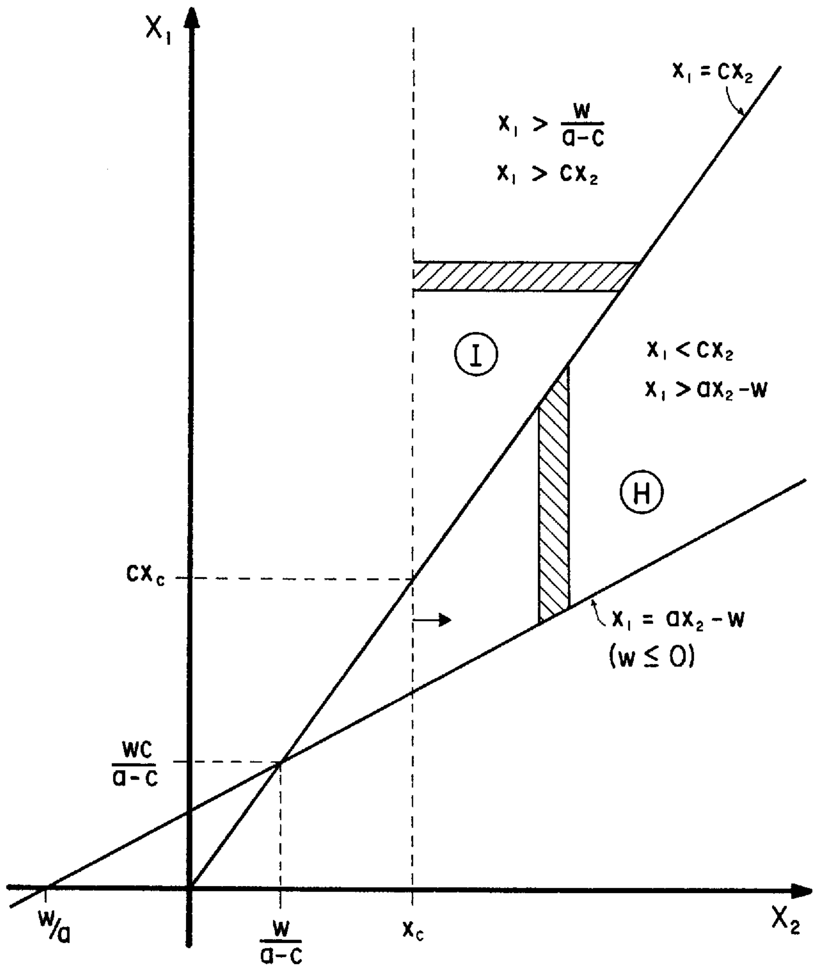


Figure 2.6 Integration Regions for the Cumulative Density Function of W, Range W⩽0, $\frac{W}{a-c} < x_c$

$$P[W \leqslant w] = \int_{x_c}^{\infty} P[ax_2 - w \leqslant X_1 \leqslant cx_2] \, f_{X_2}(x_2) dx_2 \quad \text{[Region (H)]}$$

$$+ \int_{cx_c}^{\infty} P[X_2 \leqslant x_1/c] \, f_{X_1}(x_1) \, dx_1 \quad \text{[Region (I)]} \tag{2.55}$$

Integration and summation yields:

$$P[W \leqslant w] = \frac{\beta}{\alpha a + \beta} \exp\,[-\alpha(ax_c - w)]; \; w \geqslant (a-c)\, x_c \tag{2.56}$$

A negative value of w occurs when the arriving runoff volume is greater than the volume that can be treated during the duration of the runoff event. This produces a reduction in the available storage.

It is to be noted that Equations (2.52) and (2.56) are similar expressions. In summary, the cumulative distribution function of $W = aX_2 - \mathrm{Min}\,(X_1, cX_2)$ is given by:

$$P[W \leqslant w] = \begin{cases} \dfrac{\beta}{\alpha a + \beta} \exp\,(\beta x_c) \exp\,(-\dfrac{\alpha c + \beta}{a-c} w); \; w < (a-c)x_c & \text{(a)} \\[2ex] \dfrac{\beta}{\alpha a + \beta} \exp\,[-\alpha(ax_c - w)]; \; (a-c)x_c \leqslant w \leqslant ax_c & \text{(b)} \\[2ex] 1 - \dfrac{\alpha a}{\alpha a + \beta} \exp\,[-\beta(\dfrac{w}{a} - x_c)]; \; w > ax_c & \text{(c)} \end{cases} \tag{2.57}$$

The probability density function of W is obtained by differentiating Equation (2.57)

$$f_W(w) = \begin{cases} \dfrac{\beta\,(\alpha c + \beta)}{(\alpha a + \beta)(c-a)} \exp\,(\beta\, x_c)\, \exp\,(-\,\dfrac{\alpha c + \beta}{a-c}\, w)\,; & w < (c-a)\, x_c \\[2ex] \dfrac{\alpha\beta}{\alpha a + \beta} \exp\,[-\alpha(ax_c - w)];\ & (a-c)\, x_c \leqslant w \leqslant ax_c \\[2ex] 1 - \dfrac{\alpha\beta}{\alpha a + \beta} \exp\,[-\beta\,(\frac{w}{a} - x_c)];\ & w > ax_c \end{cases} \tag{2.58}$$

2.3.2.3 Distribution of $T = \mathrm{Min}[S(n-1) + aX_3,\ b]$

The variable T represents the storage available at the arrival of the n^{th} event. This is in terms of the inter-event time and the storage available at the end of the previous event. The storage available at the end of previous event, S(n-1), is unknown because of the dynamic nature of the system. Hence a value for S(n-1) will be specified and the equations derived conditioned on this value. In this manner it will be possible to study the limits of system behaviour corresponding to previously empty, and full, storage conditions. Letting $S(n-1) = s_o$, where s_o is the assumed storage capacity at the end of the previous event, the expression for T is given by:

$$T = \mathrm{Min}(s_o + aX_3,\ b) \tag{2.59}$$

The object is to obtain

$$P[T \leqslant t] = P[\mathrm{Min}(s_o + aX_3,\ b)] \qquad (2.60)$$

The interevent time X_3 is exponentially distributed, according to Equation (2.15). The derivation in Equation (2.60) is straightforward since the only randomness is that associated with X_3. In the range $t < b$ the derived distribution technique yields:

$$P[T \leqslant t] = 1 - \exp[-\frac{\gamma}{a}(t - s_o)]; \quad s_o < t < b \qquad (2.61)$$

The pdf is given by:

$$f_T(t) = \frac{\gamma}{a} \exp\,[-\frac{\gamma}{a}\,(t - s_o)];\ s_o < t < b \qquad (2.62)$$

For $t \geqslant b$, $T = b$ always. The probability of $T = b$ is given by:

$$P[T = b] = 1 - P[T < b] \qquad (2.63)$$

Using Equation (2.61) it is obtained:

$$P[T = b] = \exp[-\frac{\gamma}{a}(b - s_o)] \qquad (2.64)$$

Summmarizing, the pdf of $T = \mathrm{Min}\ (s_o + aX_3,\ b)$ is given by:

$$f_T(t) = \begin{cases} 0 & \text{for } t < s_o & \text{(a)} \\ \frac{\gamma}{a} \exp\left[-\frac{\gamma}{a}(t - s_o)\right] & \text{for } s_o \leq t < b & \text{(b)} \\ \exp\left[-\frac{\gamma}{a}(b - s_o)\right] & \text{for } t = b & \text{(c)} \end{cases} \qquad (2.65)$$

With the distribution of T and W, the distribution a S(n), and subsequently of the overflow, can now be obtained from Equation (2.34).

2.3.2.4 Conditional distributions of available storage S(n)

To obtain the distribution of the storage and, ultimately, of the overflow, which is the major interest here, Equation (2.34) is employed:

$$S(n) = \text{Min}\ (T + W,\ b) \qquad (2.34)$$

The distribution function for S(n) is defined in terms of the various ranges associated with variables T and W, whose distributions have already been obtained. The probability law of S(n) is a conditional distribution, conditioned on the value of the storage capacity at the end of the previous event. The probability formulation is defined as follows:

$$P[S(n) \leqslant s \mid S(n-1) = s_o] = P[\text{Min}\ (T+W,\ b) \leqslant s] \qquad (2.66)$$

The variable T is bounded by b and s_o while the distribution of W is unbounded, but split among three ranges. To obtain the distribution defined in Equation (2.66) the cumulative distribution of T+W must be obtained first.

The distribution of variable T is given by Equation (2.65). The cumulative distribution of variable W is given by Equation (2.57). Notationally, the cumulative distribution of W is expressed in the following manner:

$$P[W \leq w] = \begin{cases} F_1(W) & \text{if} \quad w \leq k_1 \\ F_2(W) & \text{if} \quad k_1 < w \leq k_2 \\ F_3(W) & \text{if} \quad k_2 \leq w \end{cases} \tag{2.67}$$

where $k_1 = k_2 - cx_c$, and $k_2 = ax_c$.

The cumulative distribution of T+W can be obtained from:

$$P[T + W \leq s] = P[W \leq s-T] \tag{2.68}$$

where s represents the storage variable. The range of the distribution is $(-\infty,\infty)$. This distribution is used to define the distribution in Equation (2.66), for, if $s<b$, it is obtained:

$$P[S(n) \leq s \,|\, S(n-1) = s_o] = P[W \leq s-t] \tag{2.69}$$

The physical bounds on S(n) are no available storage, and full available storage. These are obtained from Equation (2.68). For no avaiable storage s=0, and the probability is given by:

$$P[S(n)=0 \mid S(n-1) = s_o] = P[T + W \leq 0] \qquad (2.70)$$

For full available storage s=b, and the probability is:

$$P[S(n) = b \mid S(n-1) = s_o]=1 - P[T + W < b] \qquad (2.71)$$

The cumulative distribution of T+W depends on the interrelationship of the bounds defining the various expressions for T and W. The variable W is unbounded but divided among three ranges. The variable T is bounded between two positive values. The expressions to be obtained will be grouped according to an index defined from the interaction of the bounds of T and W. The index chosen is:

$$S_b = \text{Min } (b, s_o + k_2) \qquad (2.72)$$

The term in the expression compares two volumes, the volume capacity of the storage unit, and the sum of the available storage capacity at the end of the previous event and the extra capacity made available during the additional runoff event duration. The term $k_2 = ax_c$ represents an increase in the available capacity to that at the end of the previous event. This is because of the additional event duration, x_c, which allows an additional volume of water to be withdrawn at rate a. To obtain the distribution of available storage the general expressions for the distribution of T+W are obtained first.

The distribution of T+W for $S_b = b$

Within this case several expressions for the distribution of T+W are obtained, each defined within a particular range of values of s. The first of these is illustrated in Figure 2.7, and shows the range for $s \leq s_o + k_1$. The term $s_o + k_1$ is less than s_o because k_1 is always negative. It represents a potential reduction of the initially available storage capacity due to the runoff rate arriving at the inlet trapping rate over duration x_c. This term reflects the case wherein the surface runoff rate is greater than the trapping rate. The same argument applies for $b + k_1$, where previously the tank was empty.

For the region illustrated in Figure 2.7 the cumulative distribution is obtained through Equation (2.69), and yields:

$$P[T+W \leq s] = \int_{s_o}^{\infty} F_1(s-t)dF_T(t) \qquad ; s \leq s_o+k_1 \quad (2.73)$$

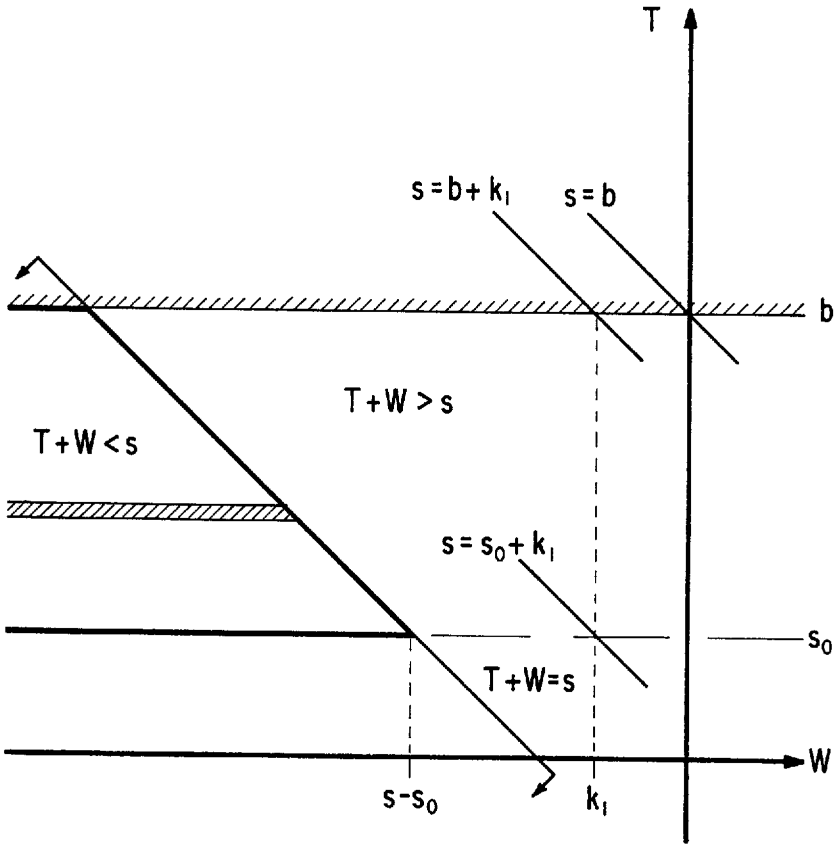


Figure 2.7 Integration Region for the Cumulative Distribution of T + W, in the Range $s<s_o + k_1$, case $S_b = b$.

This and the subsequent expressions are fully evaluated when the storage distribution is defined.

As T+W increases in value it will equate, and then exceed, $s_o + k_1$, and a different expression is obtained up to $b + k_1$. This is illustrated in Figure 2.8. The expression within this range is given by:

$$P[T+W \leqslant s] = \int_{s_o}^{s-k_1} F_2(s-t)\,dF_T(t) + \int_{s-k_1}^{b} F_1(s-t)\,dF_T(t); \quad s_o+k_1 \leqslant s \leqslant b+k_1 \quad (2.74)$$

For the next range T+W will be greater than $b + k_1$, but less than $s_o + k_2$. This is shown in Figure 2.9. The distribution is obtained from:

$$P[T+W \leqslant s] = \int_{s_o}^{b} F_2(s-t)\,dF_T(t) \qquad ; \; b+k_1 < s < s_o+k_2 \quad (2.75)$$

The next range is obtained beyond $s_o + k_2$ but before $b + k_2$. Figure 2.10 indicates that the distribution is obtained from:

$$P[T+W \leqslant s] = \int_{s_o}^{s_o-k_2} F_3(s-t)\,dF_T(t) + \int_{s_o-k_2}^{b} F_2(s-t)\,dF_T(t); \quad s_o+k_2 \leqslant s \leqslant b+k_2 \quad (2.76)$$

Finally, the last range corresponds to T+W being greater than $b + k_2$. This is shown in Figure 2.11. The distribution is obtained from.

$$P[T+W \leqslant s] = \int_{s_o}^{b} F_3(s-t)\,dF_T(t) \qquad ; \; s > b+k_2 \qquad (2.77)$$

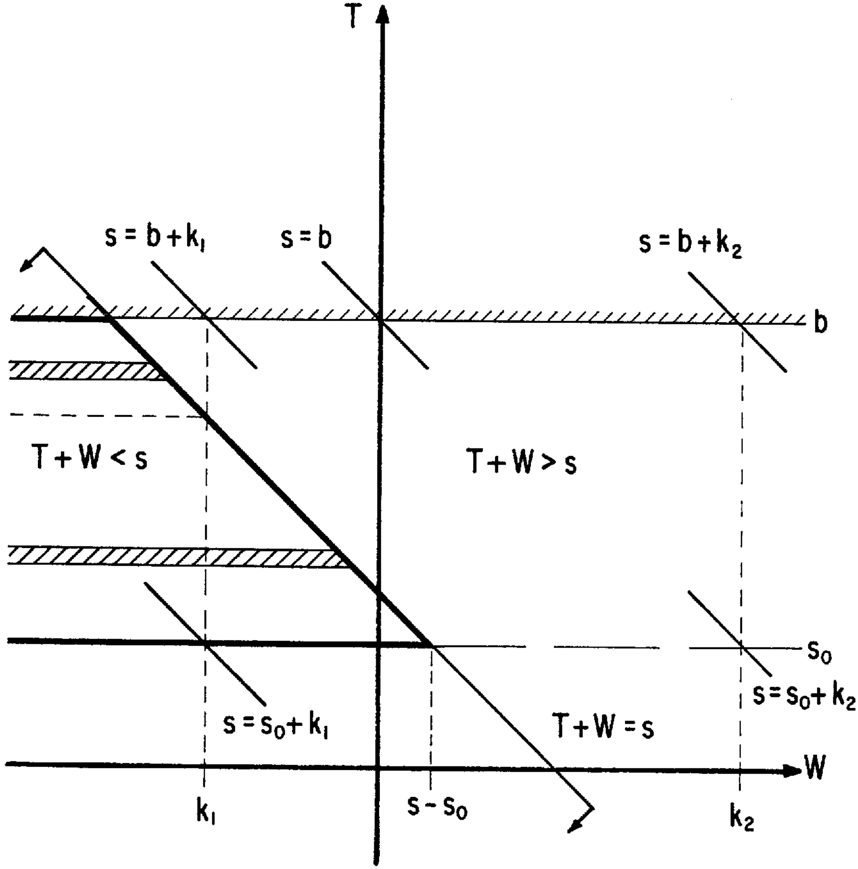


Figure 2.8 Integration Region for the Cumulative Distribution of T + W, in the Range $s_o + k_1 \leqslant s \leqslant b + k_1$, case $S_b = b$.

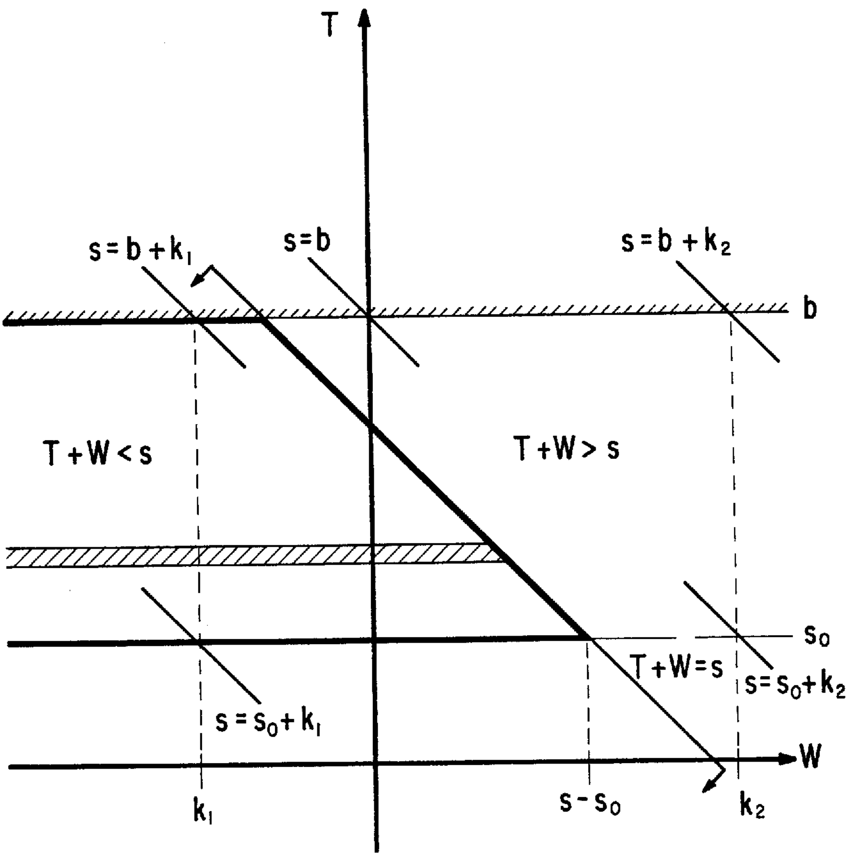


Figure 2.9 Integration Region for the Cumulative Distribution of T + W, in the Range $b + k_1 < s < s_o + k_2$, case $S_b = b$.

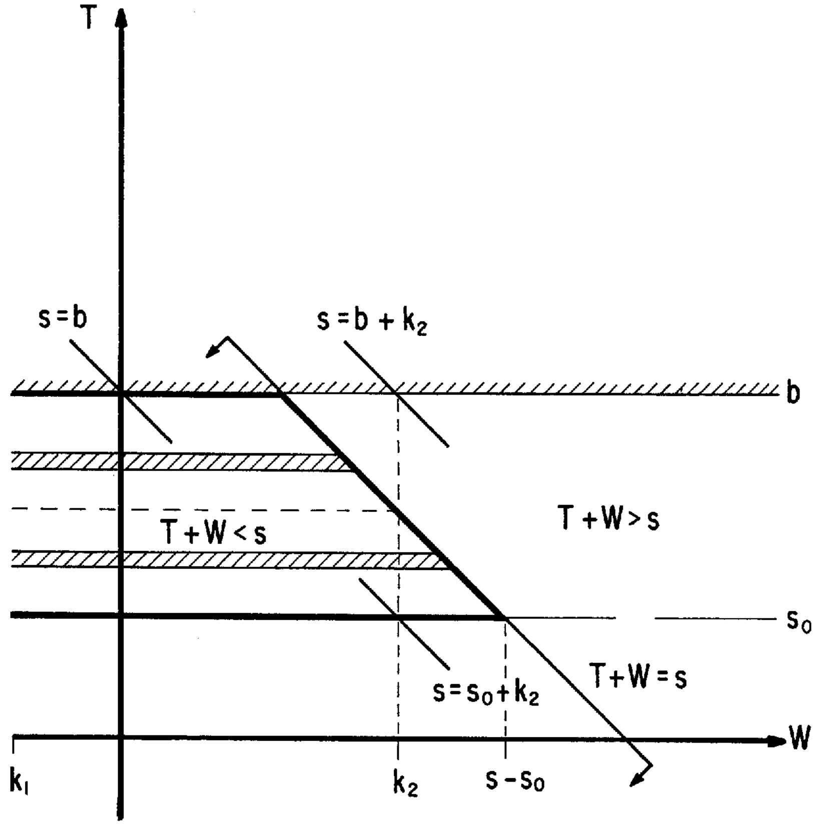


Figure 2.10 Integration Region for the Cumulative Distribution of T + W in the Range $s_o + k_2 \leqslant s \leqslant b + k_2$, case $S_b = b$.

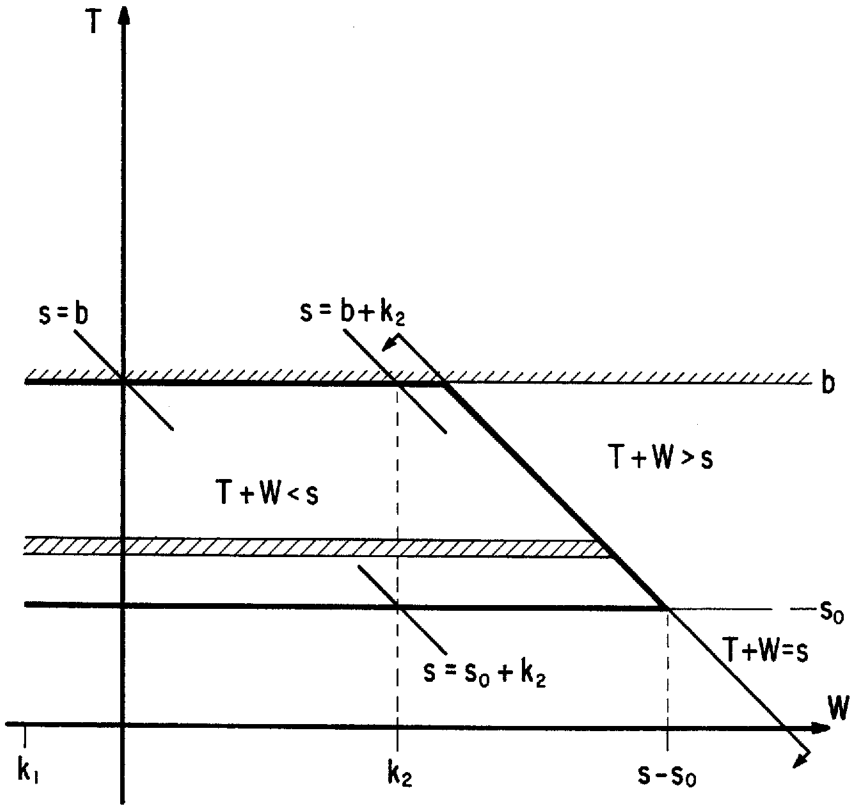


Figure 2.11 Integration Region for the Cumulative Distribution of T + W in the Range $s > b + k_2$, case $S_b = b$.

This completes the formulation for the particular value of the index. The other case occurs when $b > s_o + k_2$.

<u>The distribution of T+W=s for $S_b = s_o+k_2$</u>

Within this case some of the expressions obtained are equivalent to the expressions of the previous section. The first range, that for $s<s_o+k_1$, is obtained from Equation (2.73). This range is not affected by the index.

For the following range two possibilities exist depending on the relative valves of $b+k_1$ and s_o+k_2. If $b+k_1 < s_o + k_2$ the distribution for the range $s_o+k_1 \leqslant s \leqslant b+k_1$ is obtained from Equation (2.74). Within this particular situation the distribution for the next range is the same as shown in Figure 2.9 but now with the restriction $s<s_o+k_2$. Equation (2.75) is used to obtain:

$$P[T+W\leqslant s]=\int_{s_o}^{b} F_2(s-t)dF_T(t) \qquad ; \; b+k_1\leqslant s\leqslant s_o+k_2 \qquad (2.78)$$

The next range would correspond to $s_o+k_2<b+k_2$. Because b is not a limit for W, this case can be obtained from Figure 2.10 and Equation (2.76):

$$P[T+W\leqslant s]=\int_{s_o}^{s-k_2} F_3(s-t)dF_T(t) + \int_{s-k_2}^{b} F_2(s-t)dF_T(t) \qquad ; \qquad (2.79)$$

$$s_o+k_2<s<b+k_2$$

For the last range, $s<b+k_2$, the distribution is obtained from Equation (2.77) as it does not depend on the index.

The other possibility within this case is that $b+k_1 \geq s_o+k_2$. In this situation Equation (2.73) for the range $s<s_o+k_1$ does not change. For the range $s_o+k_1<s<s_o+k_2$ Equation (2.74) is used, within this narrower range. For the range $so+k_2 < s<b+k_1$ the situation is depicted in Figure 2.12. The distribution is given by:

$$P[T+W\leqslant s]=\int_{s_o}^{s-k_2} F_3(s-t)\,dF_T(t) + \int_{s-k_2}^{s-k_1} F_2(s-t)\,dF_T(t)$$
$$+ \int_{s-k_1}^{b} F_1(s-t)\,dF_T(t) \qquad ; \; s_o+k_2<s<b+k_1 \tag{2.80}$$

The next two ranges, $b+k_1 \leqslant s \leqslant b$, and $b<s<b+k_2$ are both obtained from Equation (2.76), which applies within these ranges. The last range, $s \geqslant b+k_2$ is obtained from Equation (2.77).

To summarize, the equations applicable to the various ranges are shown in Figure 2.13. Now proceeds the derivation of the distribution of available storage in terms of the equations of this section.

<u>Distribution of available storage S(n)</u>

The distribution of available storage is obtained from Equation (2.66), in terms of the distribution from the last section. As defined, $S(n)$ is bounded at b, but unbounded in

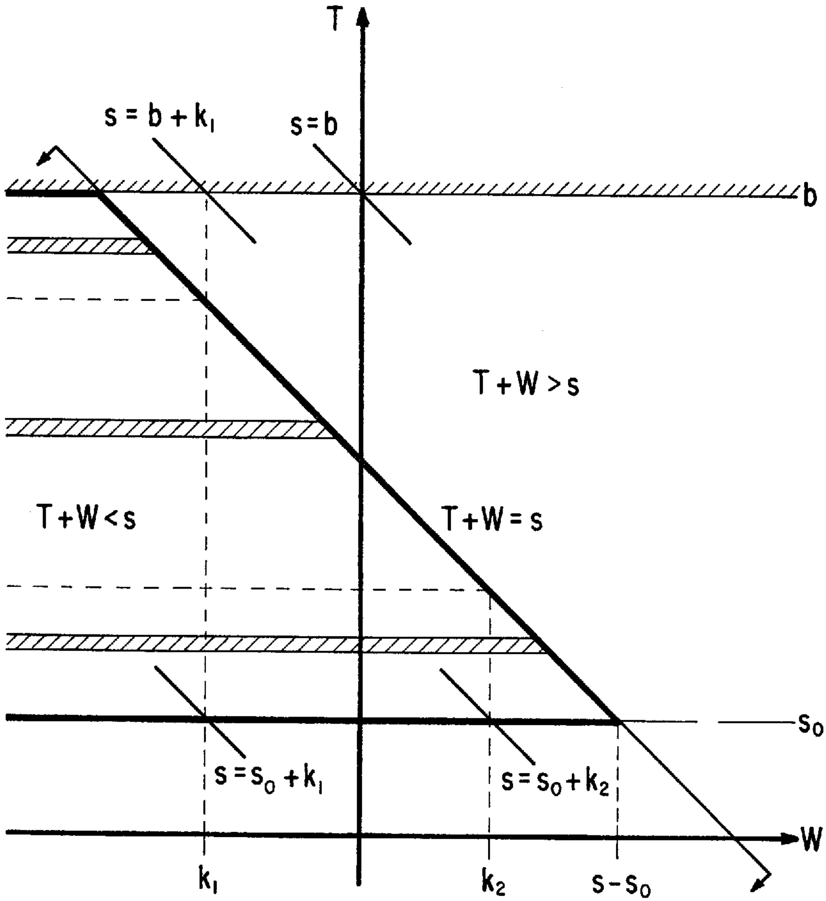


Figure 2.12 Integration Region for the Cumulative Distribution of T + W, in the Range $s_o + k_2 \leqslant s \leqslant b + k_1$, case $S_b = s_o + k_2$; $b + k_1 > s_o + k_2$.

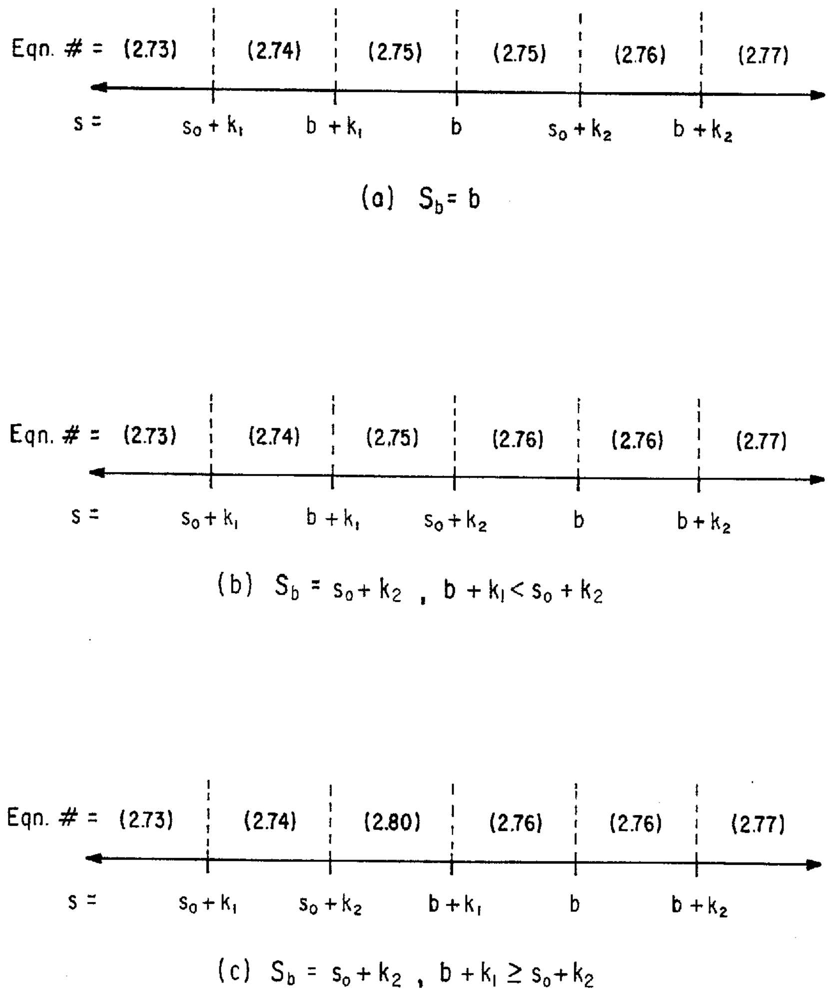


Figure 2.13 Domains of Applicability of the Equations for the Cumulative Distribution of T + W.

the negative direction. Physically, the available storage varies from full available storage to no available storage. The negative values of the storage equation are associated with overflow events but do not represent real storage, only overflow volume. Because of these bounds the storage distribution will have two point mass probabilities at each end of the domain -- those indicated in Equations (2.70) and (2.71). As in the last section the distribution of available storage is obtained according to the index S_b.

Storage distribution for $S_b = b$

The storage distribution is obtained within the bounds s=0 and s=b, with point mass probabilities at these bounds, due to the fact that the probability of T+W is unbounded. For this case the distribution obtained depends on whether the terms s_o+k_1 and $b+k_1$ are positive or negative. The equations to use are those indicated in Figure 2.13(a). It is possible for the value of s=0 to lie between $b+k_1$ and b, between s_o+k_1 and $b+k_1$, or below s_o+k_1. Each case is considered in turn.

For the situation wherein $b+k_1 < 0$ the distribution of S(n) is obtained by integrating Equation (2.75). This yields (see Appendix A for details):

$$P[S(n) \leqslant s \mid S(n-1) = s_o] = H^* \{\exp[-\alpha(s_o-s)] + \frac{\alpha a}{\gamma} \exp[-\alpha(b-s) - \frac{\gamma}{a}(b-s_o)]\} \quad ; 0<s<b \tag{2.81}$$

Where

$$H^* = \frac{\gamma\beta}{(\alpha a + \beta)(\alpha a + \gamma)} \exp(-\alpha k_2) \tag{2.82}$$

The probability of $S(n)=0$ is also obtained from the results of Equation (2.75). It is obtained through Equation (2.70). This is accomplished by setting $s=0$ in Equation (2.81) to obtain:

$$P[S(n)=0 \mid S(n-1)= s_o] = H^* \{\exp(-\alpha s_o) + \frac{\alpha a}{\gamma} \exp[-\alpha b - \frac{\gamma}{a}(b-s_o)]\} \quad ; \; b+k_1<0 \tag{2.83}$$

The other situation is for $0<b+k_1$. It is seen from Figure 2.13(a) that Equation (2.75), and thus Equation (2.81) is used for $b+k_1 \leqslant s<b$. For $s<b+k_1$ Equation (2.74) is integrated to yield:

$$P[S(n) \leqslant s \mid S(n-1) = s_o] = H \{\exp[-\frac{\gamma}{a}(s-s_o) + (\frac{\gamma}{a} - g)k_1] - \frac{ag}{\gamma} \exp[-g(s-b) - \frac{\gamma}{a}(b-s_o)]\} + H^* \{\exp[-\alpha(s-s_o)] - \exp[-\frac{\gamma}{a}(s-s_o) + (\alpha + \frac{\gamma}{a})k_1]\} \quad ; \; 0<s<b+k_1 \tag{2.84}$$

Where
$$H= \frac{\beta\gamma}{(\alpha a + \beta)(\gamma - ag)} \exp(\beta x_c) \tag{2.85}$$

$$g= \frac{\alpha c + \beta}{a-c} \tag{2.86}$$

The probability of zero available storage is obtained by setting s=0 in the previous equation:

$$\begin{aligned} P[S(n)= 0 | S(n-1)= s_o] &= H \{\exp[\tfrac{\gamma}{a} s_o + (\tfrac{\gamma}{a} - g)k_1] - \\ &\tfrac{ag}{\gamma} \exp[gb - \tfrac{\gamma}{a}(b-s_o)]\} + H^* \{\exp(\alpha s_o) - \\ &\exp[\tfrac{\gamma}{a} s_o + (\alpha + \tfrac{\gamma}{a})k_1]\} \quad ; \ s_o+k_1<0<b+k_1 \end{aligned} \tag{2.87}$$

The last situation is for $0<s_o+k_1$. From Figure 2.13(a) and the previous results it is evident that Equations (2.81) and (2.84) are used for $b+k_1<s<b$ and $s_o+k_1 \leqslant s \leqslant b + k_1$, respectively. For $s<s_o+k_1$ Equation (2.73) is integrated to yield:

$$\begin{aligned} P[S(n-1)=s | S(n-1)=s_o] &= H \{\exp[-g(s-s_o)] - \\ &\tfrac{ag}{\gamma} \exp[-g(s-b) - \tfrac{\gamma}{a}(b-s_o)]\} \quad ; \ 0<s<s_o+k_1 \end{aligned} \tag{2.88}$$

The probability of zero available storage is found by substituting s=0 in Equation (2.88):

$$\begin{aligned} P[S(n)=0 | S(n-1)= s_o] &= H \{\exp(gs_o) - \\ &\tfrac{ag}{\gamma} \exp[gb - \tfrac{\gamma}{a}(b-s_o)]\} \quad ; \ 0<s_o+k_1 \end{aligned} \tag{2.89}$$

The remaining formulation for this case is that of the probability of fully available storage capacity. This has the same expression for all the situations in this case, and is obtained through Equation (2.71). From Figure 2.13(a), Equation (2.75) is applicable, and at s=b yields, through Equation (2.71):

$$P[S(n)=b \mid S(n-1)=s_o] = 1 - H^* \{ \exp[-\alpha(s_o-b)] + \frac{\alpha a}{\gamma} \exp[-\frac{\gamma}{a}(b-s_o)]\} \quad ; S_b=b \tag{2.90}$$

Equation (2.90) exhibits a peculiarity in the sense that if $x_c \longrightarrow 0$, then $b=s_o$ since it is impossible to have $b<s_o$, that is, the storage capacity cannot be less than the previous storage capacity available. Under these conditions Equation (2.90) becomes:

$$P[S(n)=b \mid S(n-1)=b] = \frac{\alpha a}{\alpha a + \beta} \quad ; x_c=o \tag{2.91}$$

It is noted that Equation (2.91), under the same conditions, corresponds to $P[W \geqslant 0]$, obtained from Equation (2.57c). Since $W = aX_2-Z$, the condition $W \geqslant 0$ implies that the volume of water withdrawn from the unit is greater than the arriving runoff volume.

Storage distribution for $S_b = s_o+k_2$

For this case the equations indicated in Figure 2.13 (b,c) are used. First to be considered is the situation in Figure 2.13(b). The consideration of the placement of s=0 is similar to the previous case. For the situation $b+k_1<0$ Equation (2.81) applies up to $s=s_o+k_2$. For $s>s_o+k_2$ Equation (2.76) is integrated to yield:

$$P[S(n)\leq s \mid S(n-1)=s_o] = 1 + H^* \{ \exp[-\tfrac{\gamma}{a}(s-s_o-k_2) + \alpha k_2 + \tfrac{\alpha a}{\gamma}\exp[-\alpha(b-s)-\tfrac{\gamma}{a}(b-s_o)]\} - \exp[-\tfrac{\gamma}{a}(s-s_o-k_2)] - Q\{\exp[-\tfrac{\beta}{a}(s-s_o)] - \exp[-\tfrac{\gamma}{a}(s-s_o-k_2)-\beta x_c]\} \quad ; \; s_o+k_2<s<b \tag{2.92}$$

$$\text{Where} \qquad Q = \frac{\alpha a\gamma}{(\alpha a+\beta)(\gamma-\beta)} \exp(\beta x_c) \tag{2.93}$$

The probability of zero available storage is given by Equation (2.83).

The next possibility within this situation is given by $s_o+k_1<0<b+k_1$. From Figure 2.13(b) it is evident that the equations correspond to those for the case $S_b=b$, except for the range expressed in Equation (2.92). A similar observation applies to the situation $0<s_o+k_1$. The probability for full available storage capacity is obtained from Equation (2.76), in a manner analogous to that of the last section. The operation yields:

$$P[S(n)=b \mid S(n-1)=s_o] = \exp[-\tfrac{\gamma}{a}(b-s_o-k_2)] - H^* \{\exp[-\tfrac{\gamma}{a}(b-s_o-k_2) + \alpha k_2] + \tfrac{\alpha a}{\gamma}\exp[-\tfrac{\gamma}{a}(b-s_o)]\} + Q\{\exp[-\tfrac{\beta}{a}(b-s_o)] - \exp[-\tfrac{\gamma}{a}(b-s_o-k_2)-\beta x_c]\} \quad ; \; S_b = s_o+k_2 \tag{2.94}$$

The other situation is illustrated in Figure 2.13(c).

The only difference to Figure 2.13(b) is that now $s_o+k_2 < b+k_1$. This means that now only two sets of equations will be obtained. Everything will remain the same except for the expression corresponding to the integration of Equation (2.80), which yields:

$$
\begin{aligned}
P[S(n)\leqslant s \mid S(n-1)= s_o] &= 1 + H\,\{\exp[-\tfrac{\gamma}{a}(s-s_o) + (\tfrac{\gamma}{a}-g)k_1] - \\
&\tfrac{ag}{\gamma}\exp[-g(s-b)-\tfrac{\gamma}{a}(b-s_o)]\} + H^*\,\{\exp[-\tfrac{\gamma}{a}(s-s_o) + (\gamma+\alpha a)x_c] \\
&-\exp[(\alpha+\tfrac{\gamma}{a})k_1 - \tfrac{\gamma}{a}(s-s_o)]\} - \exp[-\tfrac{\gamma}{a}(s-s_o-k_2)] \qquad (2.95) \\
&-Q\,\{\exp[-\tfrac{\beta}{a}(s-s_o)]-\exp[-\tfrac{\gamma}{a}(s-s_o-k_2)-\beta x_c]\} \quad ; \ s_o+k_2<s<b+k_1
\end{aligned}
$$

The other ranges remain the same, but now only two possible locations for s=0 exist. These are illustrated in Table 2.1 where a summary is presented of the basic and derived storage equations for all cases and conditions.

The probability density function of the available storage is obtained by differentiating the cumulative distribution. However, this is of less present interest than the distribution of overflows, and only the latter will be obtained.

Table 2.1 Summary of Equations for the Distribution of Available Storage

Case	Given Condition	Basic Equation No. with Range	Final Integrated Equation No. with Range
$S_b = b$	$s_0+k_1 < 0$ and $b+k_1 < 0$	2.75 $0 < s < b$	2.81 ; 2.83(s = 0) ; 2.90(s = b)
	$s_0+k_1 < 0$ and $b+k_1 > 0$	2.74 $0 < s < b+k$	2.84 ; 2.87(s = 0)
		2.75 $b+k_1 < s < b$	2.81 ; 2.90(s = b)
	$s_0+k_1 > 0$	2.73 $0 \quad s < s_0+k_1$	2.88 ; 2.89(s = 0)
		2.74 $s_0+k_1 < s < b+k_1$	2.84
		2.75 $b+k_1 < s < b$	2.81 ; 2.90(s = b)
$S_b = s_0+k_2$ and $b+k_1 < s_0+k_2$	$s_0+k_1 < 0$ and $b+k_1 < 0$	2.75 $0 < s < s_0+k_2$	2.81 ; 2.83(s = 0)
		2.76 $s_0+k_2 < s < b$	2.92 ; 2.94(s = b)
	$s_0+k_1 < 0$ and $b+k_1 > 0$	2.74 $0 < s < b+k_1$	2.84 ; 2.87(s = 0)
		2.75 $b+k_1 < s < s_0+k_2$	2.81
		2.76 $s_0+k_2 < s < b$	2.92 ; 2.94(s = b)
	$s_0+k_1 > 0$	2.73 $0 < s < s_0+k_1$ 2.75 $b+k_1 < s < s_0+k_2$	2.88 ; 2.81 ; 2.89 (s = 0)
		2.74 $s_0+k_1 < s < b+k_1$ 2.76 $s_0+k_2 < s < b$	2.84 ; 2.92; 2.94 (s = b)
$S_b = s_0+k_2$ and $b+k_1 > s_0+k_2$	$s_0+k_1 < 0$	2.74 $0 < s < s_0+k_2$	2.84 ; 2.87 (s = 0)
		2.80 $s_0+k_2 < s < b+k_1$	2.95
		2.76 $b+k_1 < s < b$	2.92 ; 2.94(s = b)
	$s_0+k_1 > 0$	2.73 $0 < s < s_0+k_1$	2.88 ; 2.89(s = 0)
		2.74 $s_0+k_1 < s < s_0+k_2$	2.84
		2.80 $s_0+k_2 < s < b+k_1$	2.95
		2.76 $b+k_1 < s < b$	2.92 ; 2.94(s = b)

CHAPTER III

The Distribution of Storage Overflow

3.1 Introduction

An overflow from the storage unit is produced whenever the incoming runoff volume is greater than the available storage capacity. Because the term Z, the incoming runoff volume, carries a negative sign in the storage equation, an overflow condition, obtained whenever Z is greater than the available storage capacity will produce a negative value of the storage equation. The magnitude of this value is equivalent to the magnitude of the storage overflow.

Letting Y(n) denote the magnitude of the storage overflow from event n, it is related to the magnitude obtained from the storage equation as follows:

$$Y(n) = \begin{cases} -S(n) & \text{if} \quad S(n)<0 \\ 0 & \text{if} \quad S(n)\geq 0 \end{cases} \tag{3.1}$$

The cumulative distribution of Y(n) is obtained from that of S(n) through the derived distribution approach. The relationships that define the distribution of overflows are given by:

$$P[Y(n) \leq y] = \begin{cases} 0 & \text{if} \quad y<0 \\ P[S(n) \geq 0] & \text{if} \quad y=0 \\ P[S(n) > -y] & \text{if} \quad y>0 \end{cases} \qquad (3.2)$$

The fundamental expressions obtained in Chapter II are used to obtain the distribution of the overflow.

3.2 The Distribution of the Storage Overflow

The occurrence of an overflow is related to a negative value of the storage variable. The particular expression that is obtained for the probability of an overflow depends on the particular fundamental expression which applies for that case. The cases correspond to the way the basic process variables are defined and the expressions for the distribution of storage were obtained according to a given set of conditions. These conditions apply to the distribution of overflow, but on a restricted basis because the concern is now with the negative range of the storage variable.

The conditions for the distribution of the overflow are defined by the values of the terms s_o+k_1 and $b+k_1$, since these are the only ones that can be negative. Figure 2.13 indicates that the basic equations used to obtain the distributions are Equations (2.73), (2.74), and (2.75). The resulting expressions can be classified according to the scheme used for Table 2.1, but all entries are generated by three basic equations.

With the given conditions three possibilities exist for the distribution of the overflow $Y(n)$. These are illustrated in Figure 3.1, which depicts Equation (3.1). For each of the cases the distribution is obtained via Equation (3.2). The distributions are obtained, in turn, for each particular case.

<u>Distribution for condition $b+k_1 \leq 0$</u>

This condition is illustrated in Figure 3.1(a). The equations that apply are Equations (2.73), (2.74), and (2.75). With these, the storage distributions given in Equations (2.88), (2.84), and (2.81), respectively, were obtained. To obtain the distribution of storage overflow the relationship in Equation (3.1) is used to obtain:

$$P[Y(n) \leq s \,|S(n-1) = s_o] = P[S(n) \geq -s \,|S(n-1) = s_o] \qquad (3.3)$$

for $s \geq 0$

To obtain the expression in Equation (3.2) either integration of the basic equations or direct substitution of $y= -s$ in the storage equations will yield the overflow distribution. The first range corresponds to basic equation number (2.75):

$$P[T + W \leq s] = \int_o^s F_2(s-t)dF_T(t) \qquad (2.75)$$

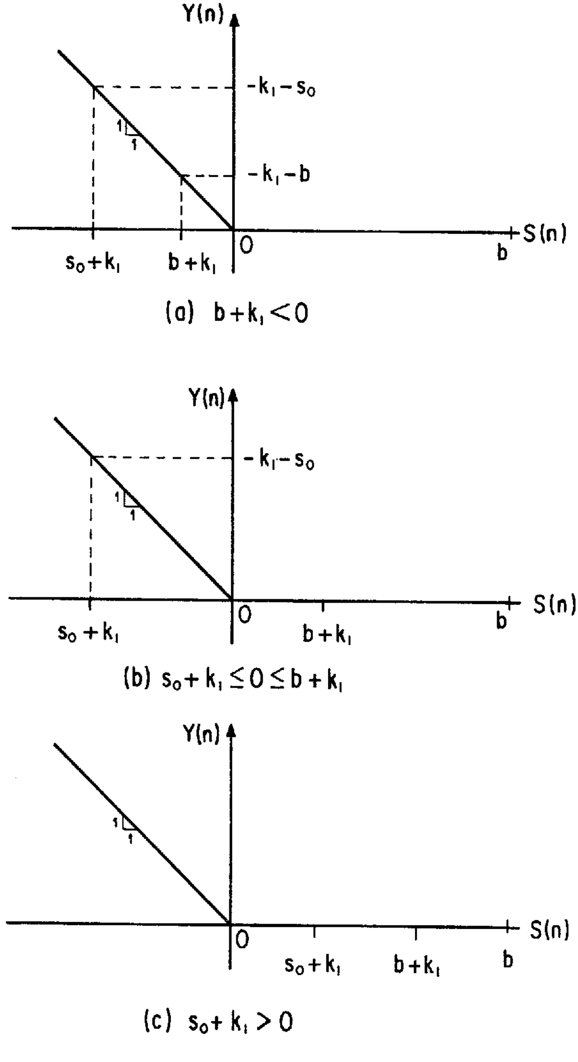


Figure 3.1 Relationship between Y(n) and S(n)

Integration of this equation, or, more conveniently, direct substitution of s= -y in Equation (2.81), which is a result of Equation (2.75), yields:

$$P[Y(n) \geq y | S(n-1) = s_o] = H^* \{ \exp[-\alpha(s_o + y)] + \frac{\alpha a}{\gamma} \exp[-\alpha(b+y) - \frac{\gamma}{a}(b-s_o)] \} \quad ; \; 0<y<-k_1-b \tag{3.4}$$

The next range corresponds to basic equation number (2.74):

$$P[T + W \leq s] = \int_0^{b+k_1} F_2(s-t)dF_T(t) + \int_{b+k_1}^{s} F_1(s-t)dF_T(t) \tag{2.74}$$

Integration of the equation, or, direct substitution of s= -y in the resulting Equation (2.84) yields:

$$P[Y(n) \geq y | S(n-1) = s_o] = H\{\exp[\frac{\gamma}{a}(y+s_o + (\frac{\gamma}{a} - g)k_1] - \frac{ag}{\gamma} \exp[g(y+b) - \frac{\gamma}{a}(b-s_o)]\} + H^* \{\exp[-\alpha(s_o+y)] - \exp[\frac{\gamma}{a}(y+s_o) + (\alpha + \frac{\gamma}{a})k_1]\} \quad ; -k_1-b \leq y \leq -k_1-s_o \tag{3.5}$$

The last range within this case corresponds to Equation (2.73):

$$P[T + W \leq s] = \int_{s_o+k_1}^{s} F_1(s-t)dF_T(t) \tag{2.73}$$

As before, substitution of $s = -y$ in the resulting Equation (2.88) yields:

$$P[Y(n) \geq y | S(n-1) = s_o] = H \; \{\exp[g(y+s_o)] - \frac{ag}{\gamma}\exp[g(y+b) - \frac{\gamma}{a}(b-s_o)]\} \quad ; \; y > -k_1 - s_o \tag{3.6}$$

The probability of no overflow is obtained from the expression:

$$P[Y(n) = 0 | S(n-1) = s_o] = 1 - P[Y(n) > 0 | S(n-1) = s_o] \tag{3.7}$$

The expression on the right is equivalent to $P[S(n) \geq 0 | S(n-1) = s_o]$. Using $y=0$ in Equation (3.4), and substitution in Equation (3.7) yields:

$$P[Y(n)=0 | S(n-1)=s_o] = 1-H^* \{\exp(-\alpha s_o)+\frac{\alpha a}{\gamma}\exp[-\alpha b-\frac{\gamma}{a}(b-s_o)]\} \tag{3.8}$$

The probability density function corresponding to the continuous part of the cumulative distribution function is obtained through differentiation of the continuous part. Differentiation of Equation (3.4), after rearranging to $P[Y(n) \leq y | S(n-1) = s_o]$, yields:

$$f_Y(y) = H^* \{\alpha \exp[-\alpha(y+s_o)] + \frac{\alpha a}{\gamma} \exp[-\alpha(b+y) - \frac{\gamma}{a}(b-s_o)]\} \quad ; \; 0 < y < -k_1 - b \tag{3.9}$$

A similar operation yields, for Equation (3.5):

$$f_Y(y) = H\{ -\frac{\gamma}{a}\exp[\frac{\gamma}{a}(y+s_o) + (\frac{\gamma}{a}-g)k_1] + \frac{ag^2}{\gamma}\exp[g(y+b) - \frac{\gamma}{a}(b-s_o)]\} + H^*\{\alpha\exp[-\alpha(y+s_o) + \frac{\gamma}{a}\exp[\frac{\gamma}{a}(y+s_o) + (\alpha+\frac{\gamma}{a})k_1]\} \quad ; -k_1-b \leq y \leq -k_1-s_o \tag{3.10}$$

Lastly, for Equation (3.6):

$$f_Y(y)=H\{-g\exp[g(y+s_o)]+\frac{ag^2}{\gamma}\exp[g(y+b)-\frac{\gamma}{a}(b-s_o)]\}; \quad y>-k_1-s_o \tag{3.11}$$

The probability of $Y(n)=0$ remains the same, as it has a fixed value. All the density functions are still conditioned on the specified value of s_o, although it has not been indicated in the notation, for simplicity.

Distribution for condition $s_o + k_1 < 0$, $b + k_1 > 0$

The distribution for this condition, shown in Figure 3.1(b), can be obtained from the equations of the last section. For the range $0<y<-k_1-s_o$ Equation (3.4) is used. For the probability of no overflow Equation (3.4) is used at $y=0$, and is substituted in Equation (3.7) to yield:

$$P[Y(n)=0|S(n-1)=s_o] = 1-H\{\exp[\frac{\gamma}{a}s_o + (\frac{\gamma}{a}-g)k_1] - \frac{ag}{\gamma}\exp[gb-\frac{\gamma}{a}(b-s_o)]\} - H^*\{\exp(-\alpha s_o) - \exp[\frac{\gamma}{a}s_o + (\alpha+\frac{\gamma}{a})k_1]\} \quad (3.12)$$

For the range $y > -k_1-s_o$ Equation (3.6) is used. The density functions are given by Equations (3.10) and (3.11), within the ranges $0<y<-k_1-s_o$, and $y>-k_1-s_o$, respectively.

Distribution for condition $s_o + k_1 > 0$

For this condition, shown in Figure 3.1(c), only one expression is obtained, corresponding to the range $y>0$ with Equation (3.6). The probability of having an overflow is obtained through Equation (3.6), with the procedure illustrated in the previous sections. The operation yields:

$$P[Y(n)=0|S(n-1)=s_o] = 1-H\{\exp(gs_o) - \frac{ag}{\gamma}\exp[gb-\frac{\gamma}{a}(b-s_o)]\} \quad (3.13)$$

The density function is that given in Equation (3.11), within the range $y>0$.

The equations appropriate for each condition together with the basic model equations are summarized in Table 3.1. The classification is made according to the scheme used in Table 2.1 of Chapter II.

Table 3.1 Summary of Equations for the Distribution of Storage Overflow

Case	Given Condition	Basic Equation No. and Range in Terms of s=-y	Final Integrated Equation Numbers		
			$P[Y(n) \geq y]$	$P[Y(n)=0]$	Density Function
$S_b = b$	$s_0+k_1<0$ and $b+k_1<0$	2.75 $0<y<-k_1-b$	3.4	3.8 $(y=0)$	3.9
		2.74 $-k_1-b<y<-k_1-s_0$	3.5		3.10
		2.73 $-k_1-s_0<y<\infty$	3.6		3.11
	$s_0+k_1<0$ and $b+k_1>0$	2.74 $0<y<-k_1-s_0$	3.5	3.12 $(y=0)$	3.10
		2.73 $-k_1-s_0<y<\infty$	3.6		3.11
	$s_0+k_1>0$	2.73 $0<y<\infty$	3.6	3.13 $(y=0)$	3.11
$S_b=s_0+k_2$ and $b+k_1<s_0+k_2$	$s_0+k_1<0$ and $b+k_1<0$	2.75 $0<y<-k_1-b$	3.4	3.8 $(y=0)$	3.9
		2.74 $-b-k_1<y<-s_0-k_1$	3.5		3.10
		2.73 $-s_0-k_1<y<\infty$	3.6		3.11
	$s_0+k_1<0$ and $b+k_1>0$	2.74 $0<y<-s_0-k_1$	3.5	3.12 $(y=0)$	3.10
		2.73 $-s_0-k_1<y<\infty$	3.6		3.11
	$s_0+k_1>0$	2.73 $0<y<\infty$	3.6	3.13 $(y=0)$	3.11
$S_b=s_0+k_2$ and $b+k_1>s_0+k_2$	$s_0+k_1<0$	2.74 $0<y<-k_1-s_0$	3.5	3.12 $(y=0)$	3.10
		2.73 $-k_1-s_0<y<\infty$	3.6		3.11
	$s_0+k_1>0$	2.73 $0<y<\infty$	3.6	3.13 $(y=0)$	3.11

3.3 Exceedance Probability

The probability of having an overflow is a measure of system performance. For specified values of the storage capacity and the withdrawal rate the probability of having an overflow from a runoff event can be assessed. Conversely, from a specified overflow probability the required value of the storage capacity and withdrawal rate can be determined. This overflow probability is called the exceedance level, and is specified by the following expression:

$$P[Y(n) > 0 | S(n-1) = s_o] \leq \varepsilon \qquad (3.14)$$

Where ε is the exceedance probability, or, within the context of reliability analysis, the associated system risk level.

Another type of exceedance probability has been considered in Chapter 2, referred to as ε_I - the exceedance probability associated with the system runoff-trapping capacity, c. Increasing ε_I reduces the design value of c, and thus the value of ε, because less runoff will enter the stormsewer system. A similar argument can be put forth if variations in ε are considered.

The expressions to use in Equation (3.14) are easily obtained from Equations (3.7), (3.12), and (3.13) through the following expression:

$$P[Y(n) > 0 | S(n-1) = s_o] = 1 - P[Y(n) = 0 | S(n-1) = s_o] \quad (3.15)$$

which is the converse of Equation (3.7).

3.4 A Special Case

Most planning models to date take no account of the controlling effect produced by the inlet trapping system in an urban area. Runoff is assumed to pass completely through the inlets on to the stormsewer system. In other cases catchment flows are treated as overland flow for the purpose of simplifying system configuration - mainly for cases where a large number of simulation runs are to be made. The statistical model proposed here can be simplified to treat these approximations.

If runoff flows overland to a storage unit this can be treated by the statistical model by allowing c, the system trapping capacity, to go to infinity. This assumes that all of the runoff volume will arrive at the storage unit.

Another simplification is to assume that the runoff duration is equivalent to the effective rainfall duration. Also, the mean event duration may be available from records or from simulation studies. This means that it is not necessary to transform the duration via Equation (2.6) because the event duration statistics are directly available from data. Thus, in Equation (2.6), $a_2 = 1$, and $x_c = 0$, and therefore $X_2 = X_2'$. This will greatly simplify the model because only a few equations will be needed.

For the cases discussed here ($c \to \infty$, $X_2 = X_2'$) the cumulative distribution of storage is given by the last entry in Table 2.1. Because, now, $k_1 = k_2 = 0$, and $S_b = s_o$ because $s_o \leq b$ always, only Equations (2.88), (2.89), (2.94), and (2.95) will describe the process. These become the following:

$$P[S(n) = 0 | S(n-1) = s_o] = H \{ \exp(-\alpha s_o) + \frac{a\alpha}{\gamma} \exp[-\alpha b - \frac{\gamma}{a}(b-s_o)] \} \quad (3.16)$$

$$P[S(n) \leq s | S(n-1) = s_o] = H \{ \exp[\alpha (s-s_o)] + \frac{a\alpha}{\gamma} \exp[\alpha(s-b) - \frac{\gamma}{a}(b-s_o)] \} \quad ; \; 0<s<s_o \quad (3.17)$$

$$P[S(n-1) \leq s | S(n-1) = s_o] = 1 - Q \{ \exp[-\frac{\beta}{a}(s-s_o)] - \exp[-\frac{\gamma}{a}(b-s_o)] \} - (1-H) \exp[-\frac{\gamma}{a}(s-s_o)] + \quad (3.18)$$

$$H \frac{a\alpha}{\gamma} \exp[-\alpha (b-s) - \frac{\gamma}{a}(b-s)] \quad ; \; s_o \leq s<b$$

$$P[S(n)=b | S(n-1)=s_o]=Q\{\exp[-\frac{\beta}{a}(b-s_o)]- \frac{\beta}{\gamma}\exp[-\frac{\gamma}{a}(b-s_o)]\} \quad (3.19)$$

The coefficients in the distribution are now given by:

$$H^* = \frac{\gamma\beta_2}{(\alpha a + \beta_2)(\alpha a + \gamma)} \quad (3.20)$$

$$H = H^{*} \tag{3.21}$$

$$Q = \frac{\alpha a \gamma}{(\alpha a + \beta_2)(\gamma - \beta_2)} \tag{3.22}$$

$$g = - \alpha \tag{3.23}$$

It is recalled from Equation (2.18) that $\beta = \beta_2/a_2$; and $a_2 = 1$ for the special case.

The special case model is applicable to situations wherein all surface runoff is trapped and the event duration statistics are available.

CHAPTER IV

The Estimation of the Storage Capacity and Withdrawal Rate

4.1 Introduction

The storage distribution derived in the previous chapter provides the basis by which the interaction between the storage capacity and treatment rate can be assessed. The quantification of this interaction is important from the design point of view because the interaction is studied through the specification of the performance level of the detention unit. Thus the designer can obtain the change in storage capacity due to a change in the withdrawal, or treatment rate. A desired combination can be obtained for some related criteria that would provide the desired level of performance. An optimal combination, taken here as one that will minimize storage and treatment costs, can be obtained if cost is the criterion.

The fundamental consideration related to the operational aspects of the storage unit is the occurrence of overflows. The overflows occur whenever the incoming runoff volume exceeds the available capacity of the storage device. On an event basis, the probability of having an overflow is directly dependent on the value of the available storage at the end of the previous event. The actual value of the previously available storage is unknown and can only be described through a probability distribution. However, consideration of the extreme storage conditions, i.e., previously empty or full, will establish the envelopes within which the system operates to allow an assessment of the range of parameters to be expected.

While the overflow distribution allows the estimation of the probability of overflow for an event, this is of less interest than the study of the long-term behavior of the system. The long-term behavior is related to the percentage of the runoff volume that is captured by the device over a long period of time. The simulation models STORM and SWWM study long-term behavior by simulating the system with a runoff record, generated from actual rainfall data. Generally, only one year of operation is simulated due to the high cost of running these programs. A relative frequency analysis of the captured volume estimates the percent of volume captured for a specified storage capacity and withdrawal rate. Simulation of the system for different combinations of storage and withdrawal rate allows the definition of a storage-treatment isoquant. A storage-treatment isoquant is the locus of storage and treatment, or withdrawal, rates that provide a certain level of volume control.

Storage-treatment isoquants in simulation studies are obtained by interpolating curves through the plotted frequencies of overflow for values of the storage capacity and the treatment rate. The statistical model proposed in this work will use the ratios of expectations as a measure of the percentage of volume control. This is defined as the flow capture efficiency.

4.2 Flow Capture Efficiency

The fraction of the part of the runoff which arrives at the unit but is not captured is defined as:

$$f = \frac{E[Y]}{E[Z]} \tag{4.1}$$

$$0 \leq f \leq 1$$

That is, f represents the long-term fraction of the runoff not captured by the detention unit. It is defined as the ratio of the expectation of the overflow from the storage unit, Y and the expectation of the volume of runoff Z arriving at the unit.

The fraction f can be interpreted as the probability of having an overflow from the system, or the percent not captured. Then 1-f represents the probability of not having an overflow, or flow capture efficiency. The concept of flow capture efficiency has been employed by Heany et al., (1977), DiToro and Small (1979), Hydroscience (1979), and Goforth, et al. (1983).

The value of E[Y] is obtained from the distribution of the overflow:

$$E[Y(n)|S(n-1)=s_o] = \int_0^\infty y f_Y(y)\, dy + 0P[Y(n)=0|S(n-1)=s_o] \tag{4.2}$$

The distributions for Y(n) have been obtained in Chapter III. The particular expression to be obtained for the expectation depends on the set of distributions that apply for the given condition. Table 3.1 shows that for all cases there are three conditions depending on the relative value of s_o, b, and k_1. Three equations for expectation will be obtained, each corresponding to a given condition.

The continuous part of the three probability density functions obtained in Chapter III for Y(n) will be denoted here as follows:

f_1 = Equation (3.9)

f_2 = Equation (3.10)

f_3 = Equation (3.11)

<u>Expectation for condition $b < -k_1,\ s_o < -k_1$</u>

For this condition Equation (3.9), (3.10), and (3.11) are used, and the expectation is obtained through Equation (4.2), which yields:

$$E[Y(n)|S(n-1)= s_o]= \int_{o^+}^{-k_1-b} yf_1dy + \int_{-k_1-b}^{-k_1-s_o} yf_2dy + \int_{-k_1-s_o}^{\infty} yf_3dy \qquad (4.3)$$

The integration yields:

$$E[Y(n)|S(n-1)=s_o] = \frac{H^*}{\alpha}\{\exp(-\alpha s_o) + \frac{\alpha a}{\gamma}\exp[-\alpha b - \frac{\gamma}{a}(b-s_o)]\} - m_I \quad ;\ b<-k_1,\ s_o<-k_1 \qquad (4.4)$$

where $m_I = E[Y_I]$, as defined in Equation (2.43); H^* is defined in Equation (2.82); $k_1 = (a-c)x_c$, and $k_2 = ax_c$.

It is noted that Equation (3.8) is contained in Equation (4.4). Using Equation (3.7), Equation (4.4) can be expressed as:

$$E[Y(n)|S(n-1) = s_o] = \frac{1}{\alpha}P[Y(n)>0|S(n-1) = s_o] - m_I \quad (4.5)$$

The volume fraction not captured is given by:

$$\frac{E[Y(n)|S(n-1) = s_o]}{m_Z} = \frac{1}{\alpha m_Z}P[Y(n) >0 \; | S(n-1)] = s_o \; - \frac{m_I}{m_Z} \quad (4.6)$$

where $m_Z = E[Z]$

If, instead, the total runoff volume $E[X_1]$ is used to define the fraction not captured the expression becomes:

$$\alpha E[Y(n)|S(n-1) = s_o] = P[Y(n) >0|S(n-1) = s_o] - \alpha m_I \quad (4.7)$$

It is noted that $\alpha = 1/E[X_1]$.

Equation (4.7) can be modified further by using the relation $m_I = \frac{1}{\alpha} - m_Z$, from Chapter II, to obtain

$$\alpha E[Y(n) |S(n-1) = s_o] = \alpha \, m_Z - P[Y(n) = 0|S(n-1)= s_o] \quad (4.8)$$

Considering the special case wherein $m_Z = 1/\alpha$, that is, $E[Z] = E[X_1]$, Equation (4.8) yields:

$$\alpha E[Y(n)|S(n-1) = s_o] = P[Y(n) > 0|S(n-1) = s_o] \qquad (4.9)$$

which indicates that the fraction not captured is equivalent to the probability of having an overflow. This is also true for Equation (4.7), but there the probability is reduced by the ratio of the mean value of park storage volume and the mean value of runoff.

Expectation for condition $b \geq -k_1$, $s_o \leq -K_1$

For this condition the expectation is given by:

$$E[Y(n)|S(n-1) = s_o] = \int_0^{-k_1-s_o} y f_2 dy + \int_{-k_1-s_o}^{\infty} y f_3 dy \qquad (4.10)$$

Integration, and division by m_Z will produce the following expression for the fraction of the load not captured:

$$\frac{E[Y(n)|S(n-1)=s_o]}{m_Z} = \frac{aH}{\gamma m_Z}\{\exp[gb - \frac{\gamma}{a}(b-s_o)] - \exp[\frac{\gamma}{a}s_o - (g-\frac{\gamma}{a})k_1]\} +$$

$$\frac{H^*}{\alpha m_Z}\{\exp(-\alpha s_o) + \frac{\alpha a}{\gamma}\exp[\frac{\gamma}{a}s_o + (\alpha+\frac{\gamma}{a})k_1]\} - \frac{m_I}{m_Z}; \qquad (4.11)$$

$$s_o \leq -k_1 \leq b$$

To obtain the corresponding expression of the fraction not captured m_Z is replaced by $1/\alpha$ in the previous equation.

Expectation for condition $b > -k_1, s_o > -k_1$

For this condition the expectation is given by:

$$E[Y(n)|S(n-1) = s_o] = \int_0^\infty yf_3dy \qquad (4.12)$$

Integration, and division by m_Z yields the fraction of the load not captured as:

$$\frac{E[Y(n)|S(n-1)=s_o]}{m_Z} = \frac{H}{m_Z}\{\frac{a}{\gamma}\exp[gb - \frac{\gamma}{a}(b-s_o)] - \frac{1}{g}\exp(gs_o)\} \qquad (4.13)$$

Substituting $1/\alpha$ for m_Z above will yield the fraction not captured in terms of the total runoff volume. Comparing this equation with Equation (3.13) it can now be expressed as:

$$\frac{E[Y(n)|S(n-1)=s_o]}{m_Z} = -\frac{1}{gm_Z}P[Y(n)>0|S(n-1) = s_o] \qquad (4.14)$$

It is noted that $g \leq 0$.

4.3 A Special Case

For the special case considered in section 3.4 ($c \to \infty$, $X_2 = X_2'$) Equation (4.14) is the only one applicable because $k_1 = 0$. The equation is now transformed to:

$$\alpha E[Y(n)|S(n-1) = s_o] = P[Y(n)>0|S(n-1) = s_o] \qquad (4.15)$$

The right-hand side of the equation is the probability of having an overflow. From Equation (3.14) this was called the exceedance probability, so that the fraction of the load not captured is equivalent to the exceedance probability, for the special case. In effect; $f = \varepsilon$. For the general case the relationship is not a direct equivalence. As an illustration consider the equation for the fraction not captured for the first condition, namely Equation (4.6). Taking the probability of having an overflow as being the exceedance probability the equation is written as:

$$f = \frac{\varepsilon}{\alpha m_Z} - \frac{m_I}{m_Z} \qquad (4.16)$$

If the total runoff volume is considered, as given in Equation (4.7), the relationship becomes:

$$f_t = \varepsilon - \alpha m_I \qquad (4.17)$$

Where f_t refers to the fraction of the load not captured in terms of the total runoff.

The fraction not captured is seen to be related to the flow capture efficiency and the statistics of the runoff process.

4.4 Equations for Flow Capture Efficiency

Equations have been derived for the estimation of the long-term fraction of the runoff not captured by storage.

These are used to estimate the flow capture efficiency. Two definitions of flow capture efficiency are possible - one corresponding to m_Z, the mean runoff volume collected by the storm sewer; and another corresponding the $E[X_1] = 1/\alpha$, the total mean runoff volume, of which only a certain fraction is available for storage. The capture efficiency using m_Z is defined as ρ, and is given by:

$$1-f = \rho \tag{4.18}$$

The capture efficiency using $E[X_1]$ is called the total capture efficiency, and is defined by $_t$ through the relation:

$$1-f_t = \rho_t \tag{4.19}$$

For each of the conditions in Section 4.2 an equation for flow capture efficiency is obtained. These are shown in Table 4.1. From these the treatment rate and storage capacity are estimated.

4.5 Storage Estimation

The equation obtained for the flow capture efficiency serve as the basis for estimating the storage capacity and treatment rate necessary to obtain a specified level of performance, in terms of the capture efficiency. The locus of storage capacity and treatment rate combinations that

Table 4.1

Equations of Flow Capture Efficiency

Condition	Capture Efficiency $\rho =$	Total Capture Efficiency $\rho_t =$
$b,\ s_o < -k_1$	$\frac{1}{\alpha\, m_Z} P_1$	P_1
$b \geq -k_1$ $s_o \leq -k_1$	$\frac{1}{\alpha\, m_Z} - \frac{1}{m_Z} P_2$	$1 - \alpha P_2$
$b,\ s_o \geq -k_1$	$1 - \frac{1}{m_Z} P_3$	$1 - \alpha P_3$

$$P_1 = 1 - H^* \{\exp(-\alpha s_o) + \frac{\alpha a}{\gamma} \exp[-\alpha b - \frac{\gamma}{a}(b - s_o)]\}$$

$$P_2 = \frac{Ha}{\gamma}\{\exp[gb - \frac{\gamma}{a}(b - s_o)] - \exp[\frac{\gamma}{a} s_o - (g - \frac{\gamma}{a})k_1]\} + \frac{H^*}{\alpha}\{\exp(-\alpha s_o) + \frac{\alpha a}{\gamma}\exp[\frac{\gamma}{a} s_o + (\alpha + \frac{\gamma}{a})k_1]\}$$

$$P_3 = H\{\frac{a}{\gamma}\exp[gb - \frac{\gamma}{a}(b - s_o)] - \frac{1}{g}\exp(gs_o)\}$$

$$m_Z = \frac{1}{\alpha} - \frac{\beta}{\alpha(\alpha c + \beta)}\exp(-\alpha c x_c)$$

produce a specific value of the capture efficiency is called the storage/treatment isoquant. The storage/treatment isoquant is obtained from the derived equations by solving for the storage capacity in terms of the treatment rate, the capture efficiency, and the runoff parameters.

The isoquants are conditioned on the value of s_o, the previously available storage capacity. Because this value is not known with certainty the isoquants are determined for specific, assumed, values of s_o. Since $0 \leq s_o \leq b$ the two extreme points of the isoquants corresponding to previously empty and previously full conditions can be obtained. Intermediate isoquants can be obtained by letting $s_o = \delta b$, where $0 \leq \delta \leq 1$. For a value of $\delta = 0$ no available storage space remains after the end of the previous event. For $\delta = 1$ the full storage space is available at the end of the previous event. For all other values of an intermediate condition is obtained.

The isoquants are defined by the requirement that the storage capacity provides a desired level of capture efficiency - an acceptable level of performance. Values of the storage and treatment rate that provides this level of performance will trace the isoquant. In a general sense defining the isoquant corresponds to finding the roots of the following equation:

$$h(b, \theta) = 0 \tag{4.20}$$

The above is an homogeneous function of the storage capacity and a parameter set $\theta = \theta[a, \rho$ (or ρ_t), α, β, γ, x_c, $c, \delta]$. The expressions for Equation (4.20) depend on the particular condition considered.

4.5.1 Storage equations for condition $b < -k_1$, $s_o < -k_1$

From Table 4.1 the expression that defines the isoquant, written according to Equation (4.20), is given by:

$$P_1(\delta b) - \rho\alpha\, m_Z = 0 \tag{4.21}$$

Where $P_1(\delta b)$ is the expression for P_1 evaluated at $s_o = \delta b$, and is given by:

$$P_1(\delta b) = 1 - H^* \{ \exp(-\alpha\delta b) + \frac{\alpha a}{\gamma} \exp[-(\alpha + \frac{\gamma}{a}(1-\delta))\, b] \} \tag{4.22}$$

The corresponding expression for the total capture efficiency is given by:

$$P_1(\delta b) - \rho_t = 0 \tag{4.23}$$

If initially the tank is empty, then $\delta = 1$, and Equations (4.21) and (4.23) can be solved explicitly for the storage capacity to yield:

$$b = \frac{1}{\alpha} \ln \left[\frac{\beta}{(\alpha a + \beta)(1 - \alpha\rho m_Z)} \right] - a x_c \tag{4.24}$$

for the capture efficiency, and:

$$b = \frac{1}{\alpha} \ln \left[\frac{\beta}{(\alpha a + \beta)(1 - \rho_t)} \right] - a x_c \tag{4.25}$$

for the total capture efficiency.

If initially the tank is full, then $\delta = 0$; and Equations (4.21) and (4.23) can be solved explicitly to yield, respectively:

$$b = \frac{a}{(\alpha a + \gamma)} \ln \left[\frac{H^* \alpha a}{(1 - H^* - \alpha \rho m_z)\gamma} \right] \tag{4.26}$$

and

$$b = \frac{a}{(\alpha a + \gamma)} \ln \left[\frac{H^* \alpha a}{(1 - H^* - \rho_t)\gamma} \right] \tag{4.27}$$

It is noted that when the tank is initially empty, the expressions obtained in Equations (4.24) and (4.25) are not in terms of γ, the inverse of the mean of the interevent time. This is so because a tank initially empty will remain empty until the next event arrives, and the process will not be a function of interevent time statistics. This holds true for all situations.

For the intermediate cases ($0<\delta<1$) no explicit solution is possible and the value of the storage capacity must be obtained by an appropriate numerical procedure.

4.5.2 Storage equations for condition $b \geq -k_1$, $s_o \leq -k_1$

For this condition Table 4.1 indicates that the isoquant for the capture efficiency is given by:

$$\frac{1}{\alpha} - P_2(\delta b) - \rho m_Z = 0 \qquad (4.28)$$

Where $P_2(\delta b)$ is the function P_2 at $s_o = \delta b$.

For the total efficiency the isoquant is defined by:

$$\frac{1}{\alpha}(1-\rho_t) - P_2(\delta b) = 0 \qquad (4.29)$$

If the storage is initially empty, $\delta = 1$ means that $s_o = b$. In this case, however, because of the constraints associated with the given condition the only possibility is that $s_o = b = -k_1$. With these values Equations (4.28) and (4.29) can be solved explicitly to yield, respectively:

$$b = \frac{1}{\alpha} \ln \left[\frac{\beta}{(\alpha a + \beta)(1 - \alpha\rho m_Z)} \right] - a x_c \qquad (4.30)$$

and

$$b = \frac{1}{\alpha} \ln \left[\frac{\beta}{(\alpha a + \beta)(1 - \rho_t)} \right] - a x_c \qquad (4.31)$$

The above expressions are the same as Equations (4.24) and (4.25). Equations (4.30) and (4.31) are used to obtain the value of the treatment rate at $b = -k_1$.

If the unit is initially full, $\delta = 0$, and this allows for an explicit solution of the isoquant in terms of the storage capacity. The solution for the capture efficiency is given by:

$$b = \frac{a}{(\gamma - ag)} \ln \{ (\frac{1}{\alpha} - \rho\, m_Z) \frac{\gamma}{Ha} + \exp[(g + \frac{\gamma}{a})k_1] - \frac{\gamma H^*}{\alpha a H} [1 + \frac{\alpha a}{\gamma} \exp[(g + \frac{\gamma}{a})k_1]] \} \tag{4.32}$$

For the total capture efficiency the term $(\frac{1}{\alpha} - \rho\, m_Z)$ is replaced by the term $\frac{1}{\alpha}(1-\rho_t)$ in the previous equation.

4.5.3 Storage equations for condition $b > -k_1$, $s_o > -k_1$

The expression in Table 4.1 for this case yields, for the capture efficiency and total capture efficiency, respectively:

$$(1 - \rho) m_Z - P_3(\delta b) = 0 \tag{4.33}$$

and

$$(1 - \rho_t) - \alpha P_3(\delta b) = 0 \tag{4.34}$$

Where $P_3(\delta)$ is the function P_3 at $s_o = \delta b$.

If, initially, the storage unit is empty, then $\delta = 1$, and Equations (4.33) and (4.34) are solved explicitly to yield, respectively:

$$b = \frac{1}{g} \ln \left[- \frac{g(\alpha a + \beta)(1 - \rho) m_Z}{\beta} \right] - \frac{\beta x_c}{g} \tag{4.35}$$

and

$$b = \frac{1}{g} \ln \left[- \frac{g(\alpha a + \beta)(1 - \rho_t)}{\alpha \beta} \right] - \frac{\beta x_c}{g} \tag{4.36}$$

If the tank is initially full, $\delta = 0$, and Equations (4.33) and (4.34) can be solved explicitly to yield:

$$b = \frac{a}{ag - \gamma} \ln \left[\frac{\gamma\, m_Z (1 - \rho)}{aH} + \frac{\gamma}{ag} \right] \tag{4.37}$$

and

$$b = \frac{a}{ag - \gamma} \ln \left[\frac{\gamma (1 - \rho_t)}{\alpha\, aH} + \frac{\gamma}{ag} \right] \tag{4.38}$$

This completes the development of the formulation for isoquant estimation. The equations obtained are termed "hydraulic" because they treat runoff volumes only. Their application is illustrated in a subsequent chapter. Lastly, the special case is considered.

4.6 A Special Case

The special situation is the simplified case that has been defined previously, namely, that in which $c \to \infty$, $X_2 =$

X_2'. Because $k_1 = 0$ for this case, only the equations given in Section 4.5.3 are applicable, and only those for the total capture efficiency. The general expression, Equation (4.34), applies, but now $P_3(\delta b)$ is given by:

$$P_3(\delta b) = \frac{\beta\gamma}{(\alpha a+\beta)(\alpha a+\gamma)} \left\{\frac{1}{\alpha} \exp(-\alpha\delta b) + \frac{a}{\gamma} \exp\left[-(\alpha + \frac{\gamma}{a}(1-\delta)b\right]\right\} \qquad (4.39)$$

For the initially empty condition, Equation (4.36), the expression now becomes:

$$b = \frac{1}{\alpha} \ln \left[\frac{\beta}{(\alpha a + \beta)(1 - \rho_t)} \right] \qquad (4.40)$$

And, for the initially full storage condition, Equation (4.38), the expression becomes:

$$b = \frac{a}{\alpha a + \gamma} \ln \left[\frac{\alpha a \beta}{(\alpha a + \beta)(\alpha a + \gamma)(1 - \rho_t) - \beta\gamma} \right] \qquad (4.41)$$

The special case is a much simpler situation and represents a considerable saving in computational effort over the general case.

CHAPTER V

Pollutant Load Model

5.1 Introduction

The water quality aspect of urban stormwater runoff is receiving considerable attention. It has become evident that runoff is a source of pollutants comparable at times to other major sources of waste flow. The fact has been established from many assessment projects, like those of the University of Cincinnati, 1970; Sartor and Boyd, 1972; McElroy et al., 1976; Heany et al., 1977; Jewell, 1980; Water Planning Division - E.P.A., 1983; and many other regional and municipal assessments.

The Nationwide Urban Runoff Program (NURP), conducted by the Water Planning Division of the Environmental Protection Agency (1983) has addressed the planning and the quantification problem related to urban stormwater pollution. They have identified the following pollutants as characterizing runoff from urban areas:

Total Suspended Solids,
Biochemical Oxygen Demand,
Chemical Oxygen Demand,
Phosphorus (total and soluble),
Nitrogen (total kjeldahl and nitrate/nitrite),
Copper, Lead, Zinc, and

Coliforms

On a national basis the major impacts seem to be associated with metals, solids (including organic), and coliforms. The possible impacts on the urban and receiving water environments are shown in Table 5.1.

The composition of the pollutant load from an urban area is a function of land use - its nature and distribution, of socio-economic factors, of local waste management policies, and hydrologic factors. These include a wide variety of interacting elements that make any location unique in terms of the water quality response to hydrologic inputs.

The major processes by which pollutants move through the urban environment are the following:

1. deposition of pollutants on the surface from the action of wind, weathering, erosion, and other sources;
2. removal of a part of the material by the action of runoff - dislocation by impact and transport;
3. collection of pollutant-laden runoff by the sewer system; and
4. transport through the sewer system.

The quantity of material that is washed off during any

Table 5.1 Some Instream Impacts of Stormwater Pollution

1. Aesthetic Deterioration
2. Dissolved Oxygen Depletion
3. Sediment Deposition
4. Excessive Algal Growth
5. Public Health Threats
6. Impaired Recreational Value
7. Ecological Damage
8. Reduced Commercial Value

Adapted from Hydroscience (1979).

given event is called the pollutant load. The loading from a particular area depends on the frequency of occurrence of the hydrologic events, the source area, the stormsewer system configuration and capacity, and the particular waste management program implemented at the site. The latter includes street sweeping efficiency, a parameter often used in load assessment.

Within an event the hydraulic and physico-chemical interaction between runoff and pollutant will determine the real time variation of concentration. The graph depicting the variation of concentration within an event is called a pollutograph. The pollutograph, either obtained from a sampling study or generated from a simulation is a useful indicator of the quality response of an urban area to a hydrologic input. However, the response to any event is a function of event parameters. The relevant parameters are, to a great extent, random variables in the sense that it cannot be predicted, in a deterministic fashion, which conditions at the beginning of an event would give rise to a particular pollutograph. The variability in the rates of pollutant deposition for all constituents, the land use variations introduced because of development, the temporal and spatial variability of rainfall, the fluctuations in sweeping efficiency - all introduce a high degree of uncertainity in the attempt to estimate the actual amount of removed pollutant.

A consistent and representative data base will reduce some of the uncertainty associated with pollutant load assessment but will not eliminate it completely. However costly, sampling programs are necessary for an adequate evaluation of any particular site. Care must be excercised in the use of sampling data, though. Regressions of water quality data with a given parameter, or a given set of parameters, however useful, are valid only for the conditions that gave rise to the given set of sampling results. If the conditions change then the coefficients in the regression will change also.

The point to be emphasized is that the urban pollutant loading mechanism is driven by factors that can be classified as random. Treating some of the relevant parameters as random variables provides additional insight into the urban runoff pollution problem. The use of statistical techniques will also suffer from limitations, however. The parameters of the statistical formulation require a relatively large data base in order to obtain statistically significant estimators. Also, most statistical formulations are based on a simple representation of the physical system.

All methods operate under certain assumptions and limitations. The use or development of a particular methodology requires an adequate understanding of these limitations in order to assess the reliability of the model. This work will use a statistical technique to assess the water quality effects from the urban area. The main advantage of the statistical approach is that it considers explicitly the underlying randomness in the parameters of interest. An added advantage is that it allows for the assessment of the long-term behaviour of the system. This is a feature of the statistical methodology that is particularly useful because it eliminates the need for costly and expensive simulations.

5.2 Stormwater Load Assessment

Most of the load models available for planning formulations are based on a relatively simple conceptualization of the runoff process. The simplification is necessary due to the nature of the planning process itself. The planning process is concerned with the system as a whole rather than with the specific mechanics of the particular elements. The mechanics are usually simplified to account only for the major features of the process. This maintains the formulation at a manageable level, allowing for the study of system behaviour under a wide range of conditions.

The efforts at load modeling have considered the related processes discussed in the last section. Good reviews of the available techniques for modeling the urban pollution processes are provided by Jewell (1980), and Patry (1983). Their efforts will not be duplicated here. However, the formulations relevant to the present work will be reviewed.

The first process of concern is the accumulation process. The major parameter associated with material accumulation is the time between runoff events, or the time between street sweepings. Linear and non-linear rates have been employed. A general expression for linear rates is given by:

$$P_o^{(n)} = r_p^{(n)} X_3^{(n)} + d^{(n-1)} \tag{5.1}$$

Where $P_o^{(n)}$ = pollutant load available at the start of the n^{th} runoff event;

$r_p^{(n)}$ = pollutant rate of accumulation between the $(n-1)^{st}$ and n^{th} event;

$X_3^{(n)}$ = interevent time;

$d^{(n-1)}$= pollutant load left over from the $(n-1)^{st}$ event.

Equation (5.1) has been used by a number of simulation models for load assessment, notably the models STORM (U.S. Army Corps of Engineers, 1977) and SWWM (Huber et al., 1975). In these models the rate of load accumulation, modified for street sweeping, is assumed constant. However, Sartor and Boyd (1972) have shown that the accumulated solids loadings for various cities in the U.S. is not linear with respect to the interevent time. This is shown in Figure 5.1. After two or three days the varying rate of accumulation tends to drop off.

The load buildup may better be described by

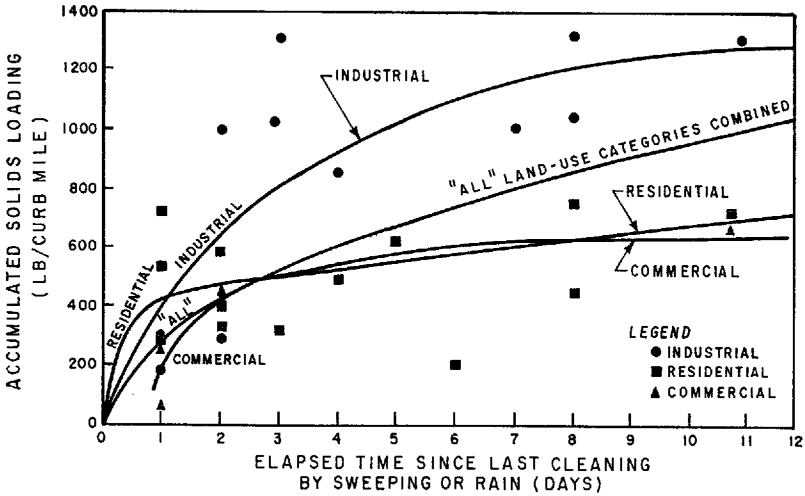


Figure 5.1 Accumulated Solids versus Elapsed Time (Sartor and Boyd, 1972)

$$P^{(n)} = P_{omax} \left[1 - \exp(-k_o X_3^{(n)})\right] \qquad (5.2)$$

Where $P^{(n)}$ = load available at start of event n;

P_{omax} = Maximum allowable accumulation;

k_o = rate coefficient;

$X_3^{(n)}$ = interevent time.

Equation (5.2) appears more realistic because it limits the value of the accumulated load rather than allowing an unbounded increase. Equation (5.2) has been employed by Ormsbee (1984), among others.

However, Whipple and Hunter (1977) have shown, for an urban area belonging to the Saddle River watershed in New Jersey, that the interevent time may be a poor indicator of the actual BOD loading. While the data is not sufficient to allow for a statistically conclusive assertion regarding the relationship between interevent time and subsequent loading, the general use of equation (5.2) and other non-linear expressions is more or less established.

The basic difficulty with these type of equations is that they are a deterministic formulation of what is, basically, a random process. Even if the total amount of pollutants available at the start of an event could be predicted with certainty the unquestioned randomness associated with the runoff process would invalidate the universality claims of any deterministic formulation. Consider, for example, the rainfall intensity. Both the energy necessary to dislodge pollutant particles and the volume of runoff needed to dilute the pollutant are functions of the rainfall intensity. The rainfall intensity shows random spatial and temporal variations during an event. The actual pollutant load will reflect these variations. These variations, while showing up in actual data, have not been accounted for in commonly available models.

The type of formulations described by Equation (5.1) and (5.2) are, nevertheless, convenient for the continuous simulation of the urban runoff pollution process. Their use in a standarized model should not be construed as an endorsement for the universal applicability of the principles involved in the formulations, even if data from certain areas appear to be described well by them. No doubt improvements will result as consistent and thorough data bases are created.

The washoff process has been the subject of numerous modeling efforts (see Jewell, 1980). The simplest formulation is that of the loading function (McElroy, 1976). Loading functions present average daily pollutant loads in terms of street curb length. The results serve as an overall assessment of general conditions, but are not event-based formulations, such as the formulation of Young et al. (1979), and thus are not useful for detention storage design.

The most commonly used approach is the first order load model. It is employed in the major stormwater planning models available at present (STORM, SWWM), and others, such as Medina (1982). The formulation assumes that the rate of washoff at any time t is proportional to the load on the watershed that is available for washoff. This is expressed as

$$\frac{dP(t)}{dt} = Kq(t)\ P(t) \cdot \text{AVAIL} \qquad (5.3)$$

Where P(t) = amount of pollutant remaining after time t
K = proportionality constant
q = runoff rate
AVAIL = availability factor

The parameter AVAIL, a function of rainfall intensity, represents the fraction of the pollutant that is available for washoff. The usual form of the expression is

$$\text{AVAIL} = a + bR^{c} \qquad (5.4)$$

Where a, b, and c are constants and R is the runoff rate. The values of a, b, and c have been found to vary between locations (Roesner, 1982). It is conservative to use AVAIL = 1.0. Likewise, the parameter k is a function of the runoff intensity. To determine K it has been assumed that a uniform runoff of 0.5 in/hr will wash off 90% of the initial pollutant load in one hour. The value of K is obtained from the solution of Equation (5.4), which is obtained from

$$\int_{P_o}^{P} \frac{dP}{P} = K \int_{o}^{t} q dt \tag{5.5}$$

The integration yields:

$$\frac{P(t)}{P_o} = \exp(-KV_t) \tag{5.6}$$

where

$$V_t = \int_{o}^{t} q dt \tag{5.7}$$

P_o is the amount of pollutant on the surface drainage area at the start of a runoff event, and V_t represents the total runoff volume up to time t. Equation (5.6) gives the fraction of pollutant load remaining on the surface. The fraction of the pollutant load washed away is given by

$$\phi(t) = 1 - \frac{P(t)}{P_o} = 1 - \exp(-KV_t) \tag{5.8}$$

Substitution of $\phi(1) = 0.90$ and $V_1 = 0.50$ in. in Equation (5.8) yields for the value of K:

$$K = 4.6 \text{ in.}^{-1} \tag{5.9}$$

Some verification for this rate was obtained by Sartor and Boyd (1972). The value given in Expression (5.9) is suggested for urbanized areas. For pervious areas a value of K = 1.4 is suggested. Roesner (1982) states that a value of K = 2.0 gave better results for the Detroit area.

Needless to say, the actual value of K is site specific. Factors such as the shape and extent of the catchment area, type of gutters and stormwater inlets, time of concentration, and the nature of the pollutant affect the particular value of K. However, the value of K given in Expression (5.9) seems to be generally accepted, in the absence of better estimates. The models STORM and SWWM use this value; also the University of Cincinnati (1970), Nix (1982), and Ormsbee et al. (1984).

Patry (1983) has utilized procedures suggested by Alley (1981) to obtain optimal estimates of K in terms of the minimum sum of squares of deviations between fitted and observed load characteristics. Analysis for suspended solids and chemical oxygen demand showed varying values of K. For thirteen events the average value of K was 11 in^{-1}, which is higher than the usually accepted value.

Equation (5.8) can be written as

$$P_o - P(t) = P_o[1 - \exp(-KV_t)] \tag{5.10}$$

Define

$$L(t) = P_o - P(t) \tag{5.11}$$

Then L(t) represents the amount of pollutant washed off the catchment at time t. Equation (5.10) expresses the "first-flush" effect. The greater the K value the smaller the required volume of runoff will be to achieve the same loading. The first-flush effect is the situation wherein a large portion of the available pollutant load is washed off the surface during the early stages of an event. While this has been observed to occur at certain locations it is by no means a general condition.

For continuous simulation and the construction of so-called pollutographs the first order model is written in discrete form (Jewell, 1980):

$$L_n = M_n \,[1 - \exp\,(-Kr_n \,\Delta t)] \tag{5.12}$$

Where: L_n = mass of pollutant washed off the subcatchment during simulation step n;

M_n = pollutant mass available at start of simulation step n;

r_n = runoff rate during step n;

Δt = simulation time step; and

K = washoff decay coefficient

Procedures for constructing pollutographs are illustrated by Wanielista (1978), and Roesner (1982). Jewell (1980) has shown that the total pollutant mass obtained from an event using Equation (5.12) is equivalent to using the model with the initial available mass and the total runoff volume:

$$X_1 = \sum_{i=1}^{n} r_i \Delta t$$

The total event pollutant load is then given by:

$$L = P_o[1 - \exp(-KX_1)] \qquad (5.13)$$

Not all areas exhibit first order pollutant washoff. Equation (5.13) is in fact an abstraction of a complex physical process. In areas with adequate data regression analysis may be preferable, and represents the other major approach to pollutant load estimation. Statistical regression procedures chose the most significant variables to represent the loading equation. Jewell (1980) has found that for suspended solids a regression of the following type gave the best statistical result:

$$SS = c_o t_1^{c1} \; t_2^{c2} \; I_3^{c3} \qquad (5.14)$$

Where: SS = suspended solids storm event loading (lb/ac);

t_1 = number of dry days;

t_2 = duration of rainfall, hours;

I_3 = average runoff intensity (in/hr); and

c_o, c_1, c_2, c_3 = regression constants.

Other authors have considered other variables for other constituents, but the overall approach is similar.

Jewell (1982) claims that Equation (5.12) cannot be statistically verified as portraying the stormwater washoff process. His conclusions were based on regression analyses of data from numerous events and locations. His analyses also seemingly showed that no specific regression formulation could be used for all areas and that within areas different pollutants sometimes required different formulations. Based on his observations, Jewell (1982) recommends the use of regression analysis with site-specific extensive data sets gleaned from thorough sampling programs as the appropriate modeling methodology. However, in many situations this would represent the idealistic rather than the realistic approach to modeling. The major difficulties in using the methodology proposed by Jewell are the limitations inherent in regression analysis, the usual lack of data in many locations, and the fact that the water quality problem in urban areas has random components. These aspects have already been commented upon earlier in this chapter. The changing nature of the runoff environment requires the use of a methodology that can, in some way, incorporate these variations.

The development of adequate data bases, coupled to the formulations of physical-statistical models is perhaps the best way to approach the urban pollutant loading problem. Within the planning framework Equation (5.13) is useful because a complex loading model is usually not required at the typical planning scales. Since some of the study areas are ungaged some sort of general methodology needs to be employed. At the present Equation (5.13) uses an acceptable formulation which has provided reasonable results.

Consideration has also been given to the hydraulics of pollutant transport through the system. Because the planning methodology to be developed here will not specifically address these processes, no review is undertaken.

5.3 Pollutant Loading Effects on the Storage Capacity

The pollutant loading generated by the Stormwater runoff is transported through the stormwater system to the storage unit. If the storage capacity is exceeded an overflow results in which a certain fraction of the load is lost to the receiving waters. The stored load is removed at a certain rate for treatment. The storage unit itself can function as a treatment device.

Medina (1981) has studied the operational behaviour of the storage device as a treatment unit using mathematical models derived from conservation principles. The system concentration response for various input concentration forcing functions and different mixing conditions was analyzed. Removal mechanisms were first order decay and sedimentation.

A number of physical/chemical treatment devices can be used for the treatment of runoff. Relevant operational characteristics can be found in Hydrocomp (1979).

The statistical methodology proposed here cannot take the specific operational characteristics of the storage unit into account. It must account for these, if possible, in a simplified fashion. Most predictive methodologies simplify some of the relevant processes. Usually it is assumed that the runoff load generated arrives intact at the storage unit. Neglected are the chemical interactions, sedimentation, re-suspension, and decay which may occur as the pollutant load is transmitted through the stormsewer.

The statistical method employed by Ditoro and Small (1979), and adopted by Hydroscience (1979), employs the following pollutant concentration function:

$$C(t) = C_o + (C_p - C_o) \exp(- \frac{t}{K_p}) \qquad (5.15)$$

Where

$C(t)$ = pollutant concentration at time t;

C_p = peak runoff pollutant concentration found at the start of the storm;

C_o = pollutant concentration found after the first flush subsides;

K_p = rate of decay of the first flush peak.

Equation (5.15) is analogous to the Horton infiltration equation in that the concentration decays to a constant value C_o as time increases. The magnitude of the first flush effect is determined by the ratio C_p/C_o. If the ratio is unity then a constant concentration is obtained from Equation (5.15). The larger the value of the ratio the more evident the first flush effect.

The measure of system performance used in the statistical method is the long term fraction of the load not captured, and has been defined as the expectation of the runoff load that overflows divided by the total runoff load (DiToro and Small, 1979). If a uniform runoff concentration is assumed then the fraction of the load not captured is equivalent to the hydraulic fraction because there is no first flush effect. The storage requirement for a given withdrawal rate will not change with the inclusion of this mean concentration. If the first flush effect exists the device is more efficient, for a given storage volume, at capturing the pollutant load than the hydraulic load. This means that first flush pollutant loadings require less storage for a given capture percentage than loading wherein the concentration is uniform.

To assess the relative influence of first flush effects DiToro and Small (1979), and Hydrocomp (1979), have used as the indicator the ratio of the actual mean storm load to the storm load predicted by using the mean concentration and runoff volume. The mean runoff load is found from:

$$M_R = \int_{q=0}^{\infty} \int_{d=0}^{\infty} \int_{t=0}^{d} \frac{C(t)}{d} \, q d f_D(d) \, f_Q(q) \, dt \, dd \, dq \tag{5.16}$$

An the mean concentration from:

$$\bar{c} = \int_{d=0}^{\infty} \int_{t=0}^{d} \frac{C(t)}{d} f_D(d) \, dt \, dd \tag{5.17}$$

The parameters of Equation (5.16) and (5.17) have already been defined in Chapter I. The concentration C(t) is obtained from Equation (5.15).

The mean concentration, as defined by Equation (5.17) is obtained by averaging over the event duration d. This cannot be interpreted as a mass average because the area beneath the C(t) curve does not yield the total pollutant mass. The mean concentration thus defined does not have the physical meaning that mass-based averaging provides.

Using the averages obtained from Equation (5.16) and (5.17) the load ratio is defined by:

$$\text{Load ratio} = \frac{M_R}{\bar{c} \, V_R}$$

where V_R is the average runoff volume.

Figure 5.2 shows the load ratio as a function of relevant event parameters. It has been assumed that runoff flows and durations are independent and exponentially distributed variables. From Figure 5.2 the maximum deviation of the load ratio due to first-flush effects appears to be on the order of 0.70.

First flush effects on interceptor performance are shown in Figure 5.3 in terms of the effect on the long term fraction of the load not captured.

The actual assessment of the relative magnitude of first flush effects is difficulted in practice by the fact that pollutographs may not exhibit a fixed concentration profile for all recorded events. The classification of these effects as small, moderate, and large may be somewhat artificial. In the absence of adequate information moderate conditions are usually assumed.

The present work will use Equation (5.10) as the basis for incorporating uneven loading effects on the storage equation.

If it is assumed that the runoff concentration is uniform and independent of the runoff volume then the fraction of the pollutant not trapped by the storage device will be the same as the fraction of the runoff volume not trapped by the device. This is expressed as follows:

$$\text{Fraction of load not captured} = \frac{\bar{C}_p E[Y]}{\bar{C}p\ E[Z]}$$

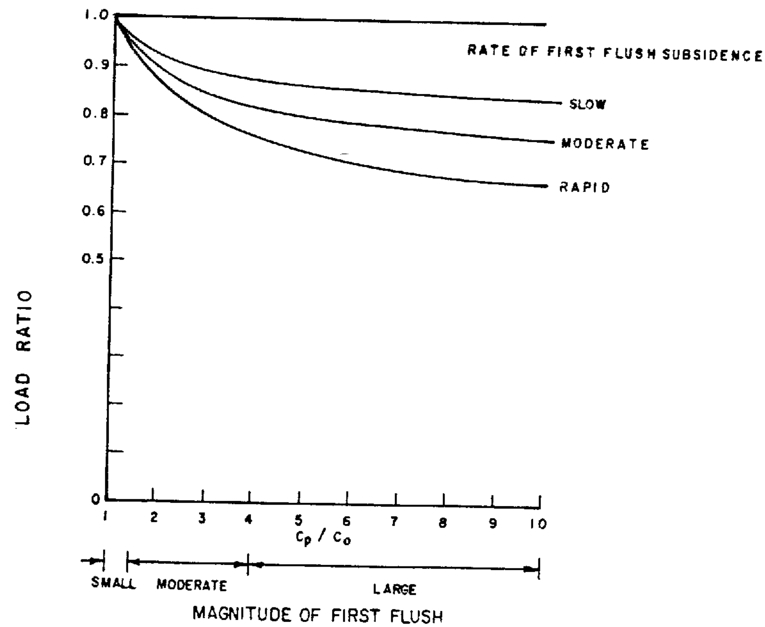


Figure 5.2. Effect of First Flush on the Load Ratio of Hydroscience Model (After Hydroscience 1979).

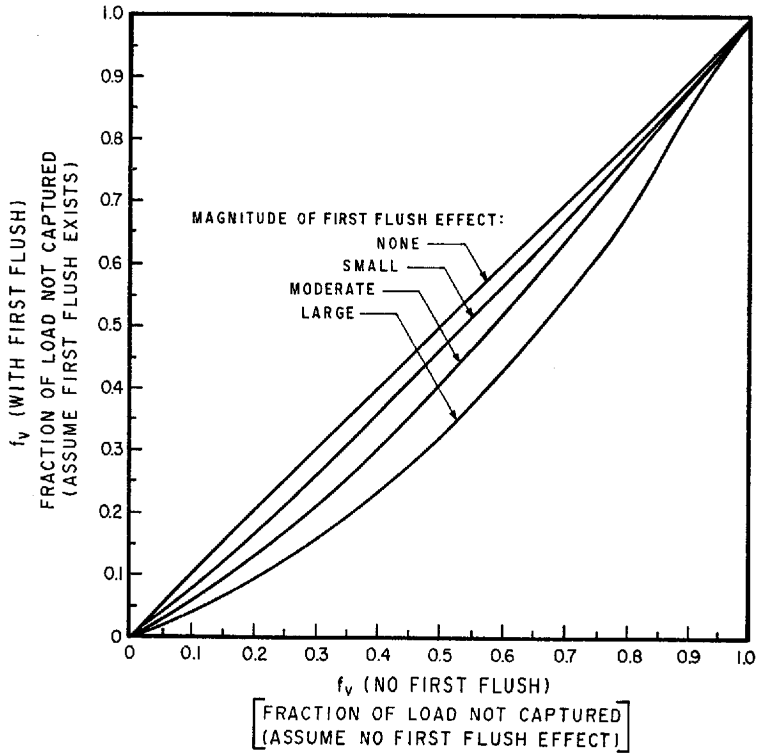


Figure 5.3 The Reduction in the Load Fraction not Captured as a Function of First Flush Effects (after Hydrocomp, 1979)

where

$\overline{Cp}$ = mean pollutant concentration;

E[Y] = expected value of the overflow;

E[Z] = expected value or the runoff volume entering the stormsewer.

Thus the equations already obtained for the fraction of the runoff volume not captured will yield the same storage requirement as if the storage tank is considered a load control device. A non-uniform load distribution will alter storage requirements because a greater proportion of the load is obtained for the same runoff volume. These effects are accounted for by the use of Equation (5.10) in the following form:

$$L = P_o \, [1 - \exp(-kZ)] \tag{5.18}$$

Here Z is the runoff volume arriving at the storage unit during a given event. As discussed earlier P_o, the load available at the start of the event, is a function of the interevent time and the left-over load from the previous event. Street sweeping should reduce the accumulated values, however, its effectiveness is highly variable. In fact, the Nationwide Urban Runoff Program (Water Planning Division - EPA, 1983) has concluded that street sweeping is generally ineffective in reducing pollutant loads. It is therefore not considered in the formulation.

Because of the randomness involved in the pollutant accumulation process it is assumed that in the long run more or less an average amount of pollutant should be present before any event. In effect this is suggested by the fact that non-linear accumulation rates seemingly become asymptotic after a few days, these being on the order of the mean interevent duration.

5.4 The Pollutant Load Model for Detention Storage

The water quality analysis is formulated in terms of the first-order load model of Equation (5.10). It estimates first-flush pollutant loads where the effect is evident. The analysis of the pollutant trapping effect of the storage device is formulated through the first order model. As has been shown, the pollutant washoff from the surface can be expressed in the following manner:

$$L = P_o[1- \exp(-K V_t)] \tag{5.10}$$

It is assumed that the initially available pollutant accumulation is constant for each event, or that it can be represented by an average value. As it has been shown that the evidence suggests that street sweeping does not greatly affects accumulation rates, and that the accumulated load reaches an asymptotic value after a few days, the assumption is reasonable. The runoff volume, V_t, is treated as a random variable.

The model represented in Equation (5.10) is adopted to represent the amount of pollutant trapped by the storage unit. The load washed off the ground surface is intercepted by the inlets and passed down to the stormsewer, eventually arriving at the storage unit. Whatever the mechanics by which the pollutants are transported through the system only a certain amount of the load will be retained by a detention unit. The pollutant trapping capacity of the detention unit depends on the storage capacity available at the start of the runoff event and the rate at which water is removed from storage for treatment. If the total runoff volume is smaller than the available capacity then all of the incoming pollutant load will be trapped. If, however, the runoff volume is larger than the capacity made available only part of the load will be trapped, the remainder being dumped to a receiving body of water.

During the event, while the available storage space is being depleted by incoming runoff, it is also being augmented by the pumping of the stored water at the given treatment rate. An overflow will occur only if the incoming runoff volume exceeds the sum of the initially available storage capacity and the volume made available by pumping over the time required to fill the basin at the incoming runoff rate. If no overflow occurs then all the hydraulic load, and hence the pollutant load, is captured by the unit.

If an overflow does occur then the part of the load corresponding to the runoff volume trapped by the unit is all that is retained. The excess runoff is diverted to the receiving water at a rate equivalent to the difference between the runoff rate and the pumping rate. This is so because pumping has been assumed to occur continuously over the event duration. Even while the overflow is occurring runoff would still be entering the tank at the treatment rate to make up for the pumped outflow. The duration of this event, and of the overflow, is given by the difference between the event duration and the time required to fill up the available storage. For the purposes of pollutant load analysis this additional incoming volume has been assumed to carry a negligible load compared to the other volumes.

These considerations suggest the following formulation, in terms of Equation (5.10), for determining the pollutant load trapped by the storage unit:

$$L(\text{lb}) = \begin{cases} P_o\,\{1-\exp[-K(T + at_o)]\} \text{ if } Z \geq T + at_o & \text{(a)} \\ P_o\,[1-\exp(-KZ)] \text{ if } Z < T + at_o & \text{(b)} \end{cases} \qquad (5.19)$$

where: T = storage available at the start of the nth event, whose distribution is given as Equation (2.65) (inches);

Z = runoff volume which arrives at the detention unit, whose distribution is given as Equation (2.38)(inches);

a = rate of water withdrawal from the storage unit (inches/hour); and

t_o = time required to fill up the available storage (hours).

Equation (5.19a) corresponds to the overflow situation wherein only the pollutant load corresponding to the hydraulic load that fills the available storage is retained. The situation in (b) corresponds to that of no overflow and the trapping of the totality of the pollutant load. As before, stationarity of the process is assumed.

The time required to fill the available storage is used to obtain the additional storage capacity that is produced while the unit is filled up. It is necessary to account for this volume because the non-linearity of the pollutant load function may produce significant pollutant washoff over this duration, and this would be trapped by the detention unit.

The complex mechanics of pollutant transport related to the hydraulics of the stormwater system are assumed to be represented by the lumped parameter K. The major interest here is to assess the relative efficiency of detention units in terms of the percent of pollutant load trapped by the storage unit.

The variables T, Z, and t_o in Equation (5.19) are random variables. It is desired to obtain an expression for the pollutant trap efficiency analogous to the expression obtained for the hydraulic trap efficiency discussed in Chapter IV. The corresponding expression for the trap efficiency of pollutants is given by:

$$\rho_{\ell} = E[L]/E[L_t] \tag{5.20}$$

Where ρ_{ℓ} is the trap efficiency, defined as the ratio of the expected value of the event pollutant load captured by storage and the expected value of the total pollutant load. The total pollutant load is defined as the load obtained from the incoming runoff volume. It can be defined in terms of the total runoff volume, X_1, or the stormsewer runoff volume, Z. In terms of the total runoff volume, the total load is given by:

$$L_{t,x} = P_o[1 - \exp(-KX_1)] \tag{5.21}$$

In terms of the stormsewer runoff volume the total load is given by:

$$L_{t,z} = P_o[1 - \exp(-KZ)] \tag{5.22}$$

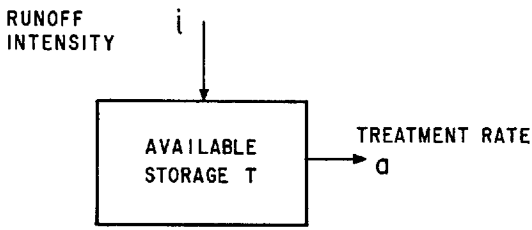
To distinguish between the two, the term of $\rho_{\ell,x}$ is applied to the trap efficiency in terms of the total runoff volume, X_1, and $\rho_{\ell,z}$ is applied to the trap efficiency in terms of the stormsewer runoff volume, Z. As indicated earlier, if the system inlet capacity is large both volumes will be about equal.

This formulation entails the assumption that the runoff volume arriving at the detention unit carries the pollutant load corresponding to that volume, assuming in effect, that no pollutant is lost in the neighborhood of the inlet or along the stormsewer.

To obtain the expectations called for in Equation (5.20) in terms of the appropriate random variables it is first necessary to define the distribution of the time required to fill the storage, t_o.

5.4.1 Definition of the time to fill available storage, t_o

The variable t_o is a function of the storage capacity available at the start of the event, the runoff intensity, and the treatment rate. The storage capacity available is filled at the net runoff rate, which is the difference between the average incoming runoff rate and the pumping, or treatment rate. A sketch of this process is shown in Figure 5.4. The average runoff intensity is the ratio between the runoff volume and the event duration, given by:



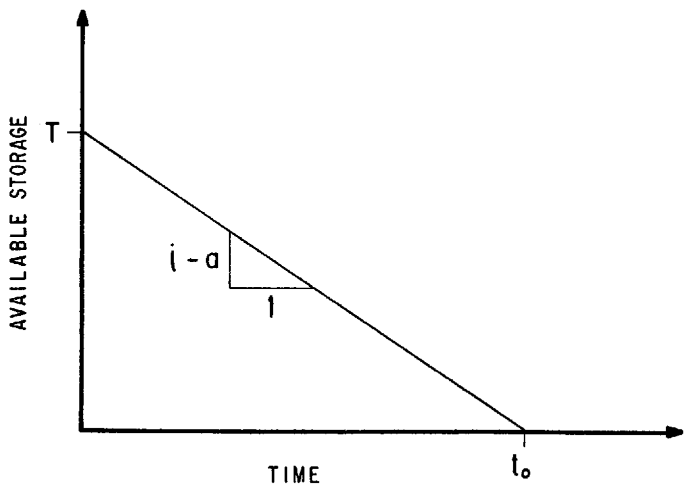


Figure 5.4 Definition Sketch for the Time Required to Fill Available Storage

$$i = {}^{Z}/X_2 \tag{5.23}$$

The time required to fill the available storage space is given by:

$$t_o = \frac{T}{i-a} \tag{5.24}$$

Because Z, X_2, and T are random variables, t_o is also a random variable. To obtain the distribution of t_o a derived distribution analysis is necessary. This is complicated by the fact that the random variables involved are defined over specific regions, thus making necessary the formulation of the distribution over several ranges of parameters. Because the distribution of t_o is employed in the formulation of the related distributions of the load model, and with the same set of random variables, joint distributions of functions of random variables have to be defined, developing into an unwieldy formulation that cannot be solved explicitly. For this reason a simplification of Equation (5.24) is attempted.

Because the runoff volume in Equation (5.23), Z = Min (X_1, cX_2), the intensity term is broken down as follows:

$$i = \begin{cases} X_1/X_2 & \text{if } X_1/X_2 < c \\ c & \text{if } X_1/X_2 \geq c \end{cases} \tag{5.25}$$

that is, the intensity term is either a function of two random variables, or a constant equivalent to the system stormwater trapping capacity, c. It is desired to simplify this arrangement by proposing that the intensity term be redefined in terms of two scalar terms, one corresponding to the mean effective rainfall intensity, and the other to the trapping rate c. This entails a redefinition of the intensity term within the context of t_o, as it cannot be properly called an intensity and be described by a constant value. Henceforth this term is defined as the average rate at which runoff fills available storage. Over the long run the unit will fill at the mean runoff rate if the system has a large capacity or at the runoff trapping rate if the capacity is limited, depending on which is greater.

Equation (5.25) is redefined as:

$$i_r = \begin{cases} E[i_m] & \text{if } i_m < c \\ c & \text{if } i_m \geq c \end{cases} \tag{5.26}$$

Where $i_m = {}^{X}1/X_2$ (5.27)

i_r = mean rate at which the detention unit fills.

In effect, Equation (5.26) can be expressed as:

$$i_r = \text{Min}\ (E[i_m],\ c) \tag{5.28}$$

With the simplification, the rate at which the available storage is reduced is in terms of the mean effective rainfall intensity, as long as the runoff trapping capacity of the stormsewer system exceeds it, otherwise the system capacity will control the runoff rate arriving at the detention unit. This avoids introducing additional random variables in the formulation. Because the general formulation is in terms of the expectation of related processes such averaging is justified for representing the long-term rate at which the unit fills.

The randomness associated with t_o is now related to the variate T, the other terms in Equation (5.24) representing the average rate at which T is depleted. The probability distribution of t_o is now defined in terms of T. However, a modification of Equation (5.28) is called for.

The intensity in Equation (5.24) must be greater than the treatment rate a, otherwise the storage unit will never fill. This is demanded by the condition $Z \geq T + at_o$ in Equation (5.19a), for if Equation (5.24) is substituted in the latter inequality it is obtained after some rearrangement:

$$Z - aX_2 \geq T \qquad (5.29)$$

Because T is a positive variate the implication is that:

$$Z/X_2 \geq a \quad , \tag{5.30}$$

and $Z/X_2 = i$.

This condition poses no problem when $i_r = c$, as it has been assumed that $c > a$. When $i_r = E[i_m]$ it is required that $i_m > a$, demanding that the expectation be a conditional expectation. The mean runoff rate is now re-defined as a conditional expectation, and Equation (5.26) now becomes:

$$i_r = \begin{cases} E[i_m | i_m \geq a] & \text{if } i_m < c \\ c & \text{if } i_m \geq c \end{cases} \tag{5.31}$$

The task now is to obtain the expression for the conditional expectation. First, the distribution of $i_m = X_1/X_2$ is obtained. The distributions of X_1 and X_2 are exponential, as given in Equations (2.13) and (2.18), respectively. The expression for the cumulative distribution of the average intensity is given by:

$$P[i_m \leq i_o] = P[X_1/X_2 \leq i_o] \tag{5.32}$$

Which can be expressed as:

$$P[i_m \leq i_o] = P[X_1 \leq i_o \, X_2], \tag{5.33}$$

yielding the following expression for the cumulative distribution in terms of the density functions of X_1, and X_2:

$$P[i_m \leq i_o] = \int_{x_2 = x_c}^{\infty} \int_{x_1 = 0}^{i_o x_2} f_{X_1}(x_1)\, f_{X_2}(x_2)\, dx_1 dx_2 \tag{5.34}$$

Substitution of Equations (2.13) and (2.18) in Equation (5.33), and integration yields:

$$P[i_m \leq i_o] = 1 - \frac{\beta}{\beta + \alpha i_o} \exp(-\alpha x_c i_o) \quad ; \; i_o \geq 0 \tag{5.35}$$

It is now desired to obtain the conditional distribution in terms of $i_m \geq a$. This is achieved by normalizing the distribution within the restricted event space of $i_m \geq a$. The necessary expression is given by:

$$P[i_m \leq i_o \,|\, i_m \geq a] = \frac{P\left[i_m \leq i_o \cap i_o \geq a\right]}{P[i_m \geq a]} \tag{5.36}$$

Which is equivalent to:

$$P[i_m \leq i_o \,|\, i_m \geq a] = \frac{P\left[i_m \leq i_o\right] - P\left[i_o \leq a\right]}{P\left[i_m \geq a\right]} \tag{5.37}$$

The expressions on the right side of Equation (5.36) are evaluated with Equation (5.34), yielding:

$$P[i_m \leq i_o | i_m \geq a] = 1 - \frac{\beta + \alpha a}{\beta + \alpha i_o} \exp[-(i_o - a)\alpha x_c] \quad ; \; i_o \geq a \qquad (5.38)$$

The probability density function is now obtained by differentiating the cumulative distribution. The operation yields:

$$f_{i_m|i_m \geq a}(i_o) = \frac{\alpha(\beta + \alpha a)\,[x_c(\beta + \alpha i_o) + 1]}{(\beta + \alpha i_o)^2} \exp\left[-(i_o - a)\alpha\, x_c\right] \quad ; \qquad (5.39)$$

$$i_o \geq a$$

With the probability density function the expectation is now obtained for the mean value of the average runoff intensity. This is given by:

$$E[i_m | i_m \geq a] = \int_a^{\infty} i_o \, f_{i_m | i_m > a}(i_o) \, di_o \qquad (5.40)$$

Unfortunately, no explicit solution exists for the previous integral. A numerical integration is employed to evaluate the expectation. To simplify the expression the numerical mean will be expressed as:

$$E[i_m | i_m \geq a] = I(a) \quad , \qquad (5.41)$$

with $I(a)$ corresponding to the expression in Equation (5.40).

Equation (5.28) now becomes:

$$i_r = \text{Min}\,[I(a),\ c] \tag{5.42}$$

Now proceeds the formulation for the expectation of the pollutant load.

5.4.2 Expressions for the pollutant trap efficiency

The first set of expressions to be obtained correspond to the expectation of the load as defined in Equation (5.19). From the results of the previous section, wherein t_o was defined as being equivalent to $T/(i_r-a)$, Equation (5.19) can be written as:

$$L = \begin{cases} P_o\{1-\exp\,[-K(pT)]\} & \text{if } Z \geq pT \\ P_o\,[1-\exp(-KZ)] & \text{if } Z < pT \end{cases} \tag{5.43}$$

Where $p = i_r/(i_r-a)$.

By defining a random variable $U = pT$ Equation (5.43) can be simplified to:

$$L = \begin{cases} P_o[1-\exp(-KU)] & \text{if } Z \geq U \\ P_o[1-\exp(-KZ)] & \text{if } Z < U \end{cases} \tag{5.44}$$

which is equivalent to stating

$$L = \begin{cases} g(U) & \text{for } Z \geq U \\ g(Z) & \text{for } Z < U \end{cases} \tag{5.45}$$

Where g( ) represents functions of random variables.

The objective is to obtain the expectation of Equation (5.45). For this the joint distribution of Z and U has to be defined. Since Z and U are independent variates because Z is a function of X_1 and X_2, while U is a function of T, the joint distribution is given as the product of the marginal distributions, that is:

$$f_{Z,U}(z,u) = f_Z(z)\, f_U(u) \tag{5.46}$$

which simplifies the analysis.

To obtain the expectation of Equation (5.45) it is convenient to use the concept of conditional expectation (Breipohl, 1970). Assuming, for the moment, that U has a fixed value, say U= u, the conditional expectation of L is given by:

$$E[L|U=u] = \int_0^u g(z)f_Z(z)dz + \int_u^\infty g(u)f_Z(z)dz \tag{5.47}$$

With the conditional expectation the total expectation is obtained by taking the expected value of conditional expectation:

$$E[L] = E[E[L|U = u]] \tag{5.48}$$

This would be obtained from:

$$E[L] = \int_u E[L|U = u]\, f_U(u)du \tag{5.49}$$

The first step, therefore, is to obtain the conditional expectation of L.

5.4.2.1 Conditional expectation of L

To obtain the conditional expectation of L the distribution of Z has to be specified. It has already been obtained as Equation (2.38):

$$f_Z(z) = \begin{cases} \alpha \exp(-\alpha z) & ; 0 \leq z \leq cx_c \\ (\alpha + \beta/c)\exp(\beta x_c)\exp[-(\alpha + \beta/c)z] & ; z \geq cx_c \end{cases} \tag{2.38}$$

The distribution is split into two ranges according to the value of cx_c. This will also split the conditional expectation into two ranges. Because the value of U is fixed at u for the first expectation, the expressions obtained will depend on whether u is greater or less than cx_c. To simplify the expressions the distribution for Z in Equation (5.49) is denoted as follows:

$$f_Z(z) = \begin{cases} f_1(z) & ; \ 0 \leq z \leq cx_c \\ f_2(z) & ; \quad z > cx_c \end{cases} \tag{5.50}$$

Defining the first case as $u \leq cx_c$, and using Equation (5.45) the conditional expectation is given by:

$$E[L|U=u] = \int_o^u g(z)f_1(z)dz + \int_u^{cx_c} g(u)f_1(z)dz + \int_{cx_c}^{\infty} g(u)f_2(z)dz \tag{5.51}$$

Because g(u) is not in terms of z the previous equation simplifies to:

$$E[L|U= u] = \int_o^u g(z)f_1(z)dz + g(u)\ P_1[Z \geq u] \tag{5.52}$$

Where $P_1[Z \geq u]$ is the complement of the cumulative distribution of $f_1(z)$ in Equation (5.50), and is given by Equation (2.36). Evaluation of Equation (5.52) with the appropriate expressions yields:

$$E[L|U=u] = \frac{KP_o}{\alpha + K} \{1-\exp[-(\alpha+ k)u]\} \quad ; \ u \leq cx_c \tag{5.53}$$

The other case corresponds to $u > cx_c$, and the conditional expectation is obtained from:

$$E[L\,|U=u]=\int_0^{cx_c} g(z)f_1dz + \int_{cx_c}^{u} g(z)f_2(z)dz + \int_u^{\infty} g(u)f_2(z)dz \quad (5.54)$$

Substitution of the appropriate expressions, and integration yields:

$$E[L\,|\,U=u]= \frac{KP_o}{(\alpha+K)} - \frac{KP_o}{(\alpha+K)\ \beta+(\alpha+K)c}\exp[-(\alpha+K)cx_c]$$
$$-\frac{KP_o}{(\beta/c+\alpha+K)}\exp[-(\beta/c+\alpha+K)u+\beta x_c] \quad ;\ u > cx_c \quad (5.55)$$

To simplify the notation the following definitions are introduced:

$$r_1 = \frac{KP_o}{\alpha+K} \quad ; \quad (5.56)$$

$$r_2 = \frac{KP_o\beta}{(\alpha+K)\left[\beta+(\alpha+K)c\right]}\exp[-(\alpha+k)\ cx_c] \quad ; \quad (5.57)$$

$$r_3 = \frac{KP_o}{\beta/c+\alpha+K} \quad (5.58)$$

The conditional expectation is now given by:

$$E[L|\,U=u]=\begin{cases} r_1 - r_2\exp[-(\alpha+k)\ u] & ;\ u \le cx_c \\ r_1 - r_2 - r_3\exp[-(\beta/c+\alpha+K)u+\beta x_c] & ;\ u > cx_c \end{cases} \quad (5.59)$$

Now proceeds the determination of the total expectation.

5.4.2.2 The expectation of the total load

To obtain the total expectation the distribution of U must be obtained. Because U = pT the distribution is readily obtained from that of T in terms of the linear transformation of the distribution function (see Benjamin and Cornell, 1971). The procedure yields, in terms of the distribution for T as given in Equation (2.65):

$$f_U(u) = \begin{cases} 0 & \text{for} \quad u \leq ps_o \\ \frac{\gamma}{ap} \exp[-\frac{\gamma}{a}(u/p - s_o)] & \text{for } ps_o < u < bp \\ \exp[-\frac{\gamma}{a}(b-s_o)] & \text{for} \quad u = bp \end{cases} \qquad (5.60)$$

The distribution of U is split among three ranges of parameter values, while the conditional expectation of the previous section is split into two. Because of this, different expressions will be obtained for the total expectation according to how the different ranges interact. Three cases arise out of the following situations:

Case I: $(ps_o \text{ and } pb) \leq cx_c$

Case II: $(ps_o \text{ and } pb) > cx_c$

Case III: $ps_o \leq cx_c \geq pb$

Each case is considered in turn.

Case I

For this case the domain of U lies below the value cx_c, and only the corresponding expression in Equation (5.59) is utilized. The total expectation would be obtained from Equation (5.49), which yields:

$$E[L] = \int_{ps_o}^{pb} E[L|U = u(\leq cx_c)] f_U(u) du \tag{5.61}$$

Integration of the equations with the distribution of Equation (5.60) and the expression for $u \leq cx_c$ in Equation (5.59) yields:

$$E[L] = r_1 - q_1 \exp[-(\alpha + K)pb - \frac{\gamma}{a}(b - s_o)] - q_2 \exp[-(\alpha + K)ps_o] \quad , \; b < \frac{cx_c}{p} \tag{5.62}$$

where

$$q_1 = \frac{r_1(\alpha + K)}{\gamma/ap + \alpha + K} \tag{5.63}$$

$$q_2 = \frac{r_1\gamma}{\gamma + (\alpha + K)ap} \tag{5.64}$$

The term r_1 is defined in Equation (5.56).

Because $s_o \leq b$ only the condition $pb \leq cx_c$ controls for this case.

Case II

For this case only the expression corresponding to $u >$

cx_c from Equation (5.59) is used in Equation (5.49) to obtain:

$$E[L] = \int_{ps_o}^{pb} E[L|U= u(\geq cx_c)]\ f_U(u)du \tag{5.65}$$

Substitution and integration yields:

$$E[L]= r_1 - r_2 - q_3 \exp\,[-\beta/c + \alpha + K)\ ps_o + \beta\, x_c] \\ - q_4 \exp[-(\beta/c + \alpha + K)\ pb + \beta\, x_c - \frac{\gamma}{a}(b-s_o)] \tag{5.66}$$

where

$$q_3 = \frac{r_3\gamma}{\gamma + (\beta/c + \alpha + K)\ ap} \tag{5.67}$$

$$q_4 = \frac{r_3(\beta/c + \alpha + K)}{\beta/c + \alpha + K + \gamma/ap} \tag{5.68}$$

The terms r_1, r_2, and r_3 are given in Equations (5.56), (5.57), and (5.58), respectively.

Because $b \geq s_o$ only the condition $ps_o > cx_c$ controls for this case.

Case III

Here both expressions in Equation (5.59) are utilized in Equation (5.49) to obtain:

$$E[L]=\int_{ps_o}^{cx_c} E[L\,|U=u(\leq cx_c)]f_U(u)du+\int_{cx_c}^{pb} E[L|\,U=u(>cx_c)]f_U(u)du \quad (5.69)$$

Integration with the appropriate expressions yields:

$$\begin{aligned} E[L]= {} & r_1 - q_2 \exp[-(\alpha+ K)ps_o] - r_2 \exp[-\tfrac{\gamma}{a}(cx_c/p-s_o)] \\ & + (q_2 - q_3)\exp[-(\alpha+ K)\, cx_c - \tfrac{\gamma}{a}(cx_c/p-s_o)] \qquad (5.70) \\ & - q_4 \exp[-(\beta/c + \alpha+ K)pb - \tfrac{\gamma}{a}(b-s_o) + \beta x_c] \quad ; s_o \leq \frac{cx_c}{p} \leq b \end{aligned}$$

where:

$$q_4 = \frac{r_3(\beta/c + \alpha + K)}{\beta/c + \alpha + K + \gamma/ap} \qquad (5.71)$$

The other terms have been defined previously. With the expressions for total expectation the mean value of the pollutant load that is retained by a detention unit during a runoff event has been obtained. Three expressions come about, each related to how the storage capacity parameters relate to a parameter in terms of the system hydraulic capacity, the system response time, and the net inflow rate to the storage unit. The next step is to obtain expressions for the pollutant trap efficiency.

5.4.2.3 Pollutant trap efficiency

The pollutant trap efficiency of the detention unit is defined as the long-term ratio of the quantity of pollutant captured by the unit and the long term pollutant load. This has been defined in Equation (5.20) as the ratio of the mean value of pollutant load trapped by storage to the mean value of the total load. It remains to obtain expressions for the mean value of the total load.

The total load has been defined in terms of the total runoff volume and the stormsewer runoff volume in Equations (5.21) and (5.22), respectively. The mean value of Equation (5.22) is obtained from:

$$E[L_{t,z}] = \int_0^{\infty} L_{t,z} \quad f_Z(z)dz \tag{5.72}$$

Substitution of Equation (5.22) and Equation (2.38), and integration, yields:

$$E[L_{t,z}] = \frac{KP_o}{\alpha + K} \{1 - \frac{\beta}{\beta + (\alpha + K)c} \exp[-(\alpha + K)cx_c]\} \tag{5.73}$$

To obtain the expectation of $L_{t,x}$ it is useful to recall that Z becomes X_1 as $c \to \infty$. For a large c, then, Equation (5.73) yields the expectation of $L_{t,x}$:

$$E[L_{t,x}] = \frac{KP_o}{\alpha + K} \tag{5.74}$$

The trap efficiency can have either of two expressions:

$$\rho_{\ell,x} = \frac{E[L]}{E[L_{t,x}]} , \tag{5.75}$$

or

$$\rho_{\ell,z} = \frac{E[L]}{E[L_{t,z}]} , \tag{5.76}$$

Either of the above expressions can be used to define the trap efficiency. Equation (5.75) would represent the overall efficiency while Equation (5.76) would represent the efficiency in terms of the stormsewer load, which is less than or equal to the total load. The expressions are easily incorporated into the equations already obtained by replacing E[L] in Equations (5.62), (5.66) and (5.70) by either of the following expressions obtained from Equations (5.75) and (5.76):

$$M_x = \rho_{\ell,x} E[L_{t,x}] \tag{5.77}$$

or

$$M_z = \rho_{\ell,z} E[L_{t,z}] \tag{5.78}$$

With the formulation of the trap efficiency expressions the storage/treatment isoquants can now be defined.

5.4.3 Storage/treatment isoquants

The storage/treatment isoquants for pollutant control are obtained in a manner analogous to the isoquants obtained in Chapter IV. Each isoquant is defined as the locus of storage capacity and treatment rate combinations that will produce a certain level of trap efficiency, in terms of the

$$\rho_{\ell,z} = \frac{E[L]}{E[L_{t,z}]} , \qquad (5.76)$$

Either of the above expressions can be used to define the trap efficiency. Equation (5.75) would represent the overall efficiency while Equation (5.76) would represent the efficiency in terms of the stormsewer load, which is less than or equal to the total load. The expressions are easily incorporated into the equations already obtained by replacing E[L] in Equations (5.62), (5.66) and (5.70) by either of the following expressions obtained from Equations (5.75) and (5.76):

$$M_x = \rho_{\ell,x} E[L_{t,x}] \qquad (5.77)$$

or

$$M_z = \rho_{\ell,z} E[L_{t,z}] \qquad (5.78)$$

With the formulation of the trap efficiency expressions the storage/treatment isoquants can now be defined.

5.4.3 Storage/treatment isoquants

The storage/treatment isoquants for pollutant control are obtained in a manner analogous to the isoquants obtained in Chapter IV. Each isoquant is defined as the locus of storage capacity and treatment rate combinations that will produce a certain level of trap efficiency, in terms of the

the specified parameter set.

Isoquant for case II

For this case Equation (5.66) is used. The procedure yields:

$$
\begin{aligned}
&q_4 \exp\{-[(\beta/c + \alpha + K)\, p + \tfrac{\gamma}{a}(1-\delta)]\, b + \beta x_c\} \\
&+ q_3 \exp[-(\beta/c + \alpha + K)\, p\delta b + \beta x_c] \qquad (5.80) \\
&+ r_2 + M_z - r_1 = 0 \qquad ; \; b > \frac{cx_c}{p\delta}
\end{aligned}
$$

Here, as well as for the next case the solution is obtained in a manner similar to that of case I.

Isoquant for case III

The procedure for this case yields:

$$
\begin{aligned}
&q_2 \exp[-(\alpha + K)\, p\delta b] + r_2 \exp\,[-\tfrac{\gamma}{a}(cx_c/p - \delta b)] \\
&+ (q_3 - q_2) \exp[-(\alpha + K)\, cx_c - \tfrac{\gamma}{a}(cx_c/p - \delta b)] \qquad (5.81) \\
&+ q_4 \exp[-(\beta/c + \alpha + K)\, pb - \tfrac{\gamma}{a}(1-\delta)b + \beta x_c] \\
&+ M_z - r_1 = 0 \qquad ; \; \frac{cx_c}{p} \leq b \leq \frac{cx_c}{p\delta}
\end{aligned}
$$

A similar set of equations can be obtained in terms of M_x, if desired.

To solve the system each of the equations is solved in turn and the one that satisfies the given range of acceptable values of the storage capacity is chosen as the solution for the given set of parameter values. The expressions that have been obtained for the isoquant and the ones to follow, can be simplified if it is assumed that the rate at which the detention unit fills up during an event is high compared to the values of the treatment rate. In this case the treatment rate will not make available an appreciable amount of storage space during an event as the unit will fill rather rapidly. Only the storage available at the start of the event is employed to determine the amount of pollutant trapped by storage. This is achieved in all former equations by taking the value of the parameter p as unity, implying, in effect, that the rate at which the unit fills is much greater than the treatment rate.

The parameter p is a factor that determines the amount by which the storage capacity is increased due to the fact that the accumulated runoff is depleted at the treatment rate. The value of this parameter has been estimated by considering mean runoff rates. The methodology is sufficiently generalized so that other procedures can conceivably be devised to estimate the value of p. Applications are discussed in Chapter VII.

5.4.4 Estimation for the extreme conditions

The extreme conditions considered are the situations wherein the reservoir is either previously full or empty. these are of interest because the isoquants corresponding to these extremes form the bounds within which the solution must lie. The situations are such that an analytical solution can be obtained.

Condition of previously empty reservoir

For this condition $s_o = b$, and all storage is available. This is given by $\delta = 1$ in the previous equations. Using $\delta = 1$ in Equations (5.79) and (5.80) allows the solution of the equations for the storage capacity:

$$b = \frac{1}{(\alpha + K)p} \ln\left(\frac{q_1 + q_2}{r_1 - M_z}\right) \quad ; \quad b < \frac{cx_c}{p} \tag{5.82}$$

$$b = \frac{1}{\beta/c + \alpha + K} \left[\ln\left(\frac{q_3 + q_4}{r_1 - r_2 - M_z}\right) + x_c\right] \quad ; \quad b \geq \frac{cx_c}{p} \tag{5.83}$$

Equation (5.80) is not utilized because its range at $\delta = 1$ becomes just $b = cx_c/p$.

Condition of previously full reservoir

This condition corresponds to $s_o = 0$, which is obtained by setting $\delta = 0$ in the isoquant equations. Now Equation (5.80) is not used because it would imply $b > \infty$ (or some large number), which is meaningless. Equations (5.79) and (5.81) are solved to yield:

$$b = \frac{a}{\gamma} \ln\left(\frac{q_1}{r_1 - q_2 - M_z}\right) \quad ; \quad b < \frac{cx_c}{p} \tag{5.84}$$

$$b = \frac{a}{(\beta/c+\alpha+K)ap+\gamma}\{\ln\{q_4/[r_1-M_z-q_2 - r_2\exp(-\gamma cx_c/ap)]$$

$$-(q_3-q_2)\exp[-(\alpha+K)cx_c-\gamma cx_c/ap]\} + \beta x_c\} \quad ; \; b \geq \frac{cx_c}{p} \qquad (5.85)$$

The solutions of this section are useful for defining the isoquants which bound the solution for any level of efficiency. Their closed-form nature makes them easy to use.

It remains to study a special case.

5.5 A special Case

The special case considered is the one that has been described in previous chapters: the case of overland flow, with available runoff statistics. For this case no time correction is applied to the event duration ($x_c \to 0$), and all runoff is available to the storage unit ($c \to \infty$). The product cx_c approaches zero. Only Equation (5.80) is used in this case as the others would imply non-positive storages. The values of the parameters in Equation (5.80) will change as a result of the simplification. The equation now has the form:

$$q_4 \exp\{-[(\alpha+K)p + \frac{\gamma}{a}(1-\delta)]\, b\} + q_3\exp[-(\alpha+K)p\,\delta\, b]$$
$$+ M_x - r_1 = 0 \qquad ; \; b > 0 \qquad (5.86)$$

The parameter r_2. as defined in Equation (5.87) is now zero. The other parameters have modified values, as given below:

$$q_3 = \frac{r_3\gamma}{\gamma + (\alpha + K)ap} \quad , \tag{5.87}$$

$$r_3 = \frac{KP_o}{\alpha + K} \quad , \text{ and} \tag{5.88}$$

$$q_4 = \frac{KP_o}{\alpha + K + \gamma/ap} \tag{5.89}$$

The parameter r_1 remains the same as Equation (5.56). The trap efficiency expression is now given by $\rho_{\ell,x}$, in Equation (5.75) because $E[L_{t,z}] = E[L_{t,x}]$ for this case.

Similar to the last section, expressions are obtained for the extreme conditions of previously empty or full storage conditions. If the detention unit is previously empty, $\delta = 1$ in Equation (5.86), indicating full available storage. Solving Equation (5.86) for b yields:

$$b = \frac{1}{(\alpha + K)\ p} \cdot \ln\left(\frac{q_3 + q_4}{r_1 - M_x}\right) \tag{5.90}$$

If the unit is previously full, $\delta = 0$ indicating no available storage. Equation (5.80) is solved to yield:

$$b = \frac{a}{\gamma + (\alpha + K)\ ap} \cdot \ln\left(\frac{q_4}{r_1 - q_3 - M_x}\right) \tag{5.91}$$

This completes the development of the water quality formulation for detention unit design. Applications of the methodology are presented in Chapter VII.

CHAPTER VI

Detention Storage Optimization

6.1 Introduction

The optimization problem that arises out of the detention storage formulation seeks to determine the most cost efficient storage location, its capacity, and treatment options. The situation is diverse enough so that several approaches are possible. Within the context of the present work it is desired to obtain the most efficient storage capacity and treatment rate, in terms of costs, for a single storage unit. This is to be accomplished in terms of the long-term operational characteristics of the storage unit.

Of the available optimization techniques dynamic programming is probably the most widely utilized for detention storage location, due to the fact that hydraulic sewer network analysis is adaptable to the recursive formulation of dynamic programming algorithms. Representative of dynamic programming approaches to optimal sizing and location of detention units is the work of Mays and Bedient (1983), Ormsbee (1984), Labadie et al. (1980), Robinson and Labadie (1981), and Bennet and Mays (1985). A good review of available techniques is presented by Ormsbee (1984).

The interest of the present work is to obtain optimal values for the storage capacity and treatment rate for efficient system operation. This optimization is performed in terms of the long-term system operation characteristics. This adds a new dimension to the study of stormwater detention systems that is not accesible to single event formulations. Simulation models have been employed to study long-term system behaviour by carrying out numerous simulations with long hydrologic records. A statistical analysis of the results defines long-term behaviour of the detention unit in terms of the storage/treatment isoquant for specified levels of efficiency. The nature of the results is such that the theory of production function analysis can be applied to the problem.

The formulation of the stormwater detention system operation in terms of a production process has been most noticeably utilized by Heany et al. (1977, 1979), and Nix (1982). Production function optimization is carried out by defining the physical production process in terms of detention basin parameters. The optimal values of detention storage parameters stand for the optimal combination of physical resource inputs. The production function is obtained by simulation.

The statistical model formulated here will use the production function approach to set up the optimization problem. One advantage of the present statistical model is that the production functions are obtained analitically, thus greatly simplifying the solution of the problem. An exposition of the general aspects of production function theory follows, with the formulation of the optimization problem that is to be solved.

6.2 Production Function Theory

A production function is a basic representation for the transformation of resources to products (de Neufville and Stafford, 1971). It also respresents the maximum output attainable through a production process by any set of inputs (Ferguson, 1975; Baumol, 1977). Given a set of resources $(X_1,X_2,...,X_n)$ the production function is expressed by

$$Z_p = g(X_1,X_2,...,X_n) \tag{6.1}$$

The output Z_p represents units of production, which need not represent monetary values. The X_n represent physical resources input to the production process.

The production function, representing at each point the maximum product obtainable for any given set of resources, is the locus of all technically efficient combination of resources. The production function does not have any particular form, but always represents the limit on what can be achieved with the resources and technology available.

A probable production function for a one input, one output process is illustrated in Figure 6.1. Point A, while feasible, does not attain the maximum allowable production, which is obtained at Z_p'. Production level Z_p' can also be

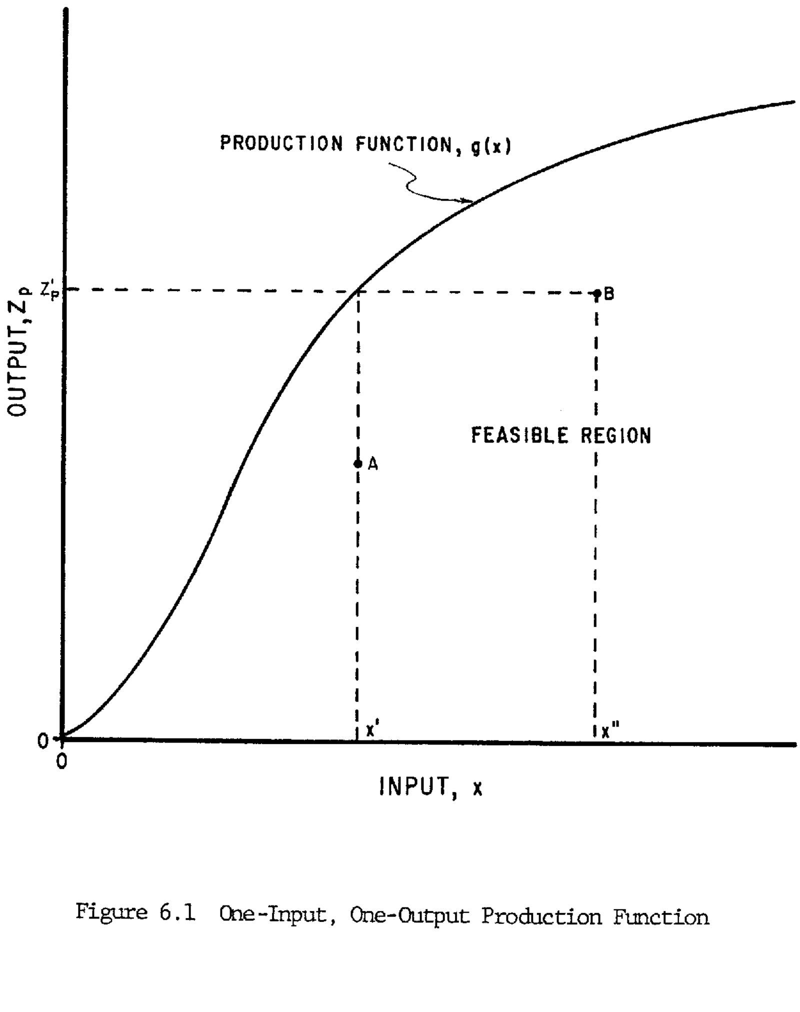


Figure 6.1 One-Input, One-Output Production Function

obtained at point B through resource use level X'', but this would be an inefficient use of the resource since the same output can be obtained through a lower resource allocation at X'.

6.2.1 Production Function Characteristics: Diminishing marginal returns

The nature of the production function can be described in terms of the change in the output produced by changes in the level of resource use. This is known as the marginal product. The marginal product MP_j with respect to each input X_j is given by:

$$MP_j = \frac{\partial Z_p}{\partial x_j} \qquad (6.2)$$

The marginal product can also be expressed in terms of finite differences as:

$$MP_j = \frac{\Delta Z_p}{\Delta x_j} \qquad (6.3)$$

The marginal product represents the contribution of an additional unit of resource x_j to the total product while the other inputs are held constant.

It has been observed that for the efficient range of system operation the marginal product of any resource will eventually diminish with an increased level of resource utilization. This is known as the law of diminishing returns (de Neufville and Stafford, 1971; Baumol, 1977). The importance of this law is that it guarantees a convex production region that facilitates the choice of an optimal production level.

Returns to scale

The change in the output occasioned by a proportional change in all inputs is known as a return to scale. It is expressed as:

$$\Delta Z_p \text{ (Return to scale)} = g[(1+\Delta)X_1, \ldots (1+\Delta)x_n] - g(X_1, \ldots, X_n) \tag{6.4}$$

Returns to scale can be increasing, decreasing, or constant, depending on the relative change of outputs as related to change in inputs.

De Neufville and Stafford (1971) report that economies of scale have been observed for engineering systems. In engineering the economies of scale are represented by:

$$\text{Cost} = (\text{constant}) \times (\text{capacity})^{\alpha} \quad ; \alpha < 1 \tag{6.5}$$

The equation shows that average costs will decrease with scale, and thus an increasing return to scale is obtained.

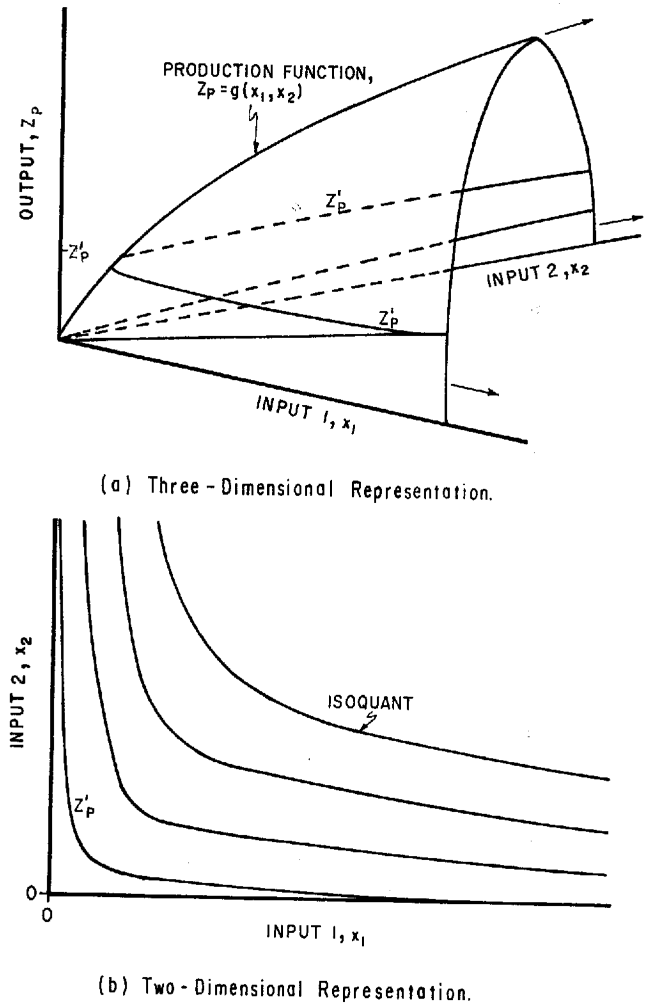


Figure 6.2 Two-Input, One-Output Production Function (Adapted from Nix, 1982)

given production process. The boundary of this surface represents the most production that can be obtained from any feasible combination of resources. If a certain level of production is fixed at Z', the locus of all efficient combinations of resources for this fixed level is called the isoquant. It is shown in two-dimensional space in Figure 6.2(b).

From the point of view of the output there is total indifference as to which point along the isoquant is chosen to represent the level of resource use. Which combination of resources is used to obtain a point on the isoquant can be a function of relative resource cost, decision maker preference, or other factors, such as availability of the resource. These other criteria define the optimal combination of resources and thus the optimal point along the isoquant.

A quantity that is related to the optimal point is the marginal rate of substitution. It is defined for the two input case as the amount by which one input must be increased and the other decreased to preserve the same output. The marginal rate of substitution (MRS) can be shown to be equivalent to the negative reciprocal of the marginal products (de Neufville and Stafford, 1971). The marginal rate of substitution of input i for input j is given by:

$$MRS_{ij} = - \frac{MP_j}{MP_i} \tag{6.7}$$

The marginal product, as defined in Equation (6.2), shows that the previous equation can be expressed as:

$$MRS_{ij} = - \frac{\partial x_i}{\partial x_j} \tag{6.8}$$

For the two-input production process discussed here the marginal rate of substitution would be expressed as:

$$MRS_{21} = - \frac{\partial x_2}{\partial x_1} \tag{6.9}$$

It is seen that, at any point, the marginal rate of substitution is equivalent to the negative slope of the isoquant. Isoquants convex to the origin exhibit the principle of diminishing marginal rate of substitution (James and Lee, 1971). Considering, as an illustration, channel improvement and reservoir storage, James and Lee indicate that as more channel improvement and less storage capacity are provided to produce a given level of flood control, the larger is the incremental channel improvement required to effect a unit reduction in flood storage.

6.3 Optimization

The resource allocation problem considered here can be solved by marginal analysis as long as the associated functions define convex regions (de Neufville and Stafford, 1971). James and Lee (1971) have illustrated the application of marginal analysis to multiple-purpose reservoir projects, as well as the application of other optimization techniques within the water resources planning environment. The application of the method is considered in two stages due to the difficulty associated with the estimation of the benefits, which, in many situations cannot be expressed in monetary terms. For example, the benefits of pollution abatement facilities.

In the first stage a particular level of production is optimized in terms of resource cost only. The locus of all optimal points for all levels of production defines what is known as the expansion path or cost effectiveness function. For each production level the expansion path intercept represents the most economical use of the resources to achieve that level. The level of optimization pursued in the present work corresponds to this first stage.

The second stage selects the best level of production. This is done in terms of the benefits associated with the production process. In many situations value judgements are involved to prescribe process benefits or to assess the nature of the benefit/cost interaction. Because of this difficulty, and because different approaches to these problems are possible, the main emphasis of the present work is on obtaining the expansion path for the first stage analysis.

The optimality criterion for the first stage analysis is that, at the optimum point, the ratios of the marginal resource cost and marginal product be equal. This is expressed as:

$$\frac{MP_i}{MC_i} = \frac{MP_j}{MC_j} \tag{6.10}$$

Considering that the ratio of marginal products is defined as the slope of the isoquant the previous equation can be expressed as:

$$\frac{\partial x_i}{\partial x_j} = \frac{MC_j}{MC_i} \tag{6.11}$$

The slope of the isoquant at optimality is equal to the ratio of marginal costs. If the resource costs are constant, then:

$$\frac{\partial x_i}{\partial x_j} = \frac{C_j}{C_i} \tag{6.12}$$

The optimum is then defined in terms of the ratio of unit resource costs. At optimality, then, the isoquant is tangent to the line of constant cost. This holds true for non-linear cost functions as well.

6.3.1 Formulation of the optimization problem

If the stormwater storage/release system is viewed as a production process the results of production function theory can be applied to obtain optimal estimates of the storage capacity and the treatment rate. The storage capacity and treatment rate can be thought of as representing the inputs to the production process. The performance level, or the output, is the trap efficiency of the storage unit. This approach has been employed by Heany et al. (1977), Heany (1979), and Nix (1982). Their efforts were aimed at obtaining the expansion path in terms of the trap efficiency of the storage unit. The second stage optimization is not addressed, for the reasons that have already been mentioned, and are further expanded upon for the particular case of storage/release systems by Nix (1982). This is not altogether negative because a particular optimization criterion is not favored, and flexibility is maintained. It, for example, would allow for a regulatory agency to set the desired production level, or system performance requirement, and thus the optimal value could be chosen from a point along the expansion path.

The optimization problem, where cost is to be minimized for a specified performance level is given as follows:

$$\text{Minimize} \quad C = g\,(a,b) \tag{6.13}$$

$$\text{such that} \quad \varepsilon = f\,(a,b) \qquad (6.14)$$
$$\varepsilon \geq \varepsilon_o$$
$$a,b \geq 0$$

Where: C = total cost, dollars,

a = treatment, or release, rate, in/hr,

b = storage capacity, in.,

ε = performance level or trap efficiency,

ε_o = specified performance level,

$g(a,b)$ = cost function for a and b, and

$f(a,b)$ = production function for a and b.

The above is a constrained minimization problem, in terms of the objective cost function and the performance level constraints. The production function corresponds to the storage/treatment isoquant for a specified performance level. In the past, the shape of the isoquant has been obtained through simulation studies, and perforce the optimization has been undertaken graphically. The models STORM and SWWM have both been used to obtain storage/treatment isoquants through simulation. As indicated in the first chapter, a frequency analysis of simulation results yields the isoquants. For a linear cost function the optimum is defined by Equation (6.12). The graphical representation of the solution is illustrated in Figure 6.3.

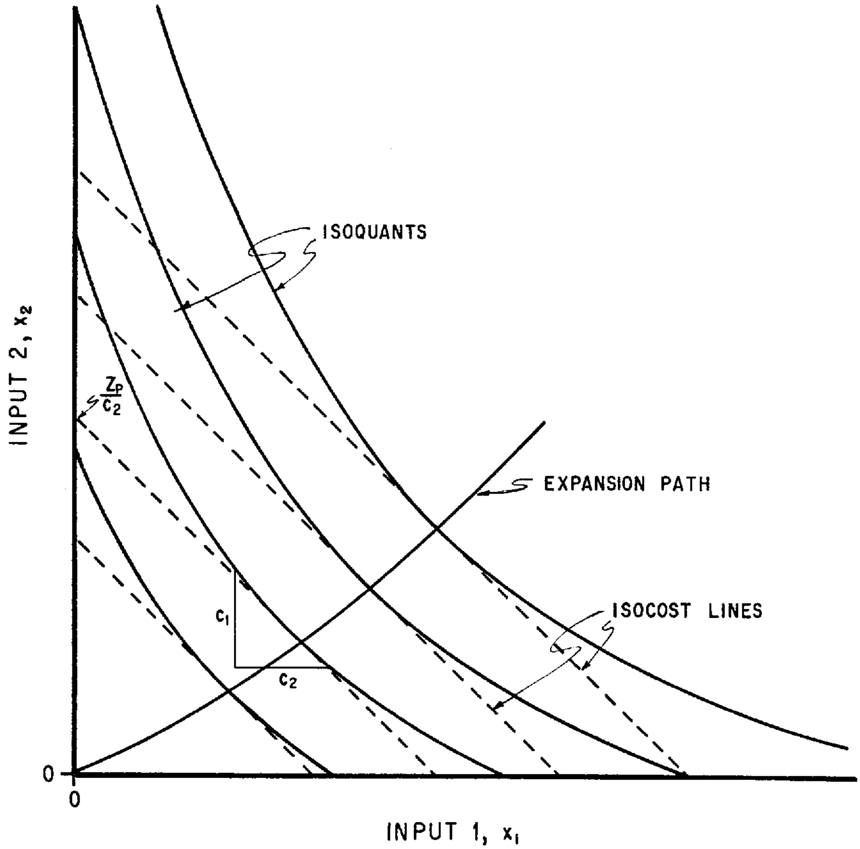


Figure 6.3 Optimization in Production Space (Heany et al., 1977)

6.3.2 Cost functions

The costs associated with storage/treatment systems can be classified in terms of capital and operation and maintenance costs. The major costs associated with stormwater storage are related to land use and engineering costs. The costs associated with treatment are related to the design, construction, and maintenance of engineering facilities. For the analysis these costs are expressed as annual costs.

Costs vary between locations, and so the estimation of local costs is preferable to using generalized cost information. Nevertheless regionalized cost information is useful for the initial assessment of the planning stage. Procedures exist for estimating costs related to storage/treatment devices (Benjes, 1976; Municipal Environmental Research Laboratory, 1976). Heany et al. (1977) developed cost functions for wet-weather treatment devices from a survey of many cities throughout the U.S.

The total input costs have been expressed as power functions (Nix, 1982). Equation (6.13) would thus be expressed as:

$$C = c_1^{\alpha_1} + c_2\, b^{\alpha_2} + c_F \qquad (6.15)$$

Where: c_1, c_2 = coefficients, c_1, $c_2 > 0$;

α_1, α_2 = coefficients, α_1, $\alpha_2 > 0$; and

c_F = total annual, fixed, storage and treatment costs.

The shape of the cost function is determined by the value of the exponents α_1 and α_2. Evidence seems to suggest that the exponents are less than or equal to unity (Heany et al., 1979; Benjes et al., 1975), indicating economies of scale. Representative parameters of the cost function are given in Table 6.1 and Table 6.2. The Water Planning Division of EPA (1983) has obtained an average pond construction cost estimate given as $C= 77.4\ S^{0.51}$, where S is the volume of storage in cubic feet, and C is the cost in 1980 dollars.

If the cost function is simplified to a linear case the equation would be given by:

$$C = C_1 a + C_2 b + C_F \tag{6.16}$$

This would entail great simplification of the cost data, but it also simplifies the optimization problem. To avoid dealing with non-linear cost functions directly, Nix (1982) transformed the production isoquant into production cost space to obtain a graphical solution. Dealing with non-linear cost functions is troublesome for graphical techniques due to the difficulties that may arise with defining points of tangency graphically. The statistical model proposed here is more efficient in this respect.

Table 6.1 Cost Functions for Wet-Weather Control Devices

		Annual Cost (S/year)					
		Amortized Capital $CA = \imath T^m$ or $\imath S^m$		Operation & Maintenance $OM = pT^q$		Total $TC = sT^z$ or sS^z	
Device	Control Alternative	$\imath$	m	p	q	s	z
Primary	Swirl concentrator	17 600	0.70	4 400	0.70	22 000	0.70
	Microstrainer	79 100	0.76	19 800	0.76	98 900	0.76
	Dissolved air flotation	113 000	0.84	28 200	0.84	141 000	0.84
	Sedimentation	291 000	0.70	72 800	0.70	364 000	0.70
	Representative Primary Device Total Annual Cost = \$1.05/m^3/day						
Secondary	Contact stabilization	280 000	0.85	69 900	0.85	350 000	0.85
	Physical-chemical	466 000	0.85	116 000	0.85	582 000	0.85
	Representative Secondary Device Total Annual Cost = \$3.93/m^3/day						
Storage	High density (37.0)/ha)	-	-	-	-	51 000	1.00
	Low density (12.4/ha)	-	-	-	-	10 200	1.00
	Parking lot	-	-	-	-	10 200	1.00
	Rooftop	-	-	-	-	5 100	1.00
	Representative Annual Storage Cost (\$per ac-in) = \$122 exp(0.16PD)[a]						

a : PD = Population Density

T = treatment rate in m^3/S

S = Storage volume in m^3

(Adapted from Heany et al.; 1977, 1979).

Table 6.2 Unamortized Capital Costs ($\$ \times 10^6$) of Storage, S(mil gal)

Type	Equation	Unit Cost S = 10 mil gal $/gal
Earthen	$C = 0.025\ S^{0.73}$	$0.013
Concrete w/o cover	$C = 0.350\ S^{0.58}$	$0.133
Concret w/cover	$C = 0.400\ S^{0.79}$	$0.250

(Adapted from Benjes, 1976)

6.4 The statistical approach

The commonly - used techniques illustrated thus far for defining storage/treatment isoquants are based on the use of simulation models. As such, these are "form-free" in the sense that they are not obtained explicitly from analytical expressions but rather from a statistical assessment of numerous simulation results. This necessitates the use of a graphical approach to optimization, which, by nature is limited to handling two resource inputs. Attempts have been made to fit analytic functions to the isoquants to overcome the graphical dependence of the procedures. Heany et al. (1977) have suggested the following equation to represent the isoquants:

$$T = T_1 + (T_2 - T_1)\exp(-KS) \qquad (6.17)$$

Where T = wet-weather treatment rate, in/hr;

T_1 = treatment rate at which isoquant becomes asymptotic to the ordinate, in/hr;

T_2 = treatment rate at which isoquant intersects the abscissa, in/hr;

S = storage volume, inches; and

K = constant, inches^{-1}

In the previous equation the constant has to be obtained with actual data or from simulation results.

Nix (1982) has used analytical production function models in order to test for the model that would best approximate the simulation results.

While these have had some success in approximating the results, they cannot be generalized. They lack the theoretical basis that would relate them to the nature of the urban stormwater process.

The statistical model developed in the present work has obtained analytical expressions for the isoquants, based on the underlying hydrological principles of the process. As such it is a generalized model. The optimization is now made easier because a graphical solution is not strictly required. Also the isoquants do not have to be obtained by extensive and costly simulations but can now be obtained in terms of a few relevant parameters.

The optimization problem is expressed in the form of Equation (6.13) and (6.14). The difference now is that the constraint is analytical. It will be shown that the isoquants are convex. The optimization by marginal analysis is now carried out by equating the slope of the isoquant to the slope of the isocost line. The values of the storage capacity and treatment rate which define the point of tangency will be the optimal values. This is the solution expressed in Equation (6.12).

The equations for the isoquants have been presented in Chapter IV. Isoquants were defined for given levels of capture efficiency, and these represent the constraints on the optimization problem. the application of these procedures with the storage/treatment isoquants is to be presented in Chapter VII.

CHAPTER VII

Application of the Statistical Model

7.1 Introduction

The development of the stormwater storage statistical model has thus far concentrated on the formulation and development of the model - mainly with the mathematical formulation of the storage/treatment isoquants. The present chapter presents the applications of the model within the stormwater management environment, and also insight into the types of processes that are capable of being represented by the statistical model. The statistical model has three major components - the hydraulic model which describes the long-term detention unit behaviour in terms of the fraction of the hydraulic load which is retained at the unit; the pollutant load model which assesses system behaviour in terms of the fraction of the available pollutant load that is trapped by the system; and the optimization model which determines the cost efficient combination of treatment rate and storage capacity necessary for efficient system operation.

The basic problem that is addressed in this research is that of developing a procedure to estimate urban stormwater detention requirements. An objective was to determine this procedure in a manner that was mathematically efficient, and that would account for the evident randomness of the related hydrologic processes. The expressions obtained for the evaluation of storage capacities, namely the storage/treatment isoquants, reflect the physical process of the temporal storage dynamics at the detention unit and also the manner in which these dynamics are temporally activated via the stochastic arrival process.

To assess the efficiency of the model in predicting the shape of the actual storage/treatment isoquant at a site it is necessary to obtain adequate long-term records in order to estimate accurately the statistical parameters required in the model. The type of information that is required is related, mainly, to the operational characteristics of the detention unit and to the runoff process, which would include pollutant transport information. Because of the inherent randomness in the occurences of runoff events a sizable record would be required to obtain reliable parameter estimates.

Because long records are usually not available in the form required by statistical approaches the results obtained from statistical formulations are usually compared to simulation results. Apart from their other uses simulation models are useful from the point of view of statistical models because they can be used to generate long records of system behaviour that lend themselves to statistical analysis. It is to this type of analysis that statistical results are compared. As has been mentioned in previous chapters the two most commonly used simulation models in detention storage assessment are the model STORM and SWWM. They have been used in storage analysis to obtain storage/treatment isoquants, both for hydraulic volume and pollutant load control. They have the capability of simulating system behaviour continuously over long periods of time. Simulations with these models have been conducted with a year's amount of hourly rainfall, although limitations in the simulation period arise because of cost constraints. Statistical analysis of simulation results yields the kind of information that statistical methods can compare with. The statistical model obtained in the present work is compared with simulation results. Good results will establish the worth of the statistical formulation and will provide a powerful tool for prompt assessment of detention storage behaviour.

Several possibilities exist regarding the application of the statistical method. Each application is to be described in turn. In a general sense the model is compared to the statistical analysis of the results obtained with the simulation models STORM and SWWM. The major objective is to obtain the storage/treatment isoquant from which efficient combinations of storage capacity and treatment rate can be obtained. The modeling results represent real-world modeling efforts from various locations of different hydrologic characteristics. Comments on model behaviour and interpretation of results are presented in each section.

7.2 Applications to Overland Flow

Overland flow represents a particular case of the general statistical model. In overland flow all of the runoff generated over the catchment may arrive at the detention unit. For this case there exist no limits to a stormsewer system, and thus no water is lost to the detention unit due to a limited inlet capacity in the face of a high-rate runoff event. From the standpoint of the storage unit, whithin the context of the storage model, it is as if the stormsewer system had an infinite capacity in the sense that all of the runoff arrives at the unit. This condition is obtained for the statistical model by assuming an infinite runoff-trapping capacity for the urban catchment. Thus, the parameter c, representing the system hydraulic trapping rate, is assumed to be infinitely large so as to intercept the highest runoff rates obtainable. This is a common assumption of standard simulation models, and is a conservative assumption which may lead to overdesign, as will be shown later on.

Another condition to be specified in this case is that the runoff statistics are available, that is, the mean value of the runoff volume, runoff event duration, and intervent time are available from given records. This means that the event duration transformation of Equation (2.6) is not needed because runoff records are available and there is no need to transform rainfall durations into effective rainfall (or runoff) durations. The two applications discussed in this section are of this kind.

With the two conditions expressed here the statistical model becomes the special case described in sections 3.4 and 4.6. The corresponding expressions of the special case are used to obtain the storage/treatment isoquant. The expression for the isoquant is given by Equations (4.34) and (4.39). Expressions for the extreme condition of storage initially empty or full are given for the special case by Equations (4.40) and (4.41), respectively. The initially empty or full condition refers to the storage condition at the end of the previous runoff event. This condition is assumed known although in reality it is a random variable whose value cannot be determined with certainty. The sensitivity of the isoquant to this parameter is to be considered.

Applications of the case discussed here to idealized catchments in Atlanta and Minneapolis are now illustrated.

7.2.1 Storage/treatment isoquants for Atlanta, Georgia

Goforth, Heany, and Huber (1983) have used the simulation model SWWM to obtain storage/treatment isoquants for a catchment in Atlanta. The 24.7 acre catchment is 37% impervious. The catchment is represented in Figure 7.1. The

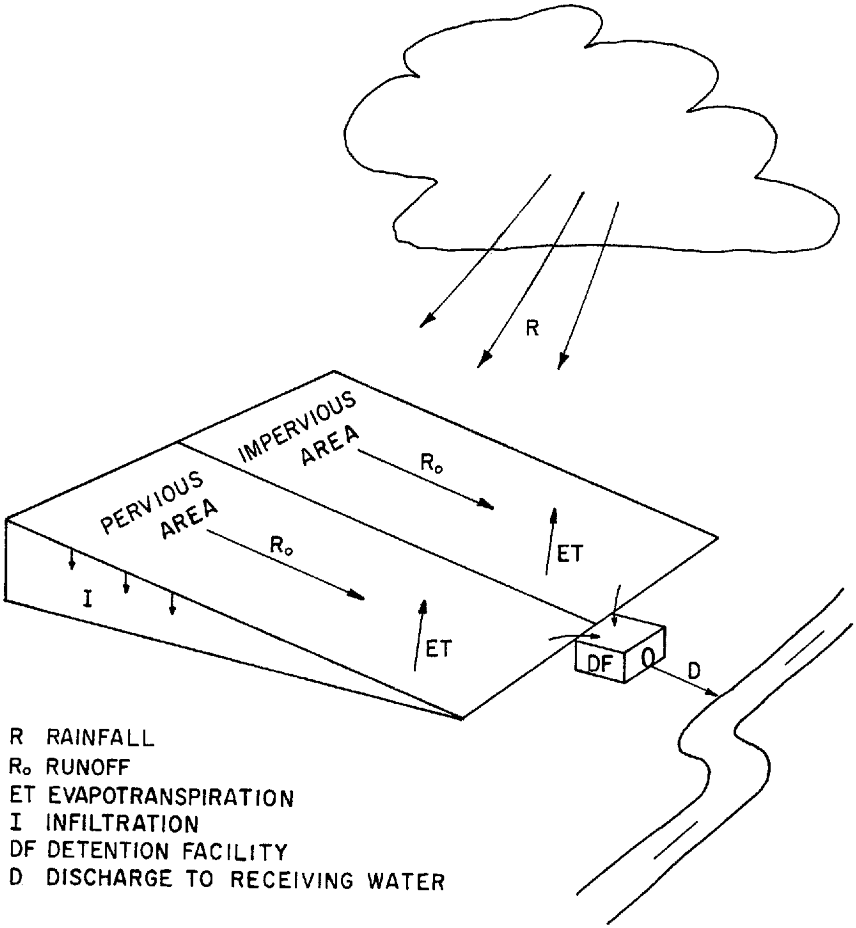


Figure 7.1 Schematic of Case Study Catchment, Atlanta (from Goforth et al., 1983)

rainfall data used to generate runoff was obtained from the National Weather Service for the city of Atlanta, and represented 24.6 years of hourly rainfall records. Because of the high cost of carrying out the simulation, a run with the full 25 years of record was not appropriate so only one year was simulated. The year was chosen so that the related rainfall statistics for that year were comparable to that of the full record.

For the simulation the hourly values of the rainfall record were transformed to runoff with a rainfall/runoff transformation of the coefficient type discussed in Chapter I. The runoff is routed to the detention unit where a portion is stored, depending on available capacity. From the detention unit water is pumped continuously at a fixed rate. For a given storage capacity and pumping rate the fraction of the runoff volume that is captured by the unit for all events was obtained to represent the trap efficiency. The locus of this efficiency is the storage/treatment isoquant.

An analysis of the runoff parameters yielded the statistics of the runoff process which form the input to the statistical model. These appear in Table 7.1. The parameters are the mean runoff volume, event duration, and interevent time. Also shown in Table 7.1 are the coefficients of variation of the parameters. The coefficient of variation is defined as the ratio of the mean and the

Table 7.1 Runoff Parameters for Atlanta Simulation

Parameter	Description
Year of record	1953
Number of events	71
Mean runoff volume, $E[X_1]$	0.223 in.; coeff. of variation (c.v) = 1.102
Mean runoff duration, $E[X_2]$	6.887 hr.; c.v. = 1.121
Mean interevent time, $E[X_3]$	124.3 hr.; c.v. = 0.937

Adapted from Goforth et al. (1983).

standard deviation of the variates. A value of unity implies that the exponential distribution can be used to describe the variable because the exponential distribution has a coefficient of variation of unity. For the parameters shown in the table the coefficients of variation are reasonably close to unity so that the use of exponential distributions for these parameters is justified. The procedure used by Goforth was to obtain a minimum interevent time, in this case eight hours, which would define independent runoff events with a coeffient of variation approaching unity.

The nature of this simulation is such that the special case of the statistical method is applicable. The most expedient use of the statistical method is that with the extreme conditions - the condition of previously empty or full storage, because of the explicit form of the solution. Safety and operational considerations would suggest the consideration of the most critical scenario - that of the previously full storage unit. The previously empty condition defines the minimum storage capacity needed to provide a specified level of capture efficiency.

Using the equations of flow capture efficiency, the results obtained are compared to simulation results and are shown as Figures 7.2 and 7.3. Figure 7.2 shows the simulation results compared to the lower bound estimates corresponding to the condition of previously empty storage, while Figure 7.3 shows the comparison to upper bound estimates

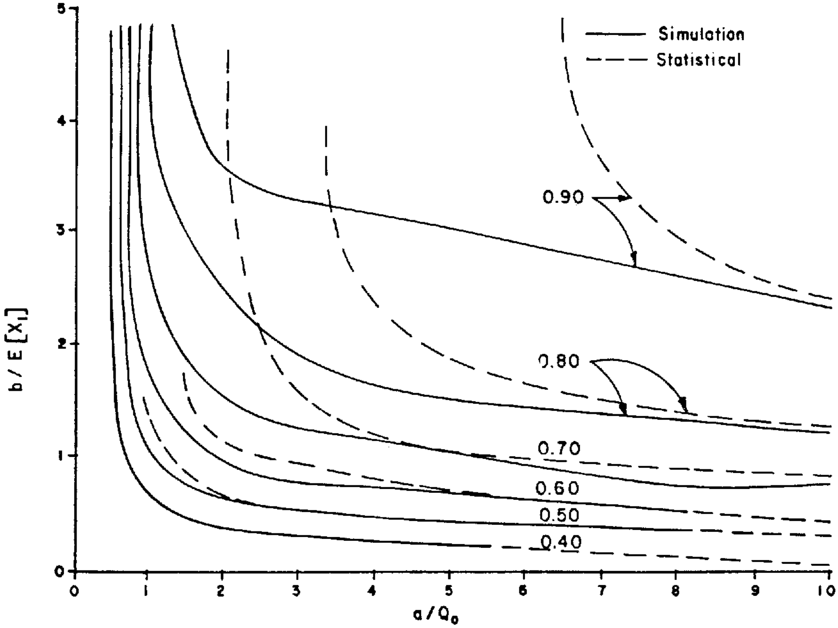


Figure 7.2. Comparison of Flow Capture Efficiency Estimates (Upper Bound) Atlanta

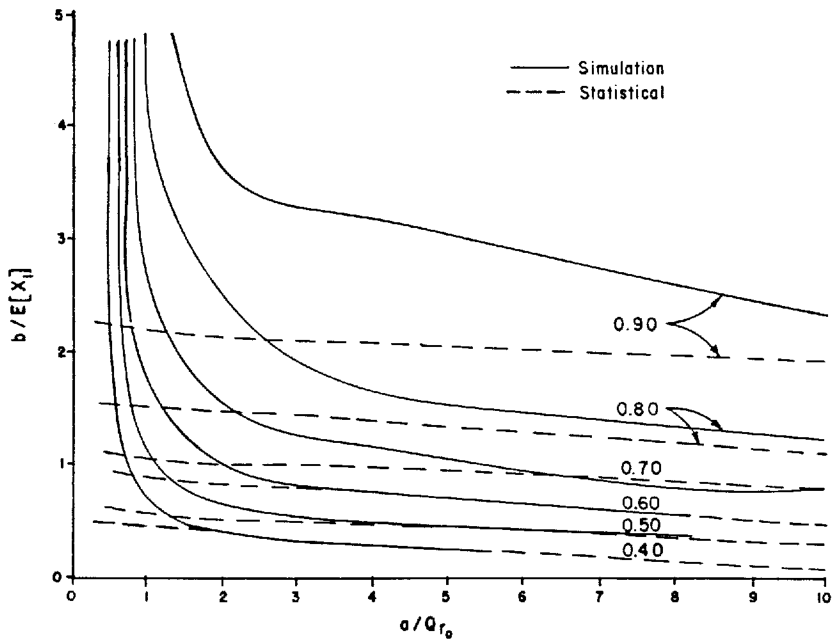


Figure 7.3 Comparison of Flow Capture Efficiency Estimates (Lower Bound), Atlanta

corresponding to the previously full storage condition. The results obtained have been normalized by the mean runoff volume, $E[X_1]$, and a parameter Q_o, defined as the ratio of the mean runoff volume and the mean interevent time:

$$Q_o = E[X_1]/E[X_3] \tag{7.1}$$

The simulation results shown in the figures exhibit some noteworthy features. The isoquants become asymptotic to a certain lower value of a/Q_o, in this case around unity. At the point where the isoquant becomes parallel to the ordinate the storage capacity is sufficient to provide the corresponding trap efficiency. Increasing the storage capacity further does not increase the capture efficiency because the treatment rate does not increase. Increasing the treatment rate reduces the storage capacity requirement, and large release rates lead to small detention capacities. Eventually the point is reached where practically no storage will be required because of the high treatment rate. Theoretically, for a specified level of trap efficiency, or control, the system is capable of operating at any point along a given isoquant - in fact isoquants can be thought of as indifference curves in a production process. The point at which the system will actually operate is obtained from the optimization procedure, and is illustrated in a subsequent section. Another important feature is that the isoquants are shown to be convex functions, thus fulfilling a prerequisite for the application of the production process optimization procedure discussed in Chapter VI.

The results obtained with the statistical method in Figures 7.2 and 7.3 indicate an excellent agreement with simulation results. As expected the upper and lower bound solutions straddle the simulation results for each control level. For lower values of the capture efficiency the statistical results show practically the same values of storage capacity for previously empty and full storage conditions. As a rule both solutions converge for large values of pumping rate, as is seen from Equations (4.40) and (4.41). This convergence occurs for lower values of a/Q_o at lower control levels, but occurs at much higher values of a/Q_o at the higher control levels. The greatest difference between the two conditions is observed for the highest trap efficiencies at the lower treatment rates. This is an effect produced by the relative magnitude of the interevent time. Because the interevent time is an order of magnitude larger than the event duration the likelihood of having the stored runoff volume substantially depleted upon the arrival of the next event is high. For the lower trap efficiencies the required detention volume, and the corresponding runoff holding capacity, are relatively small. Thus a higher likelihood exists of depleting this volume by the time the next event arrives. This is reflected in the statistical model by having the solution for both extreme conditions predict somewhat similar values of the required storage capacity over a range of pumping rates. For these rates and storage capacities it makes little difference if the detention unit is previously full as the contents will be substantially depleted by the time the next event arrives. For low pumping rates the results will not converge because the lower rates will leave more runoff in storage and detention storage requirements will increase.

For the higher trap efficiencies the storages predicted by the equations are very different for both extreme conditions, although, as stated earlier, they will converge for the higher pumping rates. For the higher trap efficiencies higher storage capacities are required. These can store higher runoff volumes which are not as likely to be depleted by the time the next event arrives. There is a higher probability of having some runoff volume left over, and thus of having the detention unit partly full over the long run. The results suggest that for higher trap efficiencies an intermediate storage level assumption would give better estimates than the extreme conditions. These conditions can be studied with the statistical model.

An example of the application of the equations for this case is presented next.

Example 7-1: lower and upper bound estimation.

It is desired to obtain the upper and lower bound storage levels for the Atlanta catchment, for a treatment rate of 0.02 in/hr, and a runoff trap efficiency of 90%. Using the parameters for Atlanta, the statistical parameters are given by $\alpha = 4.48\ \text{in}^{-1}$, $\beta = 0.145\ \text{hr}^{-1}$, and $\gamma = 0.008\ \text{hr}^{-1}$. The upper bound storage requirement, corresponding to the condition of previously full storage, is obtained from Equation (4.41):

$$b = \frac{0.02}{0.098} \cdot \ln \left[\frac{4.48(0.02)(0.145)}{(0.235)(0.098)(0.1) - 0.145(0.008)} \right]$$

yielding:

$$b = 0.50 \text{ in.}$$

The lower bound storage requirement, corresponding to the case of previously emtpy storage is obtained from Equation (4.40):

$$b = \frac{1}{4.48} \cdot \ln \left[\frac{(0.145)}{[4.48(0.02)+0.145](0.10)} \right]$$

$$b = 0.41 \text{ in.}$$

The upper bound storage represents the critical scenario, and represents the conservative storage estimation.

To study the effect on the isoquant of having intermediate storage levels use is made of Equations (4.34) and (4.39). The parameter δ in Equation (4.39) is set to represent the fraction of the storage volume that is available for storage at the end of the previous event. The equations are then solved for the isoquant in terms of this parameter. No explicit solution in terms of the storage capacity can be obtained for intermediate storage considerations, but an available numerical procedure, such as the Second Order Newton-Raphson method can be utilized to solve the equation for the storage capacity (James et al., 1985). The latter was utilized for this situation. The results obtained are shown in Figure 7.4 for the case of the 90% trap efficiency. The effects of the non-linearity in the process are evident as the higher percentages of available storage will tend to bunch up near the lower limit curve of the previously empty storage condition. The results seem to show that for higher trap efficiencies the previously empty assumption of the detention storage condition is not justified. It appears that, on the average, for the higher trap efficiencies, implying greater required storage capacity, a substantial amount of runoff may be initially available in the storage unit, at least for the lower treatment rates. The implication is that utilization of the isoquant equations for high trap efficiencies should assume that the available storage capacity upon the arrival of a runoff

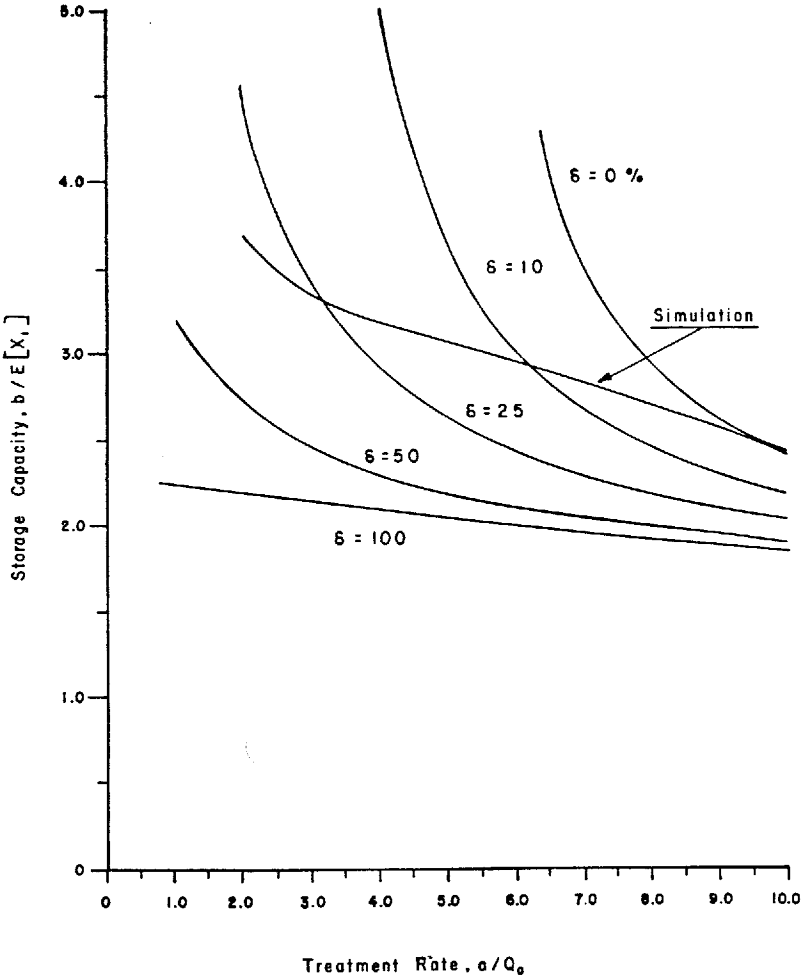


Figure 7.4. Storage / Treatment Isoquants for 90% Trap Efficiency, for Atlanta Catchment with Intermediate Storage Levels.

event is somewhat less than half the design storage capacity, given that the equations are used in the manner discussed in this section. For the case of the lower trap efficiencies the isoquants for both extreme cases are very close to each other and a distinction between levels of available storage is not warranted for practical reasons, except perhaps for large catchments.

Example 7-2: storage estimation for the intermediate case.

Utilizing the same parameters as in Example 7-1 it is desired to obtain storage requirements for the case wherein the storage level at the end of the previous event is at an intermediate value. Because this situation corresponds to the special case, Equation (4.39) is utilized.

It is assumed for this case that 25% of the storage capacity is available at the end of the previous event, thus $\delta = 0.25$.

The isoquant is obtained by substituting the parameter values in Equation (4.34). For a removal rate of 0.02 in/hr it is obtained:

$$0.1 - 0.573 \exp(-4.78b) - 0.051 \exp(-1.12b) = 0$$

from which it is obtained:

$$b = 0.44 \text{ in.}$$

This value of storage is intermediate between the two extremes obtained in Example 7-1.

7.2.2 Application to Minneapolis, Minn.

An application similar to that of the previous section is undertaken for the city of Minneapolis. The simulation of the detention unit behaviour was undertaken by Nix (1982) with the model SWWM. The nature of the catchment and the storage scheme are illustrated in Figure 7.5. The bypass is used to route excess runoff to the receiving water. Once the unit is filled runoff is bypassed and no further flows enter the basin during the duration of the event. Similar to the Atlanta simulation the system was studied with a year of representative rainfall records. It was also assumed that all runoff was available to the detention unit as, basically, overland flow. A statistical analysis of the runoff record generated with the SWWM rainfall/runoff transformation yielded the parameters shown in Table 7.2. The coefficient of variation of the parameters on the table are close to unity for the runoff duration and interevent time, but only approximately so for the runoff volume. Nevertheless it seems reasonable to assume that the exponential distribution holds for the process although the runoff process variation may indicate some other distribution.

Table 7.2 Runoff Parameters for Minneapolis

Parameter	Description
Year of Record	1971
Number of Events	93
Mean runoff volume, $E[X_1]$	0.088 in., coeff. of variation (c.v.) = 1.40
Mean runoff duration, $E[X_2]$	7.3 hr., c.v. = 1.10
Mean interevent time, $E[X_3]$	94 hr., c.v. = 0.96

Adapted from Nix (1982)

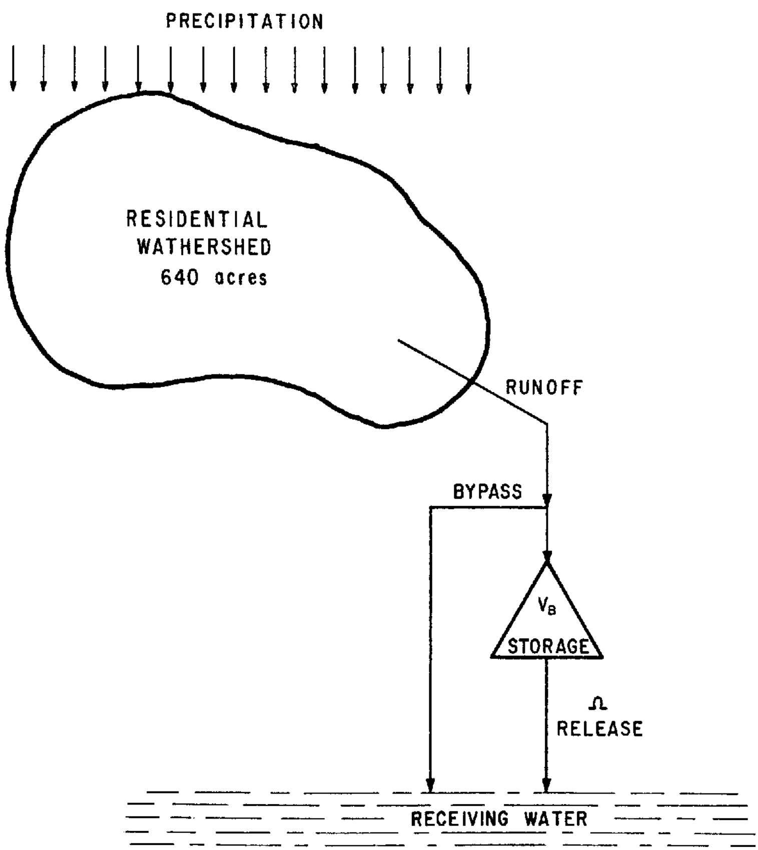


Figure 7.5 Configuration of the Stormwater Control System for the Minneapolis Catchment (from Nix, 1982)

Using the parameters of Table 7.2 the special case of the statistical method is utilized to generate storage/ treatment isoquants, as in the previous section. For the two extreme conditions - those of previously empty or full available storage - Figures 7.6 and 7.7 show the results obtained. The overall pattern is the same as in the previous section and the same type of general comment would apply, although here some further observations are called for. Results here have not been normalized as they did not appear in this form in published results.

It is noted in Figure 7.7, the case corresponding to the previously full detention unit, that some of the simulation isoquants are high enough to intercept the statistical isoquants, apparently in contradiction to the fact that the extreme conditions should straddle the simulation results. This may be due, in part, to the possibility that the underlying distributions are not strictly exponential. Another possibility is the fact that the simulation was conducted with only one year of data, and this may not be adequate, statistically speaking, to define accurate parameter estimates and long-term storage/treatment isoquants - this is a condition which would affect all the simulations presented in the present work. Most likely the situation is heavily influenced by the fact that the detention unit is operated, in the simulation, in the bypass mode. As indicated earlier, in this mode runoff is accepted by the

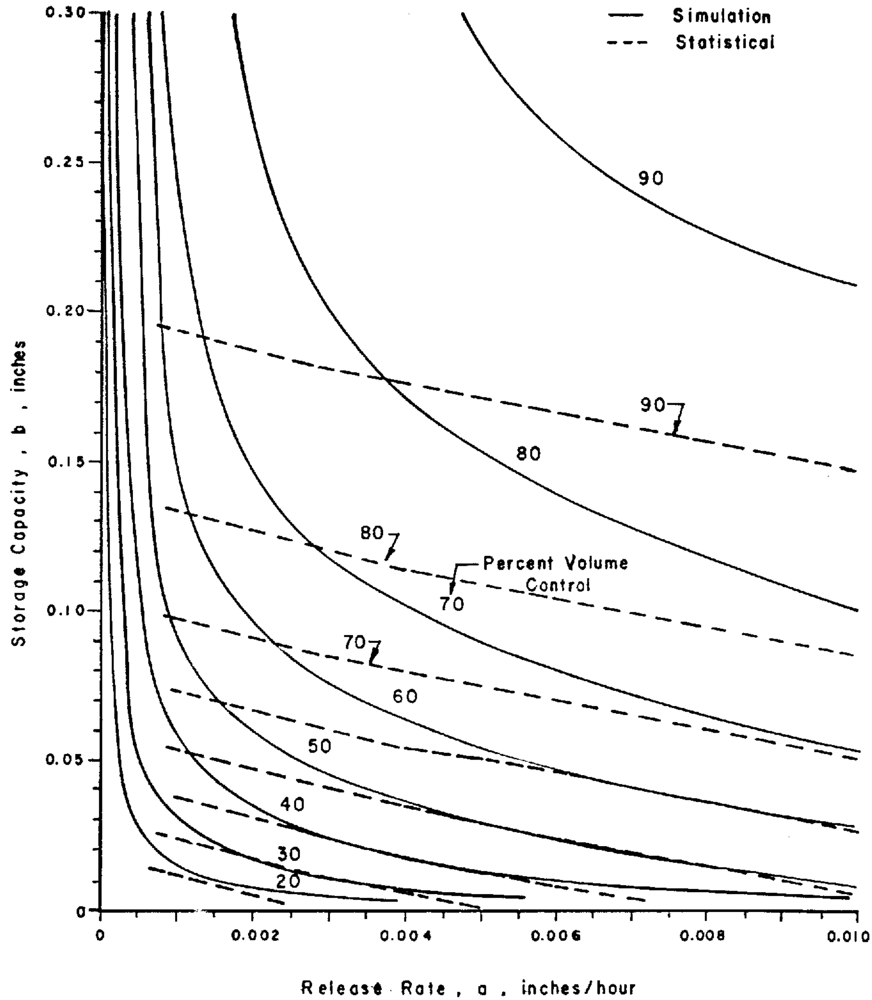


Figure 7.6. Isoquants for Runoff Volume Control for Minneapolis Catchment, Lower Bound.

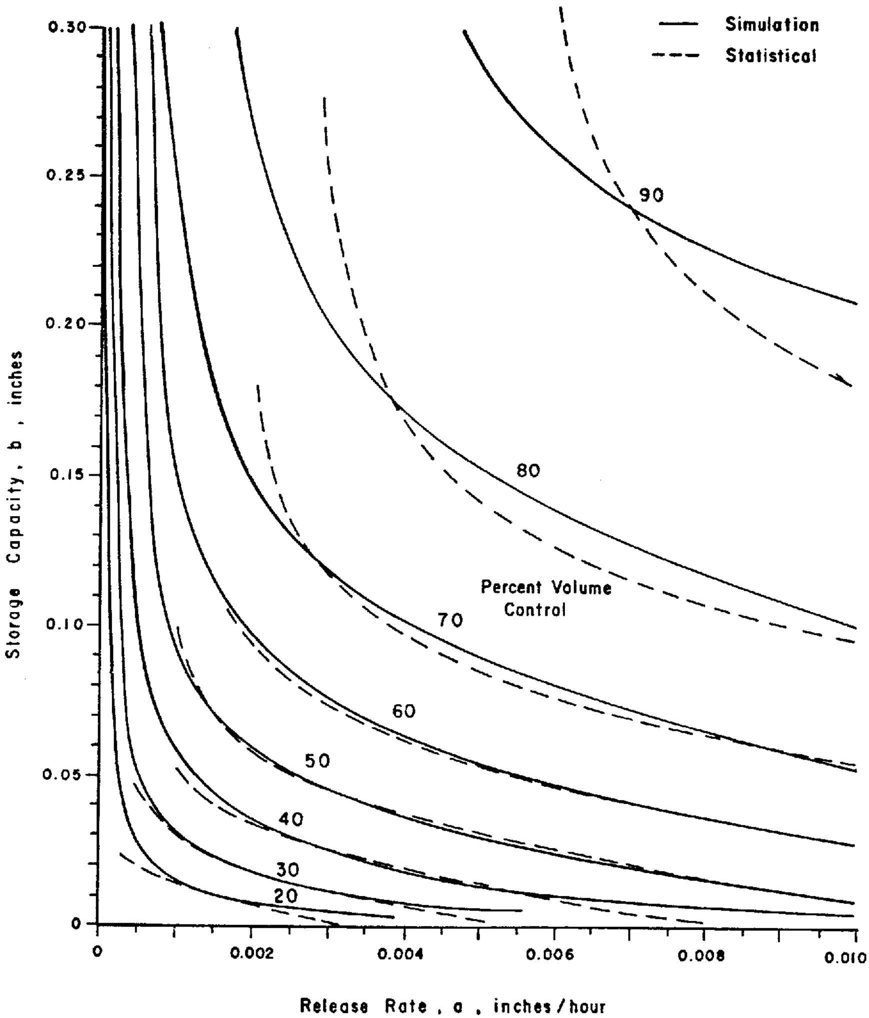


Figure 7.7. Isoquants for Runoff Volume Control for Minneapolis Catchment, Upper Bound.

detention unit up to the time it fills up. Afterwards no more inflow is accepted and the remaining runoff is bypassed. The storage equation that serves as the basis for the statistical model assumes that runoff will continue to enter the basin throughout the duration of the event - even after the unit is full. An additional runoff volume, given as the volume that is pumped from storage at the treatment rate from the time at which the reservoir is full until the cessation of runoff, can go into storage in the statistical model. For this reason the statistical model is capable of accepting a greater amount of runoff volume from an event than it would if it were to operate in the bypass mode as defined for the simulation study. The long-term implication is that, for a specified control level, the statistical model would require less storage capacity than that obtained from the simulation model for the same control level. Conversely, the simulation model operated on this particular mode would require more storage capacity for the given level of control because it retains less runoff from an event and thus needs more capacity to achieve the same level of efficiency as a system that accepts runoff continuously over the event duration. In the long run this yields the result that the isoquant obtained from the simulation model would be higher, requiring larger storage capacities, than would be the isoquants obtained from systems accepting runoff continuously. It is apparently for this reason that the simulation isoquant appears high with respect to the upper bound solution of the statistical model, although the influence from the first two observations may also be present.

It is, of course, expected that some discrepancy exists between the approaches. After all, the simulation model is a detailed approach that accounts for event variability at very short time intervals, while the statistical approach deals with long-term effects involving the major process parameters in a lumped fashion. Because of this it is all the more remarkable that the results have shown such good agreement and consistency.

The sensitivity of the isoquant to variations in the storage level at the end of the previous runoff event has already been analysed for the case of the previous section. The same general observations would hold true here and the analysis will not be duplicated. For the particular case of the bypass mode considered in the present section the results show that the condition of previously full available storage may be recommended as the condition to assume for isoquant estimation.

Good results have been obtained which establish the usefulness and overall validity of the statistical approach as compared to simulation results for the case of the hydraulic load. Isoquants for pollutant load control are discussed in the following section.

7.2.3 Isoquants for pollutant control

Two types of pollutant control assessment procedures are possible with the statistical model. In the first of these an uniform pollutant concentration is presumed to exist throughout the duration of the discharge. For this case the storage/treatment isoquant is exactly equivalent to the hydraulic storage/treatment isoquant because the constant concentration terms will cancel out upon taking the ratios of expectations of pollutant load, yielding the ratios of the hydraulic load defined in Chapter IV. The second formulation employs the first-order pollutant washoff model which has been characterized as defining "first flush" loading conditions. As indicated in the review of Chapter V, other formulations have been employed, mostly based on regressions, but the first-order model is deemed adequate for the present purposes. Indeed, its applicability to situations is more or less established, although it is somewhat difficult to fit the first-order decay parameter of the formulation to actual data (Jewell, 1983).

The statistical model, as formulated in Chapter V is now applied to several locations and compared to simulation results.

Pollutant control isoquant for Minneapolis

The results obtained in Chapter V are applied to a catchment in the city of Minneapolis - the same catchment as that of the last section. The equations that are applicable to this situation correspond to those of the special case, and are given, for the general case by Equation (5.86), and for the extreme conditions of previously empty or full storage by Equations (5.90) and (5.91), respectively. The results are shown in Figure 7.8 for the pollutant trap efficiencies of 50%, 70%, and 90%. A full range of previous available storage conditions is included for the 90% isoquant - the extreme conditions and intermediate values. Only three isoquants have been constructed because they illustrate the general behaviour of the model. At each specified treatment rate the conditional expectation of the runoff intensity is obtained in order to determine the rate at which the storage unit is being filled by the incoming runoff. The expression to be evaluated by the model is given in Equation (5.42), the intensity term being defined by Equation (5.40). Table 7.3 shows the values of the mean runoff intensity that are obtained from Equation (5.40).

The same type of comment that was made for the hydraulic isoquant applies here in a general sense. There is much greater sensitivity to the fraction of the available storage capacity that is assumed available at the end of the previous event for the higher trap efficiencies, as was the case for the hydraulic load. For the higher trap efficiencies the simulation results indicate that the system is in an intermediate storage condition, on the average, at the

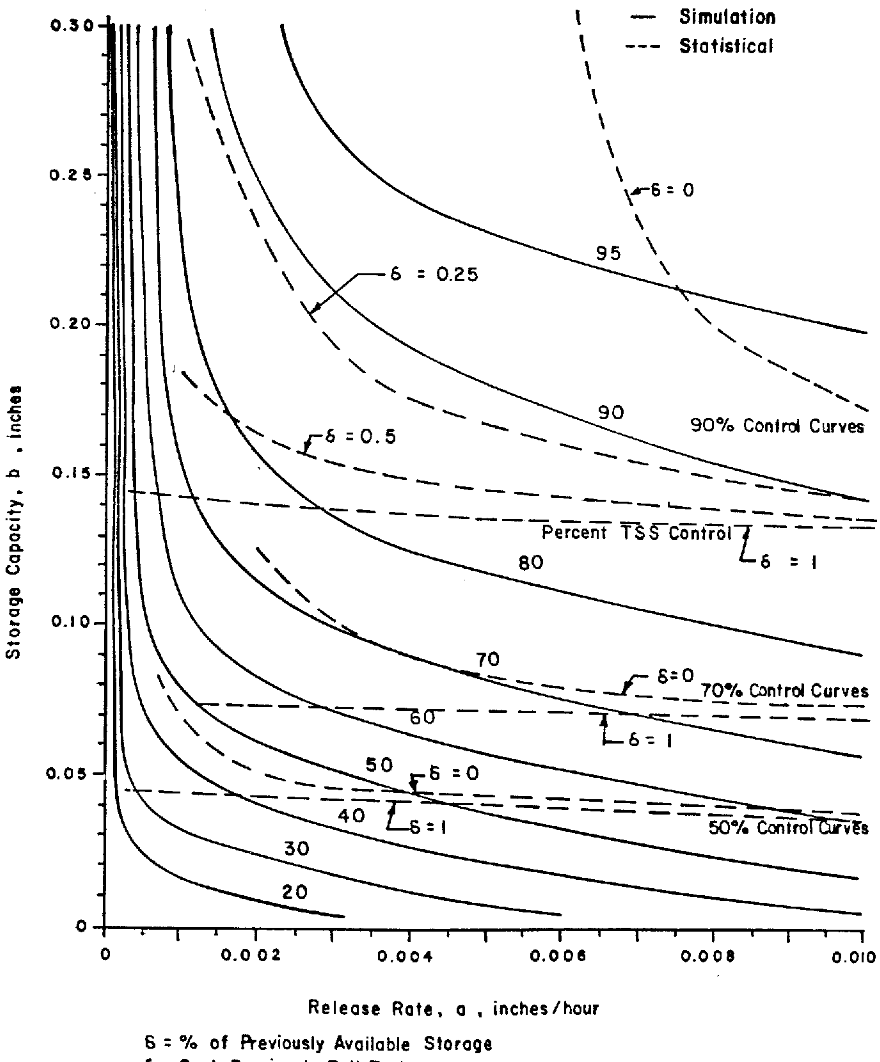


Figure 7.8. Pollutant Control Isoquants for Minneapolis Catchment.

Table 7.3 Conditional Mean Runoff Intensity for Minneapolis

Treatment Rate (in/hr)	I(a) (in/hr)
0.001	0.109
0.002	0.117
0.003	0.125
0.004	0.133
0.005	0.141
0.006	0.149
0.007	0.157
0.008	0.165
0.009	0.172
0.010	0.180

end of each event on the order of less than 50% available storage. For the lower trap efficiencies the storage condition at the end of the previous event does not affect the results appreciably over a range of treatment rates but it does make a difference for low treatment rates, depending on the particular trap efficiency level chosen.

Example 7-3: Storage requirement for pollutant control.

It is desired to obtain the storage capacity required to control 70% of the Total Suspended Solids (TSS) load obtained from the Minneapolis catchment. Runoff is removed from the unit at the rate of 0.006 in/hr. The special case condition is in effect.

The statistical runoff parameters are given by α=11.36 h^{-1}, β= 0.137 hr^{-1}, and γ= 0.011 hr^{-1}. The conditional mean runoff intensity is obtained from Table 7.3 as I(a) = 0.149 in/hr. It is used to obtain the parameter p in Equation (5.43) as p = 1.04. The first order washoff parameter is given as K = 4.6 in^{-1}.

The isoquant for this case is defined by Equation (5.86), with the extreme cases given by Equations (5.90) and (5.91). The values of the coefficients are obtained by direct substitution of the known parameters in the appropriate expressions. The following values are obtained for the coefficients:

$$r_1 = 0.288 \ P_o \ lb$$
$$r_3 = r_1$$
$$q_3 = 0.029 \ P_o \ lb$$
$$q_4 = 0.260 \ P_o \ lb$$
$$M_x = 0.202 \ P_o \ lb$$

The value of P_o drops out of the isoquant, so its value need not be specified.

If it is assumed that the storage unit is previously empty Equation (5.90) is used to obtain:

$$b = 0.0602 \ln (3.345)$$

or :

$$b = 0.072 \text{ in.}$$

If the critical condition of previously full storage is employed, Equation (5.91) is used to obtain:

$$b = 0.0542 \ln (4.53)$$

or :

$$b = 0.082 \text{ in.}$$

If an intermediate previous storage condition is specified, say $\delta = 50\%$, meaning a half-full unit at the end of the previous event, the storage capacity is obtained from Equation (5.86):

$$0.260 \exp(-17.5b) + 0.029 \exp(-8.3b) - 0.086 = 0$$

from which it is obtained:

$$b = 0.075 \text{ in.}$$

If the latter is the design value to be specified, this would imply a storage capacity of 4 acre-ft.

The isoquants obtained for the statistical method show a somewhat flat response over a range of treatment rates of the figure. The equations produce a decreasing curve with increasing treatment rate for all cases but the rate of decrease is slower than that obtained with the simulation for this particular case. The possible explanation of this effect has to do with the manner in which the simulation study is set up, and points to a pitfall that arises in using simulation studies with short records to predict long-term system behaviour. The simulation study has used a year's lenght of rainfall records to determine, by relative frequency analysis, the shape of the isoquant. This record is representative, in terms of the statistics, of the total available record. The problem with this approach is that the event occurrences are fixed for all simulation runs. The effect of increased treatment rates on relative storage requirements is obtained relative to a fixed realization of the hydrologic process, with increasing treatment rates being measured against fixed runoff rates. Thus, a relatively rapid decrease in storage requirement could occur with this approach as the treatment rate is increased with respect to prescribed runoff rates. In an actual situation the system would be operating against the full sample space of hydrologic event possibilities, which may include events of an extreme nature, or simply events, which exceed the intensity of the events within the particulart data set employed in the simulation. The statistical load model, in taking account of these possibilities via the statistical distribution of the associated hydrologic variables, exhibits a slower decrease in storage requirement with increasing treatment rates. This is interpreted as reflecting the fact that increased treatment rates do not reduce the likelihood of obtaining high intensity runoff rates (yielding a large load over a short duration) to the extent implied by the simulation results. In this respect the statistical results appear more realistic.

Comparing the isoquants produced by the first order pollutant load model and the isoquants produced by the hydraulic model (which can be interpreted as the uniform concentration model) it is seen that the pollutant load model produces isoquants that require, over a range of treatment rates, less storage capacity than the isoquants of the hydraulic model. The first-flush condition described by the first-order load model will produce a greater pollutant load with the same runoff volume as would the uniform concentration model. For a given trap efficiency the first-order model would require less runoff volume to produce the specified control level and thus less storage capacity would be required. However, this is not true for all situations. Table 7.4 compares the results, for the Minneapolis data, of the storage requirements produced by the first-order and uniform concentration models. This is done in terms of the ratio of storage requirements for both models for a given treatment rate and control level. For the lower trap efficiency more storage is required by the pollutant load model as the treatment rate increases, as the storage ratio is greater than unity. This phenomenon has been observed by Nix (1982), and is attributed to the fact that relatively large treatment rates (actually, lower ratios of storage capacity to treatment rate) relieve the basin fast enough to allow it to capture less concentrated loads at the end of many events, and thus the increase in storage requirements. For the higher trap efficiencies the system works better at capturing the pollutant load than the runoff load because of the first flush effect, and less storage is required.

The planner has to make the choice about the objective of the detention system that is designed - whether it is primarily for pollution control or runoff volume control. This in turn is dependent on the occurrence of first-flush effects at the site. If first-flush effects are evident and

Table 7.4 Comparison of Runoff and Pollutant Control Storage Requirements for Previously Full Storage

	Ratio of Pollutant load storage requirement to runoff volume storage requirement	
Treatment rate (inches/hour)	90% control	50% control
0.001	-*	0.67
0.002	-	0.89
0.004	-	1.20
0.006	0.96	1.68
0.008	0.93	2.61
0.010	0.97	5.34

* - Points for these rates were along the asymptotic part of the isoquant and gave very high values of the storage capacity, and thus fell outside the useful range.

pollution control is the desired objective then the isoquants corresponding to the first-order model should be generated.

The use of the statistical method is illustrated in a second application, to the city of Denver, Colorado.

<u>Pollutant control isoquant for Denver</u>

As part of the nationwide combined sewer overflow study, Heany et al. (1977) developed Biochemical Oxygen Demand (BOD) control curves for several gaged urban catchments in the Nation. The isoquants for BOD control were obtained with the simulation model STORM, whose general mechanics were discussed in Chapter I. The stormwater system was operated in the off-line mode wherein runoff flows directly to the treatment plant at the treatment rate and is diverted to the storage unit whenever the runoff rate exceeds the treatment plant capacity. The statistical model is applicable to this situation for the case of uniform pollutant concentration, or the runoff volume model. It is also applicable for the case of the first-order load model but only approximately so because, for the off-line case, the runoff pollutant load does not go in its entirety to the storage unit first. A part of it will go directly to the treatment plant. This amount, over the runoff event duration at the given treatment rate is relatively small for events which require diversion and use available storage space, so that the statistical model can be applied to this type of configuration.

The parameters for the model were obtained from the Hydroscience study (Hydroscience, 1979). These correspond to the parameters of the statistical study. They are described in Table 7.5. For generating the isoquants the first-flush condition was utilized in the model STORM, with the first-order decay coefficient discussed in Chapter V.

The previous applications of the statistical model have shown that for the higher trap efficiencies it is reasonable to assume that some left-over runoff volume will be available at the end of the previous event. To that effect the simulation results are compared to the special case of the statistical load model for the extreme conditions of previously full and empty storage. The procedure is the same as in the previous section, with new values of the conditional mean runoff intensity generated with the Denver data. The results are shown in Figure 7.9. Only one set of isoquants is illustrated, corresponding to the 90% BOD trap efficiency. Isoquants for the other control level gave more or less similar results.

The results again show good agreement with the simulation results. The simulation isoquant is near the statistical isoquant of the extreme condition. Some discrepancies may arise due to the approximations involved in simulating the off-line case and the fact that the Denver runoff data

Table 7.5 Runoff Parameters for the Denver Simulation

Parameter	Description
Year of record	1960
Mean runoff volume, $E[X_1]$	0.078 in.; coefficient of variation (c.v.) = 1.49
Mean runoff duration, $E[X_2]$	4.8 hr.*; c.v. not available
Mean interevent time, $E[X_3]$	119 hr.; c.v. not available

Adapted from Hydroscience (1979).

* - This parameter was estimated from the rainfall records.

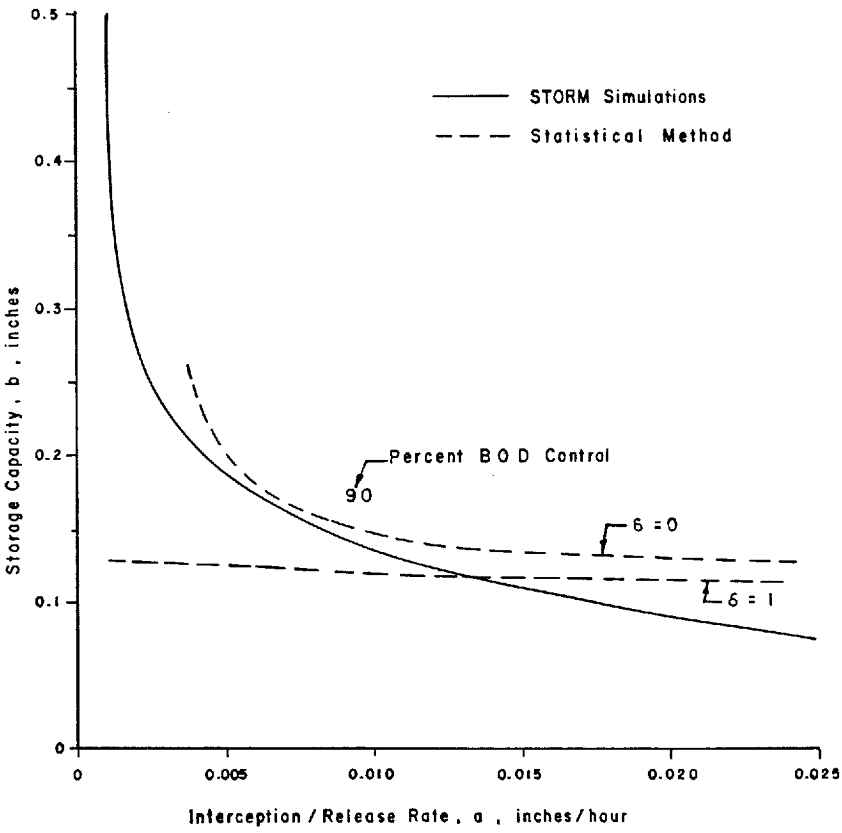


Figure 7.9. BOD Control Isoquant for Denver; Comparison of Lower and Upper Bound Statistical Results to Storm Simulation.

shows the mean runoff volume with a coefficient of variation different from unity, which may indicate a different distribution than the exponential, in this case a gamma. Nevertheless the agreement is quite good and provides a further validation of the statistical model.

The applications of the following section considers the situation wherein the system runoff trapping capacity is limited, and how this affects the shape of the isoquant.

7.3 Applications to Systems of Limited Runoff-Trapping Capacity

The general formulation of the statistical model incorporates a parameter that sets a limit on the amount of runoff that the detention unit can accept during any runoff event. This parameter has been defined as the runoff trapping capacity of the stormsewer system. It has been defined as a constant for any particular system representing, in a lumped fashion, the total capacity of the system for capturing the runoff volume. By definition this parameter represents the overall potential trapping capacity. The inlets to the stormsewer are hydraulic control devices whose capacities are functions of hydraulic and geometrical properties. The trapping capacity term is a representation of these devices, or, more precisely, a representation of the potential capability of these for capturing runoff flows. Such a potential is capable of being defined because runoff depths are not allowed to increase indefinitely over the urban catchment. Since flow accross the inlets is a function of flow depth over the inlets, the prescription of maximum allowable depth also specifies a maximum allowable inlet flow, which for this type of application can be represented by the trapping capacity term.

A further application along this line arises for the case of controlled inlet flow, as related to the concept of dual drainage modeling discussed in Chapter II. In this procedure the inlet capacity is controlled by an in-situ device that regulates flow, allowing flow into the inlet up to a certain maximum rate. The excess runoff is diverted to park storage. This procedure has proved effective in controlling surcharge of the stormsewer and has been applied successfully to some locations in Canada (Roussel et al., 1986; Wisner et al., 1986). The trapping rate as defined in the statistical model can represent inlet control devices as employed in the dual drainage concept.

In a theoretical sense, the trapping capacity term should be a term representative of the hydraulic and hydrologic interactions of the catchment, but the interplay of the processes is quite complex and present efforts can only account for the major characteristics associated with urban stormflow which are relevant to the deterministic and event-based nature of most models. The runoff trapping capacity term, c, is, in a certain sense, a statistical parameter - a mean rate of runoff arrival at the detention unit that is controlled by the hydraulic transmission properties of a particular system under a certain hydrologic input. It also represents, theoretically, the long-term potential trapping capacity of the stormwater collection system, so that the term is not only a spatial parameter but represents temporal effects as well. This would seem to suggest the development of a parameter estimation procedure to assess the value of the term. The term is not merely a rainfall-runoff transformation - it incorporates collection system properties that affect the detention unit's long-term performance.

The present effort is not directed at establishing parameter estimation procedures for the capacity term. Rather, it is directed at defining the parameter and assessing the manner in which it affects the hydraulic or pollutant control isoquants, and thus the design and cost of detention units. The basis for representing the trapping system through a parameter have been established and the solution obtained would be applicable to situations which can be represented as such.

7.3.1 Application to Atlanta

The general statistical model is to be applied to some of the catchments that have already been analyzed with the special case formulation, now assuming the existence of a runoff collection system of limited capacity. The first application is to the 24.7 acre catchment in Atlanta. To chose a realistic value of the density of inlets to the stormsewer system the inlet density for a 26 acre urban catchment in Canada is assumed to exist for the area. This is given by Wisner et al. (1982) as 1.78 inlets per acre, and is considered a very high density. For a 24.7 acre catchment this would give about 44 inlets.

The actual capacity for each inlet is a function of inlet type and the particular geometric arrangement of the inlet environment. The capacity is also affected by in-situ field conditions as they develop during the event. This would correspond to debris accumulation over the gratings and obstruction of approach flow by vehicles or other objects, both over the inlet and along the approach gutter. To a certain extent the latter represent random effects and are seldom accounted for in hydraulic models. The runoff trapping capacity term of the statistical model is a measure of the overall catchment runoff capture efficiency. If its true value is known it would be a measure of all of the aforementioned effects. Until field studies are available to estimate this parameter accurately suitable estimates must be obtained for analysis. It will be assumed for this application that inlet flow control devices have been installed to regulate flows into the sewer system. These were actually installed in the 26 acre catchment described by Wisner and Kassem (1982). One of the devices tested was an orifice plate that allowed a maximum inflow of one cubic foot per second (cfs). Flows exceeding this capacity were carried further downstream as carryover flow. Eventually, the excess volume is routed to surface storage. Assuming that all inlets have a maximum capacity of 1 cfs the total runoff trapping capacity for the Atlanta catchment with 44 inlets is about 1.77 inches per hour, in terms of the total catchment area.

The runoff data for the city of Atlanta has already been presented, and is utilized again for this example. Because the runoff data is available no transformation has to be made to obtain the runoff event duration statistics. The results of Chapter IV are used to obtain the isoquants, but a choice must be made of the appropriate equation to use from the three general expressions available. Generally, since the equations are to be solved for the storage capacity each equation is solved in turn for a value of the storage capacity and a check is made with the range of each equation to determine which solution satisfies its constraints. The one that does is the appropriate solution for the given set of parameters. As discussed in Chapter IV the expressions to solve for the isoquants are differentiated by the value of the parameter k_1. The parameter k_1 is given by the following expression:

$$k_1 = (a - c)x_c \tag{7.2}$$

where a is the treatment, or withdrawal, rate from the storage device, c is the system runoff trapping capacity rate, and x_c represents the time base factor that is used to transform the effective rainfall duration into the runoff duration, an estimate of which is the catchment time of concentration, as discussed in Chapter II. The term k_1 is always negative, as it has been assumed that the system runoff trapping capacity is always greater than the design treatment rate, otherwise there would be no need for a detention unit. The negative of Equation (4.1) would represent a potential amount of runoff volume that could be made available over the duration of the runoff event beyond the effective rainfall duration. The term a-c is the negative of the difference between the potential rate at which the system can transmit runoff and the rate at which the stored runoff is depleted from the detention unit. For the present example actual runoff data has been generated so that it is not necessary to use x_c, and its value is taken as zero. For this situation the equation that is used to obtain the storage/treatment isoquant is either of Equations (4.33) and (4.34), because $k_1 = 0$. Expressions for the required storage capacity for the extreme conditions are given in Equations (4.35) through (4.38).

The shape of the isoquants obtained from the appropriate expressions are similar in shape to the ones obtained for the special case, but they will be lower than those obtained from the special case because of the finite runoff trapping capacity of the stormsewer system. This implies lower storage requirements on the detention system. This suggests a further application of the methodology in the sense that variations in storage requirements due to variations in system runoff trapping capacity can be assessed to determine asymptotic values of the trapping capacity for which negligible variations in storage requirements are obtained. Such asymptotic rates would establish design capacities beyond which no additional increases in storage requirements are obtained.

With a finite value of the trapping capacity there will exist a positive probability of exceeding this rate. This is obtained from Equation (2.29). For the parameters of the present illustration Equation (2.29) predicts less than 2% chance of exceeding the trapping rate of 1.76 inches/hour, which indicates a fairly high capacity system. An alternative use of Equation (2.29) is to specify an exceedance level and solve for a value of c, the trapping rate, which is used to set the inlet flow control mechanisms.

A number of inlet control devices are discussed by Wisner et al. (1986) whose capacities can be used in the present example to assess the effects of varying system capacities on the storage/treatment isoquants, although any particular design value could be used. To illustrate the effects of limited system hydraulic capacities the isoquants for trap efficiencies of 50% and 90% will be obtained. They will be obtained only for the extreme case of previously full storage condition because this will suffice to illustrate the variations of storage requirements produced by systems of limited capacity. The equation to use is either of Equations (4.35) and (4.36). The difference between the equations is that Equation (4.35) defines the isoquant in terms of the runoff volume arriving at the detention unit, and Equation (4.36) is in terms of the total runoff volume, which is equal to or greater than the runoff volume arriving at the detention unit. The results obtained with Equation (4.35) appear in Figure 7.10, and those obtained with Equation (4.36) appear in Figure 7.11. It is seen that with increasing runoff control less storage would be required because less volume eventually gets down to the detention unit. Table 7.6 compares the results obtained in terms of the ratios of required storage capacities for specified trap efficiency levels and treatment rates, together with values of the inlet exceedance probability.

The results show that reducing the runoff trapping capacity of the system reduces storage requirements but increases the chance of having the system runoff trapping capacity exceeded. This excess volume would be diverted to park storage, or suitably disposed of. The mean excess

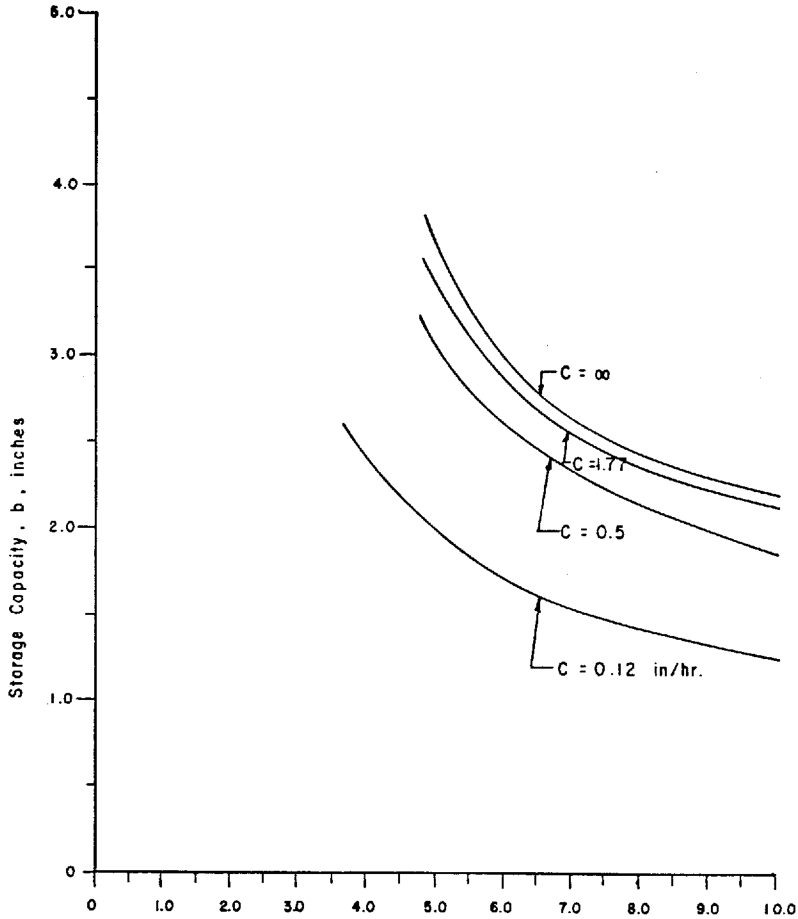


Figure 7.10. Runoff Control Isoquants for different Levels of System Stormwater Trapping Capacity, in Terms of the Mean Stormsewer Runoff Volume, for the Atlanta Catchment.

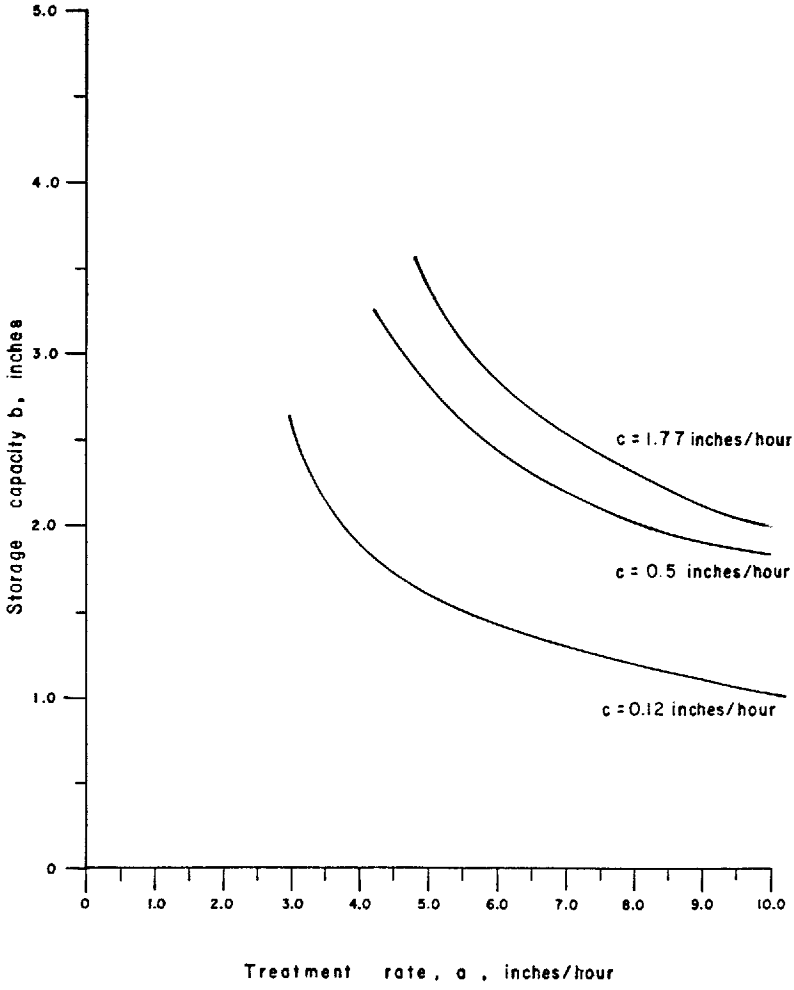


Figure 7.11. Runoff Control Isoquants for Different Levels of System Stormwater Trapping Capacity, in Terms of the Total Runoff Volume, for the Atlanta Catchment.

Table 7.6 Comparison of Storage Capacity Requirements for Systems of Limited Runoff Trapping Capacity for the Atlanta Catchment

Normalized treatment rate	Ratio of unlimited system capacity storage requirement to specified limited system capacity storage requirement. Runoff trapping capacity (inches/hour) 1.77	0.50	0.12*
5.0	1.04	1.18	1.81
6.0	1.04	1.15	1.72
8.0	1.03	1.14	1.70
10.0	1.04	1.14	1.75
Trapping system exceedance prob.	1.8%	6.1%	21.2%

* - Hypothetical rate corresponding to 0.1 cfs per inlet for 30 inlets. The other rates are from Wisner et al. (1986).

runoff can be estimated with Equation (2.43). Since real-life runoff collection systems are limited capacity systems, which may be of relatively high capacity, the probability always exists of having an event that would exceed the capacity. For the traditional system design procedures this probability would be obtained in terms of the recurrence associated with design rainfall intensities. The statistical methodology proposes an event-based procedure for obtaining the probability of exceeding system capacity in terms of the actual runoff parameters of the catchment.

Example 7-4: storage estimation for system of limited trapping capacity.

It is desired to obtain storage capacity requirements for the Atlanta catchment, for a system of runoff trapping capacity of $c = 0.50$ in/hr, and for a normalized treatment rate of $a/Q_o = 8$. Since $Q_o = 0.00179$ in/hr, this implies a treatment rate $a = 0.014$ in/hr. The storage estimate is chosen to correspond to the 90% trap efficiency level, in terms of the runoff volume arriving at the detention unit. Assuming that the design is to be made for the most critical condition — that of previously full storage — Equation (4.37) is used to determine the storage capacity.

The parameters of the coefficients have already been defined in Example 7-1. With these, the following values are obtained for the coefficients of the storage equation.

$$g = -\ 4.91\ \text{in}^{-1}$$

$$H = 0.073$$

$$m_Z = 0.21\ \text{in}$$

Substitution of all terms in Equation (4.37) yields:

$$b = -\ 0.182\ \ln\ (0.164 - 0.116)$$

or

$$b = 0.55\ \text{in}$$

In situations wherein pollutant load control is the major concern the use of inlet control devices would be advantageous because only the first stages of the runoff event are captured by the detention unit and they would carry the larger amount of the pollutant load. This could result, depending on the conditions, in reduced storage requirement. The results obtained in this section show the extent to which storage requirements may be altered by considering limited-capacity systems.

It is desired to transform the rainfall event duration into a runoff event duration via the transformation of Equation (2.6). It is assumed that the effective rainfall event duration is given for this case by the rainfall duration, as an approximation. The transformation of Equation

(2.6) would determine the catchment runoff duration, or the hydrograph time base. It is assumed in the statistical model that the runoff trapping capacity is being exerted throughout the catchment over the duration of the runoff event as given by Equation (2.6). As such the potential amount of runoff that can be trapped by the system over the event duration represents the maximum runoff volume available to the detention unit via the runoff conveyance system. This is so because the actual runoff durations over any particular inlet may vary, but are not expected to be greater than the catchment hydrograph duration. To achieve the transformation the parameters of Equation (2.6) have to be estimated. Because the rainfall duration is assumed to be equivalent to the effective rainfall duration only the parameter corresponding to the catchment time of concentration, x_c, is estimated. The other parameter, a_2, is assumed equal to unity. If rainfall-runoff records were available a regression analysis would provide an estimate of the transformation parameters. For the following application an estimate of the time of concentration will be used as the parameter estimate.

7.3.2 Application to Minneapolis

For this application the trapping system of limited capacity is considered, as well as the effective rainfall duration transformation. To account for the limited capacity system a procedure similar to the one employed for the Atlanta catchment is followed. The inlet density (typical values are obtained from Wisner et al., 1986), and the total system runoff trapping capacity are estimated assuming the existence of inlet flow control, and the isoquants are evaluated. If inlet control does not exist the trapping rate is estimated from maximum allowable runoff depths over the catchment. The runoff depths are translated into inlet flow rates from available rating curves, which in this situation are likely to be described by the orifice flow equation. Conversely, several design values may be employed, and the trapping rate most appropriate converted to inlet capacity specifications.

The event duration transformation is also to be applied. In a sense the event duration statistics that have been obtained from the simulation models STORM and SWWM are effective rainfall duration statistics because of the manner in which the rainfall runoff transformation is achieved. The coefficient method that has been employed to obtain isoquants from simulation studies merely transforms the rainfall event duration into effective duration by applying a runoff coefficient and a variable infiltration rate. The effective event duration is assumed to represent the actual runoff duration. Although the model SWWM has refined hydrograph generation and routing capabilities, Nix (1982) employed STORM generated runoff data, using the coefficient method, as input to the storage/treatment block of SWWM to determine the isoquants because of the cost involved in actually routing a hydrograph for each of the total number of events necessary to estimate the isoquant.

In the hydrologic process the effective runoff duration is not necessarily equivalent to the overland runoff duration due to the physical process of runoff flow. To account for this difference the linear runoff transformation has been proposed. To account for these effects the parameters of the transformation are estimated and the transformation is applied in this application. The parameter to be used is the time of concentration, as discussed previously. These are only estimates but what is desired is to assess the relative effect of these considerations on storage capacity requirements.

The time of concentration was not directly available for the catchment under consideration. It has been estimated by proportioning the catchment area with the average catchment area and average time of concentration for 48 urban catchments in the Nation, obtained by McCuen et al. (1984).

The equations to be employed for this case are those given in Article 4.5 of Chapter IV. The procedure has already been outlined. The isoquants can be obtained either in terms of the actual runoff volume arriving at the detention unit or in terms of the total event runoff volume. The parameters to be employed appear in Table 7.7. The results obtained, in terms of the runoff volume arriving at the detention unit, are shown in Figure 7.12. As a general rule, the larger event durations have greater potential for capturing higher runoff volumes due to the runoff trapping rate operating over the event duration. The implication is that larger runoff volumes can now be trapped by the system. This can be offset by the fact that the system will be pumping water for a longer duration, and may actually reduce storage requirements for low intensity events. This is illustrated in Table 7.8, which compares the results obtained with the general case to those obtained with the special case in terms of the ratio of storage capacity requirements.

A further application is considered for the case of pollutant loads to assess the effect of the limited capacity system on the pollution control isoquants.

The parameters to be used in the application appear in Table 7.9. To simplify the analysis the pollutant trap efficiency isoquants are obtained only for the case of 20% storage capacity available at the end of the previous event. The isoquants appear in Figure 7.13. Only three isoquants have been obtained, as representative of the overall condition. Table 7.10 compares the results obtained under the present conditions with those of the special case in the form of ratios of required storage capacities for the given trap efficiency levels and treatment rates. Again the

Table 7.7 Parameters of the Statistical Model for the Case of Limited Runoff Trapping Capacity, for the Minneapolis Catchment

Parameter	Value
Catchment area	640 acres
Mean runoff volume	0.088 inches
Mean event duration	7.3 hours
Mean interevent time	94 hours
System runoff trapping rate	0.50 inches/hour
Estimated time of concentration	0.60 hours
Percent of previously available storage capacity	20

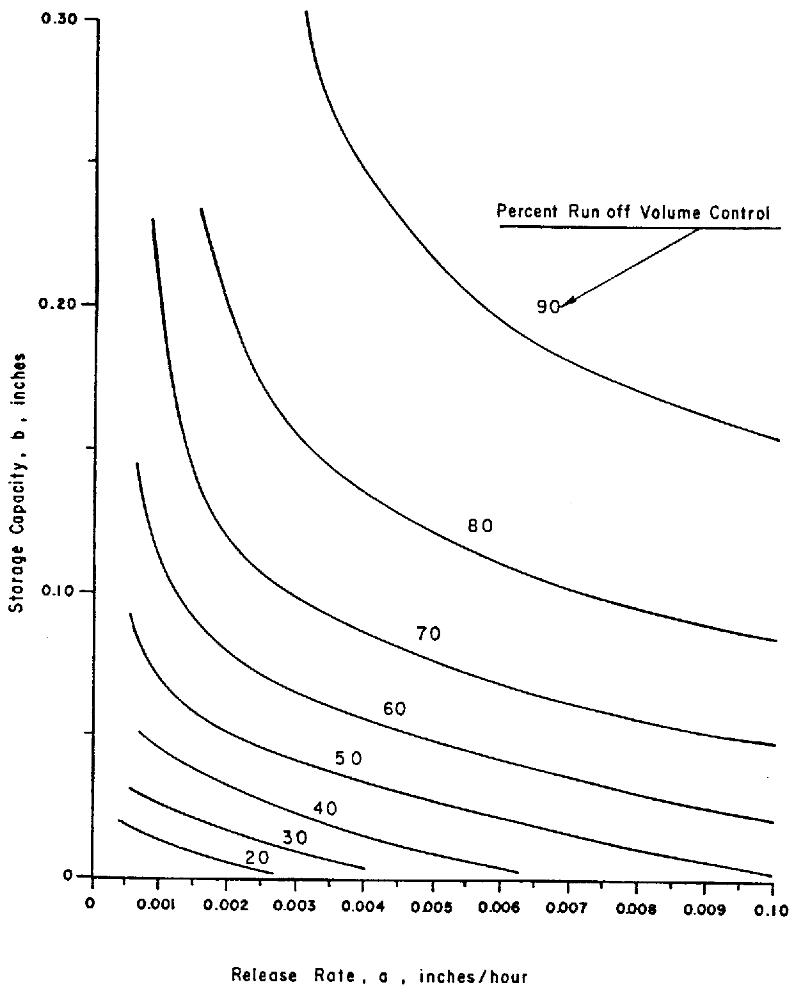


Figure 7.12. Isoquants for the Minneapolis Catchment, with Limited Runoff Trapping Capacity.

Table 7.8 Comparison of Storage Requirements for Combination of Runoff Rate and Time of Concentration, for a Treatment Rate of 0.005 inches/hour, in Terms of Storage Ratios for Two Trap Efficiencies, for the Minneapolis Catchment

Type of Storage ratio:	Unlimited capacity system to 0.5 in/hr trapping rate system	0.5 in/hr rate to 0.5 in/hr rate with time of concentration 0.6hr.
Trap efficiency		
90%	1.05	0.98
50%	1.07	1.10

Table 7.9 Parameters for the Pollution Control Isoquants for Systems of Limited Runoff Trapping Capacity, for the Minneapolis Catchment

Parameter	Value
Catchment area	640 acres
Mean runoff volume	0.088 inches
Mean event duration	7.3 hours
Mean interevent time	94 hours
System runoff trapping rate	0.12 inches/hour
Estimated time of concentration	0.6 hour
percent of previously available storage	20
First-order pollutant washoff rate	4.6 hr^{-1}

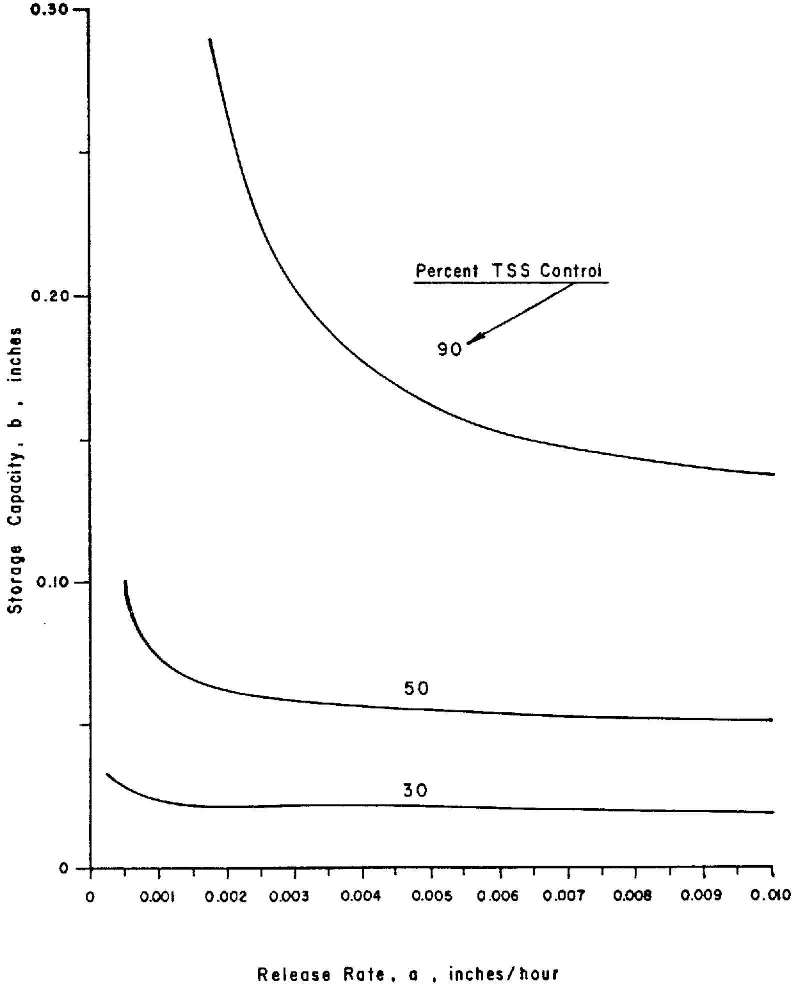


Figure 7.13. Isoquants for Suspended Solids Control for Systems of Limited Trapping Capacity, for the Minneapolis Catchment.

Table 7.10 Comparison of Storage Requirements for Pollutant Load Control, for the Minneapolis Catchment, in Terms of Storage Capacity Ratios, for Selected Treatment Rates

	Ratio of storage for unlimited system capacity to storage of limited system capacity of 0.12 inches/hour		
Treatment Rate (in/hr):	0.002	0.006	0.010
Trap efficiency level			
90%	1.08	1.08	1.10
60%	1.04	1.05	1.07
30%	1.02	1.04	1.06

results show that systems with limited runoff capture capacity require less storage.

Example 7-5: pollutant trap efficiency.

For this application the design of a particular system is to be evaluated in terms of its pollutant trap efficiency.

It has been assumed that the pollutant washoff coefficient in the first order model is given by $K = 4.6\ \text{in}^{-1}$, corresponding to the 90% washoff presumed to occur during the first hour of the runoff event. Although this value is widely used, in reality the value of this parameter is defined by on-site conditions and the nature of the particular pollutant under consideration. If different pollutants yield different coefficients then the trap efficiency for each can be assessed.

Patry (1983) has found values of K that differ significantly from the common value. For Suspended Solids (SS) Patry found a K of $11.0\ \text{in}^{-1}$. For Chemical Oxygen Demand (COD) a value of $17.1\ \text{in}^{-1}$ was obtained. These values are assumed to hold for the present example.

Considering the Minneapolis catchment it is desired to estimate the pollutant trap efficiency in terms of SS and COD, for a stormwater detention of capacity $b = 0.10$ in, and runoff removal rate of $a = 0.005$ in/hr. The runoff statistics have been presented in Example 7-3.

It is assumed that the system runoff trapping capacity is $c = 0.5$ in/hr; the time of concentration $x_c = 0.6$ hr; and the percent of previously available storge $\delta = 75\%$. The mean runoff intensity is given by $I(a) = 0.141$ in/hr. The value of p is obtained at 1.07.

The appropriate isoquant equation is chosen from the three available cases. This depends on the value of cx_c. Because $pb = 0.107$ in. is less than $cx_c = 0.3$ in., Equation (5.62), corresponding to Case I is utilized. The values of the coefficients, with the SS washoff coefficient of $K = 11.0 \text{ in}^{-1}$, are given below:

$$r_1 = 0.49\ P_o \text{ lb}$$
$$q_1 = 0.45\ P_o \text{ lb}$$
$$q_2 = 0.41\ P_o \text{ lb}$$

The value of P_o need not be specified as it will cancel upon obtaining the trap efficiency.

The previously available storage capacity is given by $s_o = 0.075$ in.

Substitution in Equation (5.62) yields the mean value of the SS load trapped by storage:

$$E[L] = 0.49\ P_o - 0.45\ P_o \exp(-2.45) - 0.041\ P_o \exp(-1.79)$$

yielding:

$$E[L] = 0.44\ P_o \quad \text{lb}$$

The trap efficiency is obtained from Equation (5.76), in terms of the actual pollutant load arriving at the detention unit. This load is obtained from Equation (5.73), which yields for SS:

$$E[L_{t,Z}] = 0.49\ P_o \quad \text{lb}$$

The SS trap efficiency is given by the ratio of loads as:

$$\rho_{\ell,z} = 89.8\%$$

Proceeding similarly for the case of COD, now with K = 17.1 in^{-1}, the following new parameter and coefficient values are obtained:

$$\begin{aligned}
r_1 &= 0.60\ P_o \quad \text{lb}\\
q_1 &= 0.56\ P_o \quad \text{lb}\\
q_2 &= 0.040\ P_o \quad \text{lb}\\
E[L] &= 0.57\ P_o \quad \text{lb}\\
E[L_{t,Z}] &= 0.60\ P_o \quad \text{lb}
\end{aligned}$$

The COD trap efficiency is:

$$\rho_{\ell,z} = 95\%$$

The unit is more efficient at capturing the COD load.

The next step in the analysis is to determine which storage/treatment rate combination is more efficient in terms of the required control efficiency and overall cost.

7.4 Detention Storage Optimization

The optimization problem has been formulated in Chapter VI in terms of production function theory. The objective was to minimize costs while providing a specified runoff, or pollutant, control level in the form of the trap efficiency. The isoquants serve as the constraint of the process. The costs are related to the storage and treatment costs in terms of storage capacity and treatment rate to be provided. The variables to be optimized are the storage capacity and treatment rate. While other parameters could be considered as representing decision variables as well, such as the system runoff trapping capacity and the capture efficiency itself, the most important are the ones chosen because they represent the major decision elements in the system, and they lend themselves to an efficient, and manageable, optimization formulation. For the reasons given in Chapter VI the actual required values of the other parameters are chosen by regulating agencies or other entities.

The objective of the optimization is the determination of the optimal expansion path, as defined in Chapter VI. The expansion path represents the locus of efficient, in terms of minimum cost, combinations of storage capacity and treatment rate providing a certain amount of flow or pollutant capture efficiency. The planner then makes use of the expansion path within the larger framework of the facilities planning problem. The major advantage of the statistical model within the context of this problem is that it provides for, generally, closed-form solutions for the storage/ treatment isoquant constraints of the optimization problem, without the need to perform frequency analysis of simulation results.

Nix (1982) has fitted mathematical production functions to the isoquants obtained from simulation in an effort to obtain analytical expressions for the isoquant. Also, Heany (1977) has employed a trascendental equation as a general expression to use for isoquants. However useful, these lack the physical and theoretical basis that the statistical model provides, specially for areas where simulation studies are not conducted to validate the use of a prescribed mathematical function. It is in part for this reason that graphical optimization is undertaken in the absence of adequate analytical expressions. One of the difficulties associated with graphical optimization techniques is that of defining the point of tangency between the isocost lines and the isoquant, especially for non-linear isoquants. Nix (1982) has developed a graphical optimization technique in cost space to circumvent this difficulty, but some of the inconveniences in working with graphical techniques remain.

The analytical nature of the isoquants facilitates the optimization procedure as the problem can be formulated analitically without the need for graphical solutions, although these can still be applied, now more efficiently because any point along the isoquant can be analytically obtained. A relatively simple procedure that yields the expansion path is to equate the slope of the isoquant to the slope of the isocost curve, and then solve for the optimal values of storage capacity and treatment rate. The slope of the isoquant is obtained from the first derivative of the function. Some difficulty exists in the algebra due to the non-linear nature of the isoquants, and to the fact that several expressions may be required to define the isoquants for the general case. Nevertheless a considerable saving in effort is achieved in the long run over the graphical techniques.

The cost function associated with the optimization problem has already been defined. For the analysis in this section the representative costs associated with storage of stormwater and treatment will be utilized, but non-linear cost functions can also be utilized. The representative costs pertain to average conditions. It is possible, even likely, that local costs will vary, and coefficients will have to be determined appropriately. The representative annual costs for the city of Minneapolis, obtained from the study by Nix et al. (1977), are shown in Table 7.11, along with other parameters. The costs represent average costs and incorporate all related costs that add on yearly, such as operation and maintenance costs.

The optimal solution can be found either in terms of the flow capture efficiency or the pollutant capture efficiency. The applications will be illustrated for the Minneapolis catchment, considering the different situations that have been analyzed thus far. The expansion path for the runoff volume control isoquant is presented in Figure 7.14. The expansion path for the pollutant control isoquant is presented in Figure 7.15. The optimal combination of treatment rate and storage capacity are obtained along the expansion path. In Figure 7.16 the difference between the two approaches, in terms of the total cost as a function of the control level, is illustrated.

It is seen that the pollutant control formulation will yield lower overall costs than that for runoff control. This is due to the first flush condition regarding pollutant loads. In situations wherein the pollutant concentration is approximately uniform over the event duration higher control costs are to be expected because greater runoff volumes must be trapped, requiring larger storage capacities.

Example 7-6: formulation of an optimization problem.

In this example an optimization problem is formulated

Table 7.11 Unit costs of Stormwater Management for the city of Minneapolis

Item	Value
Urbanized area	215,000 acres
Population density	7.92 persons per acre
Annual unit cost for secondary treatment	$ 9810 /acre-in/hr
Annual unit cost of storage	$ 219 /acre-in

Costs are in 1977 dollars.

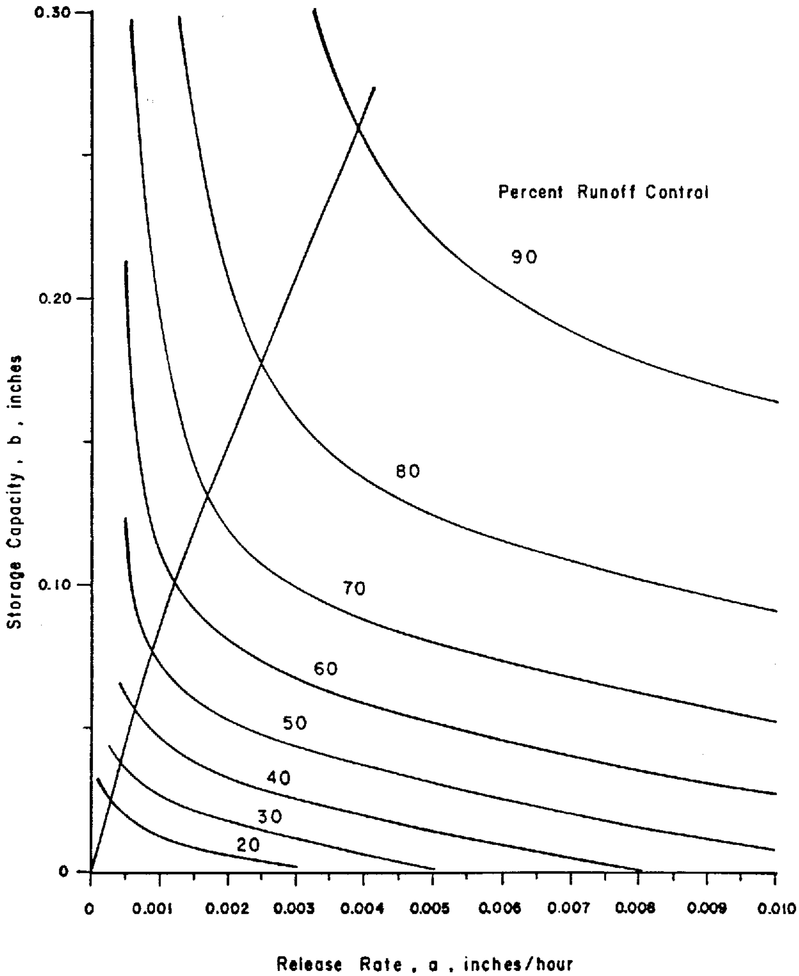


Figure 7.14. Expansion Path for Runoff Volume Control, for the Minneapolis Catchment.

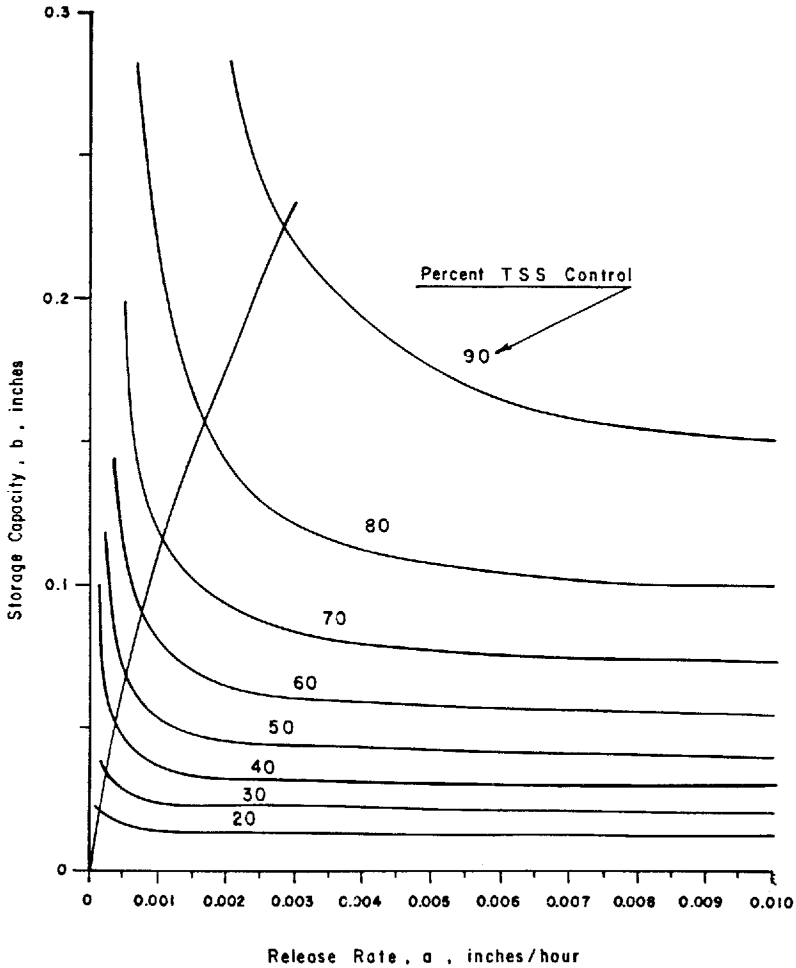


Figure 7.15. Expansion Path for Pollutant Load Control, for the Minneapolis Catchment.

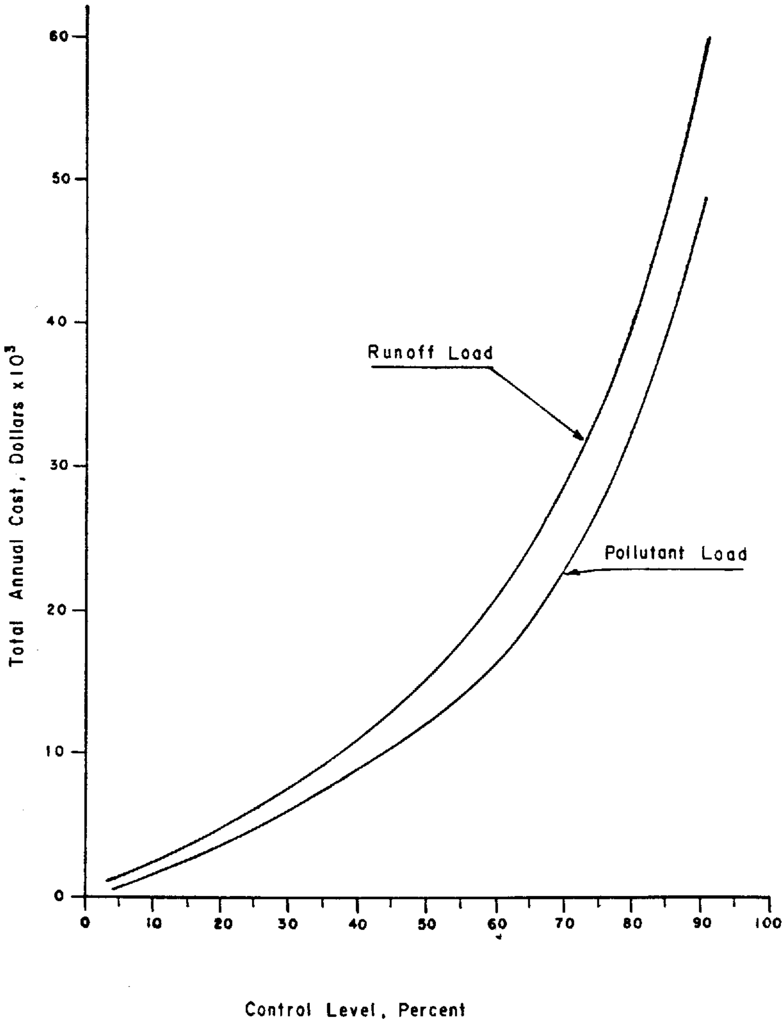


Figure 7.16. Total Annual Costs for Levels of Runoff and Pollutant Control, for the Minneapolis Catchment.

to obtain a point on the expansion path for a detention unit, for the Minneapolis catchment. For simplicity the special case is considered (corresponding to a system of unlimited runoff trapping capacity). This is done for runoff volume control, to illustrate the formulation used to obtain Figure 7.14.

The statistical parameters for the Minneapolis catchment have been presented in earlier examples. It is assumed that 25% of the storage capacity is available at the end of the previous event.

The formulation of the general optimization problem is given in Equations (6.13) and (6.14). The cost function to be minimized is defined with the unit costs of Table 7.11. For the 640 acre catchment the total yearly cost in dollars is given by:

$$C = 6.78\ (\times\ 10^6)a + 1.402\ (\times\ 10^5)b$$

where a is the treatment rate and b the storage capacity.

The constraint of the formulation corresponds to the storage/treatment isoquant, in terms of the detention unit trap efficiency, which is assumed to be ρ = 80%. For this case the isoquant is obtained from Equations (4.34) and (4.39). Substitution of the catchment parameters in these equations yields:

$$H[82.9a \exp(-11.36b - \frac{0.0082}{a}b) + \exp(-2.84b)] = 0.2$$

where

$$H = \frac{0.0015}{(11.36a + 0.137)(11.36a + 0.011)}$$

and $a, b \geq 0$.

The solution would yield the optimal values of a and b for the specified trap efficiency; and represents a point on the expansion path corresponding to the 80% trap efficiency.

This application completes the illustrations with the statistical model. Overall the applications have shown good agreement with simulation results and have provided insight into the nature of the stormwater detention problem, with tools that are relatively straightforward to apply and avoid the need of extensive simulation.

The following Chapter will present the conclusions to be obtained from this study and the recommendations for further research.

CHAPTER VIII

Conclusions and Recommendations

8.1 Conclusions

A statistical model has been formulated for the study of stormwater detention units. The results obtained through the statistical approach have compared favorably with the results obtained from detailed simulation studies. The following general conclusions are obtained from this study:

a) The statistical approach is proven to be a valid approach to stormwater detention planning. Results have compared favorably to those obtained from simulation models, in terms of the storage/treatment isoquants for runoff and pollutant load control.

b) The use of exponential distributions for process variables may be somewhat limiting in some cases, but the analytical tractability obtained with exponential distributions is a major advantage of the formulation.

c) Because of its analytical, largely closed-form formulation, the statistical model can be easily incorporated into comprehensive land use planning frameworks.

d) The formulation of the optimization problem is greatly facilitated with the formulation of analytical device performance measures, in the form of storage/treatment isoquants. These represent the constraints in the optimization problem.

e) The use of statistical models is particularly useful when long-term effects are being considered, since simulation models are limited in the number of simulation runs that can be made, specially in the case of multivariate distributions.

8.2 Recommendations

Several recommendations can be made for further research on urban stormwater management. A comprehensive listing of these is given by Heany (1986), whose efforts will not be duplicated here. Needs involve all aspects of stormwater planning, such as data collection and modeling. Statistical models share these needs, but also have particular requirements.

The statistical method has been shown to be a viable approach to planning, addressing the fundamental randomness of the associated hydrologic processes. Recommendations for further research are given below:

a) A particular area that is in need of further study is pollutant load estimation. Efforts need to be undertaken to account for the effects of temporal and spatial variabilities of surface pollutant load sources. The dynamic nature of the urban environment requires an accounting of the parameters which would help describe these variabilities. Regressions cannot properly account for the variabilities, unless these are periodically updated with fresh data whenever significant land use changes occur.

b) Additional efforts should be directed towards the incorporation of pollutant removal functions for the detention unit. At present, only limited results have been obtained with statistical models due to the difficulty of incorporating the particulars of pollutant removal mechanisms within the general conceptualization of the runoff process found in most statistical formulations. Usually a simple removal coefficient is utilized.

c) Other aspects for which attention is recommended include those related to the formulation of adequate receiving water concentration distributions, based on the actual physics of pollutant mixing and transport.

d) Further development of statistical methods should see a more detailed representation of the urban stormwater collection system, and thus a more complex formulation of the runoff process. It is expected that these developments will eventually allow evaluation of urban best management practices for runoff control to the extent that is presently being accomplished by largely deterministic approaches (Schueler, 1987). The need to study the long-term behaviour of stormwater detention units, and to account for the evident randomness of the hydrologic processes that drive the system, make the statistical approach to urban stormwater management a rewarding field of endeavor.

e) In Puerto Rico very little has been done to assess the urban runoff problem. No appreciable effort has been undertaken to study the problem from the point of view considered in this study. Studies along this line are necessary in view of the high level of urban development and the potential environmental impact of uncontrolled storm flows.

REFERENCES

Akan, A.O., Unsteady Gutter Flow into Grate Inlets, M.S. thesis, Dept. of Civil Engineering, University of Illinois at Urbana-Champaign, Aug. 1973.

Alley, W.M., "Estimation of Impervious Area Washoff Parameters", Water Resources Research, 17(4), 1981, pp. 1161-1166.

Baumol, W.J., Economic Theory and Operations Analysis, 4th ed., Prentice Hall, 1977.

Benjamin, J.R., and Cornell, C.A., Probability, Statistics, and Decision for Civil Engineers, McGraw Hill, 1970.

Benjes, H.H., "Cost Estimating Mannual-Combined Sewer Overflow Storage and Treatment", EPA-600/2-76-286, U.S. Environmental Protection Agency, Dec., 1976.

Bradley, S.P., Hax, A.C., Magnanti, T.L., Applied Mathematical Programming, Addison-Wesley, 1977.

Brater, E.F., and Sherril, J.D., "Rainfall-Runoff Relations on Urban and Rural Areas", Environmental Protection Agency, EPA-670/2-75-046, May 1975.

Breipohl, A.H., Probabilistic Systems Analysis, Wiley, 1970.

Burke, C.B., "Highway Extension and Research Project for Indiana Counties (Herpic)" Purdue University, 1981.

Chan, S., and Bras, R., "Urban Stormwater Management: Distribution of Flood Volumes", Water Resources Research, Vol. 15, No. 2, April 1979, pp. 371-382.

Chow, Ven Te, Editor, Handbook of Applied Hydrology, McGraw Hill, 1964.

Chow, V.T., and Yen, B.C., "Urban Stormwater Runoff: Determination of Volumes and Flowrates", Environmental Protection Agency, EPA/600/2-76-116, May 1976.

Delleur, J.W., "Recent Advances in Urban Hydrology in the U.S.A", Internat. Symp. on Urban Hydrology, Hydraulics, and Sediment Control, H.J. Sterling, and R.W. Bevore, Eds., University of Kentucky, 1981.

Dendrou, S.A., Talavage, J.J., and Delleur, J.W., "Urban Storm Drainage Systems Planning", Purdue University, Water Resources Research Center, Technical Report No. 101, May, 1978.

De Neufville and Stafford, Systems Analysis for Engineers and Managers, McGraw Hill, 1971.

DiToro, D.M., and Small, M.J., "Stormwater Interception and Storage", Journal of Environmental Engineering, ASCE, Vol. 105, No. EE1, Feb. 1979, pp. 43-54.

Eagleson, P.S., "Climate, Soil, and Vegetation 2. The Distribution of Annual Precipitation Derived from Observed Storm Sequences", Water Resources Research, Vol. 14, No. 5, Oct. 1978, pp. 713-721.

Ferguson, C.E., The Neoclassical Theory of Production and Distribution, Cambridge University Press, 1975.

Giggey, M.D., and Smith, W.G., "National Needs for Combined Sewer Overflow Control", Journal of The Environmental Engineering Division, Vol. 104, No. EE2, April 1978, pp. 331-366.

Goforth, G.F., Heany, J.P., and Huber, W.C., "Comparison of Basin Performance Modeling Techniques", Journal of Environmental Engineering, ASCE, Vol. 109, No. 5, Oct. 1983, pp. 1082-1098.

Heany, J.P.; Huber, W.C.; Field, R.; and Sullivan, R.H., "Nationwide Cost of Wet-Weather Pollution Control", Journal of the Water Pollution Control Federation, Vol. 51, No. 8, Aug. 1979, pp. 2043-2053.

Heany, J.P., Huber, W.C., Medina, M.A., Murphy, M.P., Nix, S.J., and Hasan, S.M., "Nationwide Evaluation of Combined Sewer Overflows and Urban Stormwater Discharges, Vol. II, Cost Assessment and Impacts", EPA-600/2-77-064, Environmental Protection Agency, March 1977.

Heany, J.P., "Research Needs in Urban Storm-Water Pollution", Journal of Water Resources Planning and Management, ASCE, Vol. 112, No. 1, Jan., 1986, pp. 36-47.

Howard, C.D.D., "Theory of Storage and Treatment-Plant Overflows", Journal of Environmental Engineering, ASCE, Vol. 102, No. EE4, Aug. 1973, pp. 709-722.

Howard, C.D., Flatt, P.E., and Shamir, U., "Storm and Combined Sewer Storage Treatment Theory Compared to Computer Simulation", Environmental Protection Agency, EPA-600/2-81-125, July 1981.

Huber, W.C., Heany, J.P., Medina, M.A., Peltz, W.A., Sheikh, H., and Smith, G.F., "Storm Water Management Model User's Manual, Version II", EPA-672/2-75-017, March 1975.

Huber, W.C., Heany, J.P., Medina, M.A., Peltz, W.A., Sheikh, H., and Smith, G.F., "Storm Water Management Model Users Manual, Version III", EPA-600/2-84-109A, Sept. 1984.

Hydroscience, Inc., "A Statistical Method for Assessment of Urban Runoff", Environmental Protection Agency, EPA 440/3-79-023, May 1979.

James, M.L., Smith, G.M., and Wolford, J.C., Applied Numerical Methods for Digital Computation, 3rd. ed., Harper & Row, 1985.

Jewell, T.K., Urban Stormwater Pollutant Loadings, Ph.D. dissertation, University of Massachusetts, 1980.

Jewell, T.K., Adrian, D.D., "Improved Techniques for Modeling Stormwater Quality", Urban Stormwater Quality, Management and Planning, Yen, B.C., Ed., Water Resources Publications, Littleton, Co., 1982.

Labadie, J., and Grigg, N., "Urban Stormwater Management", Stochastic Approaches to Water Resources, Vol. II, Ed. by H.W. Shen, Water Resources Publication, Fort Collins, Colorado, 1976.

Loganathan, G.V., and Delleur, J.W., "Multiple Objective Planning of Land/Water Interface in Medium Size Cities: Application to West Lafayette, Indiana", Purdue University, Water Resources Research Center, Technical Report No. 144, March 1982.

Loganathan, G.V., and Delleur, J.W., "Effects of Urbanization on Frequencies of Overflows and Pollutant Loadings from Storm Sewer Overflows: A Derived Distribution Approach", Water Resources Research, Vol. 20, No. 7, July, 1984, pp. 857-865.

Loganathan, G.V., Delleur, J.W., and Segarra, R.I., "Planning Detention Storage for Stormwater Management", Journal of Water Resources Planning and Management, ASCE, Vol. III, No. 4, Oct., 1985, pp. 382-398.

McElroy, A.D., Chiu, S.Y., Nebgen, J.W., Aleti, A., and Bennett, F.W., "Loading Functions for Assessment of Water Pollution from Nonpoint Sources", EPA/600/2-76-151, May 1976.

McElroy, F.T., Mattox, C.F., Hartman, D.W., and Bell, J.M., "Sampling and Analysis of Stormwater Runoff from Urban and Semi-Urban/Rural Watersheds", Tech. Rep. No. 64, Purdue University Water Resources Center, West Lafayette, Ind., Sept. 1976.

Medina, M.A., "Hydrologic and Water Quality Modeling for Instream Flow Strategies", University of North Carolina Water Resources Research Institute, UNC-WRRI-82-183, Dec., 1982.

Medina, M.A., Huber, W.C., and Heany, J.P., "Modeling Stormwater Storage/Treatment Transients: Theory", Jour. of the Environmental Engineering Division, ASCE, Vol. 107, No. EE4, August 1981, pp. 781-797.

Medina, M.A., Huber, W.C., and Heany, J.P., "Modeling Stormwater Storage/Treatment Transients: Applications", Jour. of the Environmental Engineering Division, ASCE, Vol. 107, No. EE4, August 1981, pp. 799-815.

Melville, W.A., and Bell, J.M., "Simulation versus a Probabilistic Approach for Evaluating Stormwater Treatment Alternatives", Purdue University WaterResources Research Center, Technical Report No. 105, June, 1979.

Morris, H.M., and Wiggert, J.M., Applied Hydraulics in Engineering, Second ed., Ronald Press, 1972.

Nix, S.J., Heany, J.P., and Smolenyak, K., "Summary Characterization of Urban Wastewater Management Options", Urban Stormwater Management Workshop Proceedings, Field, R., Ed., Environmental Protection Agency, EPA-600/9-78-017, Dec., 1977.

Nix, S.J., Analysis of Storage-Release Systems in Urban Stormwater Quality Management, Ph.D. dissertation, University of Florida, 1982.

Ordon, C.J., "Volume of Stormwater Retention Basins", Journal of the Environmental Engineering Division, ASCE, Vol. 100, No. EES, Oct., 1974, pp. 1165-1177.

Ormsbee, L.E., Delleur, J.W., and Houck, M.H., "Development of a General Planning Methodology for Stormwater Management in Urban Watersheds", Purdue Univ. Water Resources Center, West Lafayette, Ind., Tech. Report No. 163, March 1984.

Pagán, I., Statistical Analyses of Spatial and Temporal Storm Rainfall Characteristics in Puerto Rico, Final Technical Report to the Dept. of Interior, Univ. of Puerto Rico Water Resources Research Institute, Project No. 374103, Sept. 1984.

Patry, Gilles G., Real Time Forecasting of Urban Quantity and Quality in Urban Catchments, Ph.D. dissertation, University of California, 1983.

Raasch, G.E., "Urban Stormwater Detention Sizing Technique", International Symposium on Urban Storm Runoff, University of Kentucky, July 23-26, 1979.

Restrepo-Posada, P., and Eagleson, P.S., "Identification of Independent Rainstorms", Journal of Hydrology, Vol. 55, 1982, pp. 303-319.

Robinson, D.K., and Labadie, J.W., "Optimal Design of Stormwater Drainage Systems", 1981 International Symposium on Urban Hydrology, Hydraulics, and Sediment Control, University of Kentucky, Lexington, KY, July 27-30, 1981.

Roesner, L.A., "Quality of Urban Runoff", Urban Storm water Hydrology, edited by D.F. Kibler, Ed., American Geophysical Union, Water Resources Monograph 7, 1982(a).

Roesner, L.A., "Urban Runoff Process", Urban Stormwater Hydrology, edited by D.F. Kibler, American Geophysical Union, Water Resources Monograph 7, 1982(b).

Roussel, R., Pigeon, J.C., and Noixeux, J.R., "Application of the OTTSWMM Model for a Relief Sewer Study in Laval, Quebec", Proc. of Stormwater and Water Quality User's Group Meet., Orlando, Florida, EPA/600/9-86/023, 1986.

Sartor, J.D., and Boyd, G.B., "Water Pollution Aspects of Street Surface Contaminants", Rep. EPA-R-272-081, Environmental Protection Agency, Nov. 1972.

Schueler, T.R., "Controlling Urban Runoff: A Practical Manual for Planning and Designing Urban BMPs", Metropolitan Washington Council of Governments, Washington, D.C., July, 1987.

Schwarz, R.B., Adams, B.J., "Distributed Storage for Urban Runoff Control: An Analytical Model", Second International Conterence on Urban Storm Drainage, Yen, B.C., Ed., Urbana, Illinois, June, 1981.

Small, M.J., and DiToro, D.M., "Stormwater Treatment Systems", Journal of the Environmental Engineering Division ASCE, Vol. 105, No. EE3, June, 1979, pp. 557-569.

Terstriep, M.L., Stall, J.B., "Urban Runoff by Road Research Lab Method, Proc. ASCE, J. Hyd., Nov. 1969, No. HY6.

Terstriep, M., and Stall, J., "The Illinois Urban Drainage Area Simulator, ILLUDAS", Illinois State Water Survey, Urbana, Bulletin 58, 1974.

Todorovic, P., and Yevjevich, V., "Stochastic Process of Precipitation", Hydrology Paper No. 35, Colorado State University, Sept., 1969.

University of Cincinnati, "Urban Runoff Characteristics", Environmental Protection Agency, 11024 DQU, Oct. 1970.

U.S. Army Corps of Engineers, Storage, Treatment, Overflow, Runoff Model, "STORM", User's Manual, Hydrologic Engineering Center, Aug. 1977.

Viessman, W., Knapp, J., Lewis, G., and Harbaugh, T., Introduction to Hydrology, Second Edition, Harper & Row, 1977.

Wanielista, M.P., Stormwater Management, Quantity and Quality, Ann Arbor Science, 1978.

Water Planning Division, "Results of the Nationwide Urban Runoff Program", Vol. 1, Final Report, U.S. Environmental Protection Agency, Dec., 1983.

Wisner, P.E., Cheung, P.W., "Parks Against Storms", Second International Conference on Urban Storm Drainage, Yen, B.C., Ed., Urbana, Illinois, June, 1981.

Wisner, P.E., Kassem, A.M., and Cheung, P.W., "Park Against Storms", Urban Stormwater Quality Management and Planning, edited by B.C. Yen, Water Resources Publications, Littleton, Colorado, 1982.

Wisner, P.E., Kochar, C., Fraser, H., and Rampersad, C., "Application of Inlet Control Devices and Dual Drainage Modeling for New Subdivisions", Proc. of Stormwater and

Water Quality User's Group Meet., Orlando, Florida, EPA/600/9-86/023, 1986.

Whipple, W., Hunter, J., "Effects of Storm Frequency on Pollution from Urban Runoff", Journal of the Water Pollution Control Federation, Vol. 49, No. 11, Nov. 1977, pp. 2243-2248.

Yen, B.C., Wenzel, H., Mays, L., and Tang, W., "Advanced Methodologies of Design of Storm Sewer Systems", University of Illinois Water Resources Center, UILU-WRC-76-0112, Aug. 1976.

Yen, B.C., Ed., Storm Sewer System Design, Dept. of Civil Engineering, Water Resources Center, University of Illinois, 1978.

Young, G.K., Bondelid, T.R., and Athayde, D.N., "Urban Runoff Pollution Method", Journal of the Water Resources Planning and Management Division, ASCE, Vol. 105, No. WR2, Sept. 1979, pp. 353-369.

Yu, Yun-Sheng, Seitz, R.D., and Kaul, S.K., "Hydraulic Efficiency of Urban Stormwater Inlets", International Symposium on Urban Runoff, University of Kentucky, July 23-26, 1979.

APPENDIX A

The major analytical advantage of working with exponential distributions is the fact that integrals of exponential functions yield exponential functions. The integrations for the cumulative distributions and expectations throughout the present work have benefited from this property.

All the integrations performed in this work are of the following general type:

$$\int_{\xi}^{\tau} h(p)\ \exp[g(p)x]dx = \frac{h(p)}{g(p)}\ \exp[g(p)x]\Big|_{\xi}^{\tau} \qquad \text{(A-1)}$$

where h(p) and g(p) are expressions in terms of the process parameter set ρ; x is the random variable, and ξ and τ are the limits of integration.

The other type of integration is for the expectation of the random variable. The general expression for this type of integral is given by:

$$\int_{\xi}^{\tau} xh(p)\ \exp[g(p)x]dx = [x - \frac{1}{g(p)}]\frac{h(p)}{g(p)}\ \exp[g(p)x]\Big|_{\xi}^{\tau} \qquad \text{(A-2)}$$

The actual form of the parameter function depends on the particular formulation that is evaluated. The particular algebraic details for each expression are not presented, but each is evaluated through some form of the above two expressions.