Computation of Gradually Varied Flow in Channel Networks with Hydraulic Structures

by

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Acronyms

BiCGSTAB	Bi-Conjugated Gradient Stabilizer with Preconditioner Method
DSM	Direct Step Method
GVF	Gradually-Varied Flow
HEC-RAS	Hydrologic Engineering Center- River Analysis System
iLU	incomplete LU factorization
PR	Puerto Rico
StdSM	Standard Step Method
SSM	Simultaneous Solution Method
UDT	Utah Department of Transportation

Justification

The first major irrigation system was built during Egypt's First Dynasty, close to 3100 B.C. as a diversion of flood waters of the Nile River [1]. Irrigation canals transport water from a source, such as a natural river or a reservoir, to a crop field or a community. Irrigation canals are vital for agriculture. One-sixth of irrigated crop land produces one-third of the world's harvest of food crops [2]. Food production is a global concern in a world of growing population and limited resources. Sustainability of food production depends on sound and efficient water use and conservation practices consisting mainly of irrigation development and management [3]. It is of upmost importance for farmers to control the water distribution in irrigation systems. Hydraulic structures such as weirs and gates must be set at specific levels to distribute the correct amount of water for crop production and water conservation.

This research will develop a computer model to determine the water levels and discharges in complex irrigation channel networks including hydraulic structures to control water distribution. This computer model will provide farmers information on flow levels and volumetric discharges, helping then to make decisions on irrigation water for their crops. The agricultural community will be able to apply of water conservation and management more efficiently. The scope of research will be to model irrigation channel networks that may be composed of several channel branches and loops. The research will be limited to subcritical flow conditions on the channel system. Analysis and/or design of hydraulic structures in irrigation system, such as lateral weirs, gates and inverted syphons, will be included. The design computer tool will have the capacity of modeling any channel configuration, including series, parallel and network channel systems and solve for flow and water levels simultaneously.

Hydraulic structures are an essential component in real-life scenarios of irrigation channels. An example of a hydraulic engineering applications is the Lajas Valley Irrigation System, located on the southwest of Puerto Rico (PR). This system has an annual discharge of 30,830 acre-ft of water supply for agriculture and potable water supply. Also, it provides irrigation water to 330 intakes and four water treatments plants. This system impacts approximately 100,000 people in its high season of tourism and vacation periods [4]. Irrigation districts are vital for sustainability in Puerto Rico and other countries.

Previous publications

Gradually varied flow (GVF) has been studied and researched since 19th century. Chaudhry and Schulte [5] were one of the first pioneers to develop an algorithm for parallel channels in 1986. Their algorithm solve for water depths and discharges at different sections for steady-state and GVF conditions based on two fundamental equations: energy equation and continuity equation. The nonlinear matrix system is solved simultaneously using the Newton-Raphson method. In order to increase accuracy, and reduce computer time and storage, they found a way to transform the resulting Jacobian matrix into a banded matrix for series and parallel channels. In a similar manner, Chaudhry and Schulte [6] extended their previous algorithm to solve GVF conditions in a channel network, however, the matrix became sparse and more difficult to solve. This channel network algorithm is based on the same methodology as the parallel-channel algorithm; but can solve for multiple channel configurations, such as series, parallel and networks. The two models were applied to an idealized channel network. Results were compared with those obtained by the fourth-order Runge-Kutta method for each channel configuration, which were almost identical as described by the author.

A decade after the publication of Chaudhry and Schulte algorithm, Naidu *et. al.* [7] presented an algorithm for GVF computations in tree-type channel networks. This algorithm computes the water surface profile under the same flow conditions as Chaudhry and Schulte. The solving technique for this algorithm decomposes the channel network into smaller units that are solve using the fourth-order Runge-Kutta method, and connects all the solutions using the Shooting Method. This technique does not involve solving simultaneously a large matrix system; opposite to the one presented by Chaudhry and Schulte. The model was applied to an idealized tree-type channel network as Chaudhry and Schulte study. Their algorithm was more efficient than the direct method using the Newton-Raphson technique by an order of magnitude.

Reddy and Bhallamudi [8] developed an algorithm to compute water surface profiles in steady, GVF of channel networks. Their algorithm computes the same two variables as Chaudhry and Schulte; flow depths and discharges at a cyclic looped channel network. Although Reddy and Bhallamudi's algorithm is very similar to the one from Chaudhry and Schulte, their solving methodology is very different. Reddy and Bhallamudi's algorithm is based on three principles: classifies the computations in an individual channel as an initial value problem or a boundary value problem; determines the path for linking the solution from individual channels; and the Netwon-Raphson iterative technique is used for obtaining the network solution [8]. Therefore, this algorithm does not have to solve large matrix systems. The model was tested with the idealized channel network presented in Chaudhry and Schulte, and Naidu *et. al.* study. The efficiency of their algorithm compared with Naidu *et. al* [7] technique.

Sen and Garg [9] developed a model for unsteady flow in channel networks using St. Venant equations. Sen and Garg's algorithm uses the finite difference method to solve the system of equations for all branches of the network simultaneously. The algorithm does not require any special node numbering schemes. The number of equations to be solved reduce to four times the number of branches in the network, resulting in a significant reduction in storage requirements and solution time. The model was applied to two idealized channel networks, one looped network and one branch network.

In a different manner, Islam *et al.* [10] conducted a comparison of two channel network algorithms. The two algorithms had different techniques for separating end-node variables for each branch; forward-elimination and branch-segment transformation equation. Both algorithms model steady and unsteady flows in branched and looped channel networks. The St. Venant equations are discretized using the four-point implicit Preissmann scheme and the nonlinear matrix system is solved using the Newton-Raphson method. The model was applied to similar channel networks as Sen and Garg. Islam *et al.* [10] concludes that the algorithm that uses the branch-segment transformation equation is found to be at least five times faster and require less computer storage than the algorithm using forward-elimination method. In a similar manner, Islam *et al.* [11] extended his previous work and developed a hydraulic simulation model for irrigation channel networks. The model uses the same discretization technique for the St. Venant equations, as his previous work; but solves the nonlinear matrix system using the sparse matrix solution techniques. Similarly, this algorithm models the same flow conditions and channel configuration as its previous algorithm. In addition, this algorithm is capable of simulating different boundary conditions; such as discharge and stage hydrographs, rating curves, and uniform flow. At the moment of this research, Islam *et al.* [11] study was the only algorithm that include the analysis of different hydraulic structures; such as weirs, sluice gates, drops/falls, pipe outlet, and imposed discharge. Also, it was the only algorithm to include a user friendly graphical user interface for entering and editing channel network description and boundary conditions. Even though Islam *et* *al.* affirms that its algorithm is capable of solving complex channel networks, the results presented were only for a simple network of four channels connected in series from the Kangsabati irrigation project at West Bengal, India. Their results were similar to the ones from a HEC-RAS model and performed satisfactory for most of the irrigation event at the irrigation project. This case does not represent a challenging example for other solutions procedures.

At the beginning of this decade, Zhu *et al.* [12] develop an algorithm for gradually-varied subcritical flow in channel networks. This algorithm simulates the same flow conditions as Islam *et al.* [11] and Sen and Garg's [9] algorithm; and solves the system using the same techniques as Islam *et al.* [11] first algorithm. The algorithm treats backwater effects at the junction points on the basis of junction-point water stage prediction and correction. The model was applied to two hypothetical channel networks and a real-life river network in South China, in which the simulated results compared well with ones from literature and measurements.

Objectives

The main objective for this research is to develop a computer model to determine discharge and water levels in complex irrigation channel networks including hydraulic structures to control water distribution. The solution algorithm will solve the mass and energy equation for GVF plus additional equations to analyze and/or design lateral weir, sluiced gates and inverted syphons. To the research knowledge, the addition of analysis and/or design of hydraulic structures for irrigation systems as part of a simultaneous solution has not been proposed before. As part of this research, the following tasks will be completed:

1. *Development of a computer algorithm*: The software will be able to solve a wide variety of configurations of irrigation canals (series, parallel or network systems), including design and/or analysis of hydraulic structures commonly found in such systems (lateral weir, sluiced gates and inverted syphons).

2. *Development of a graphical user interface (GUI)*: The GUI will allow a user friendly interaction with the numerical model.

3. *Model application*: The new tool will be used in a case study. Efforts will be made to collect information of "Arenal Tempisque" irrigation district located in Guanacaste, Costa Rica. This is a major irrigation project for crop production in this Central

American country. The system if formed by three irrigation sub-districts with a total of eighty-nine irrigation channels. Dr. Alejandra Rojas from the Department of Agricultural Engineering of the University of Costa Rica (UCR) has collaborated previously with the chairman of this research. Access to field data and new data measurements could be granted through this partnership.

Methodology

This section consists on describing the proposed methodology for the proposal. First, the governing equations of the gradually varied flow in open channel will be presented with its assumptions. Second, the system of equations for the channel network will be proposed with the boundary conditions of the system. Third, design of some of the hydraulic structures that might be present on the channel network will be discuss. This include the design of lateral weir, inverted syphon and sluice gates. The numerical method selected for the solution of the system will be discuss in more detail. Finally, some preliminary results from a case study of a looped channel network will be presented.

Governing equations

Gradually varied flow (GVF) occurs when the rate of variation of depth with respect to distance is small. The friction losses have to be considered, since the analysis of GVF is usually done for long channels. The GVF is based on the following assumptions [13]:

1. The slope of the channel bottom is small. This slope may be assumed small if less than 10%, therefore the flow depth measured vertically or normal to the bottom are approximately the same.

2. The channel is prismatic and there is no lateral inflow or outflow. A prismatic channel refers to a channel that its cross section and bottom slope does not change with distance. If the channel has different cross section or bottom slope, it may be divided into piecewise prismatic channels. If there is lateral inflow/outflow the continuity equation must be modified.

3. The pressure distribution is hydrostatic at all channel section; this is because the streamlines are more or less straight and parallel.

4. The head loss may be determined by using the equations for head losses in uniform flow.

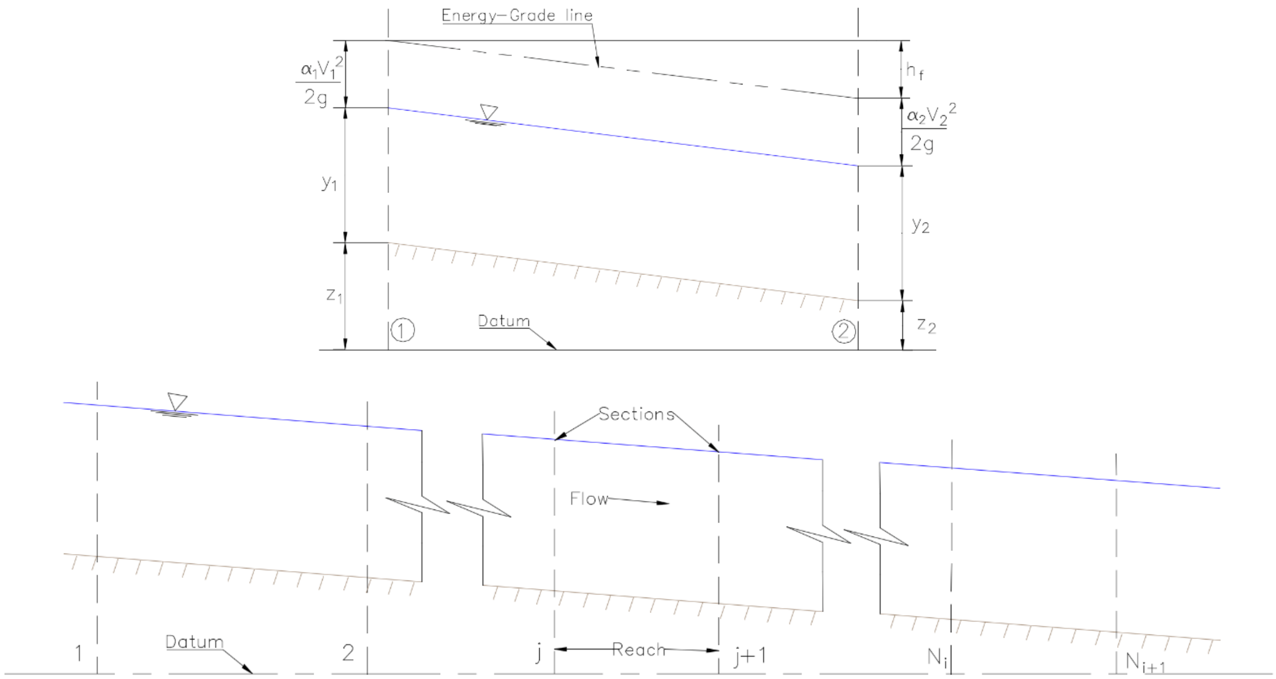

Figure 1: Definition sketches for the governing sketches. (Adapted from [13])

This research requires the simultaneous solution of a large number of non-linear equations plus verification of channel regimes and, if needed, equations describing the flow across hydraulics structures. The governing equations are the energy equation, between two consecutive sections of the same channel (Eq. 1), and the continuity equation between two consecutive channels (Eq. 2). The energy and continuity equation using Figure 1are [13]:

$$F_{i,k} = z_{i,j+1} - z_{i,j} + y_{i,j+1} - y_{i,j} + \alpha_i \frac{Q_{i,j+1}|Q_{i,j+1}|}{2gA_{i,j+1}{}^2} - \alpha_i \frac{Q_{i,j}|Q_{i,j}|}{2gA_{i,j}{}^2} + \frac{1}{2}\left(x_{i,j+1} - x_{i,j}\right)\left(\frac{Q_{i,j+1}|Q_{i,j+1}|n_i{}^2}{C_o{}^2 A_{i,j+1}{}^2 R_{i,j+1}{}^{1.33}} + \frac{Q_{i,j}|Q_{i,j}|n_i{}^2}{C_o{}^2 A_{i,j}{}^2 R_{i,j}{}^{1.33}}\right) = 0 \quad \text{Eq. 1}$$

$$F_{i,k+1} = Q_{i,j+1} - Q_{i,j} = 0 \quad \text{Eq. 2}$$

where:

Q = Rate of discharge (Longitud^3/time),

z = Elevation of the channel bottom above a specified datum(elevation),

y	=	Flow depth (longitude),
α	=	Velocity-head coefficient (dimensionless),
g	=	Acceleration due to gravity (longitude/ time^2),
x	=	Horizontal distance (longitude),
A	=	Flow area (longitude^2),
n	=	Manning's roughness coefficient (dimensionless),
R	=	Hydraulic radius (longitude)
C_o	=	Dimensional coefficient of Manning's equation (dimensionless), where for SI units equals 1.0 and for English units is 1.49,
i	=	Subscript that refers to the number of the channel,
j	=	Subscript that refers to the section number of the channel i, and
k	=	Subscript that refers to the equation number on the matrix system.

On the Eq.1 the last term on the right side is an approximation of the head loss, and may be computed by the average of the friction slopes. Also, to be able to account for the reverse flow, which is a flow direction opposite to the assumed one, the discharge term on the energy equation must be expressed as $Q_{i,j}\left|Q_{i,j}\right|$ instead of ${Q_{i,j}}^2$. Figure 1 show a representative sketch for this notation.

The energy equation (Eq. 1) and the continuity equation (Eq. 2), will be solved simultaneously for a network of channel cross sections to determine the water levels and the channel discharges. Equations for hydraulic structures within the irrigation system will also be included. The system is formed by a large number of non-linear equations. The proposed simultaneous solution method (SSM) computes GVF on complex channel systems, such as irrigation districts ([5], [6], and [13]). This method utilizes the Newton-Raphson iterative procedure for the solution of a system of nonlinear equations. To better understand the concept, the SSM will be explained for a system of simple looped channel network.

Channel networks

One of the main difference between the algorithm for series and looped networks of channels is that in looped networks the discharge in each individual channel are not known. Therefore, the continuity equation (Eq. 2) for each reach must be included to obtain the necessary number of equations to solve the system. The SSM for GVF in looped channel networks, as Figure 2, can be expressed as a linear matrix system, such as Eq.3.

$$[A]\{\Delta\} = \{F\} \qquad \text{Eq. 3}$$

where:

$[A]$ = Jacobian Matrix,

$\{\Delta\}$ = Vector of flow depth and discharge corrections, and

$\{F\}$ = Vector of energy and continuity equation.

The vector of energy and continuity equations $\{F\}$ consists of an arrangement of these equations (Eq. 1-2) that depends on type the channel that is being consider. For the channels that are before and after the looped channels (consider as channel in series) (channel i and $i+3$ on Figure 2), the energy equation is written for first reach, followed by the continuity equation for the same reach. This is then repeated for all the reaches (N_i) of channel i or $i+3$ in a consecutive manner. For the looped channels (consider as channel in parallel) (channel $i+1$ and $i+2$ on Figure 2), the energy equation for the first reach of channel $i+1$ is written, followed by the continuity equation for the same reach of channel $i+1$. Then, the energy equation for the first reach of channel $i+2$ is written, followed by the continuity equation for the same reach of channel $i+2$. This process is repeated in the same manner for all the reaches on both channels. It is crucial for this procedure that the channels in parallel have the same amount of reaches.

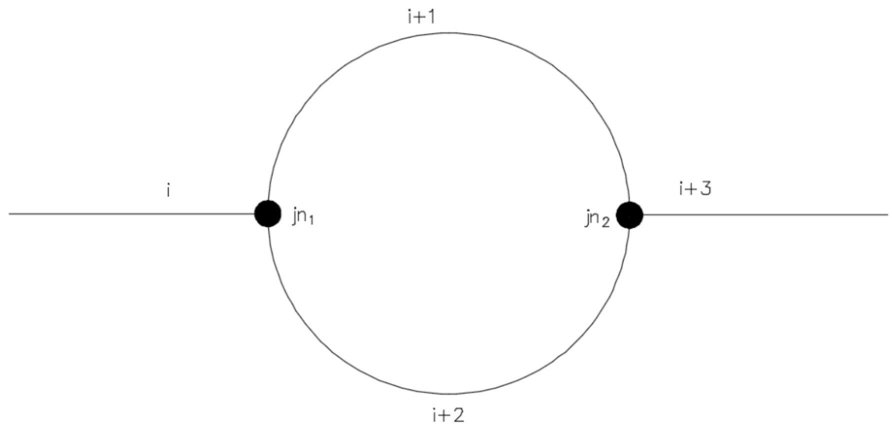

Figure 2: Example of a looped channel network.

The Jacobian matrix $[A]$ consists of the partial derivatives of the energy and continuity equations with respect to the flow depth and the discharge. The assembly of this matrix follows the same pattern as the vector of energy and continuity equation for the channel in series and in parallel. First, the partial derivative of the energy equation with respect to flow depth is written for section *j* of channel *i*. The second term in the same row will be the derivative of the energy equation with respect to discharge for the section *j* of the same channel *i*. Next, on the same row, the partial derivative of the energy equation with respect to flow depth is written for section *j+1* of channel *i*. The last term of this row will be the partial derivative of the energy equation with respect to the discharge for section *j+1* of the same channel *i*. The following row of the Jacobian matrix will consist of the partial derivatives of the continuity equation with respect of the discharge, since the partial derivative with respect of the flow depth is cero. These partial derivatives are shown in Eq. 4 and Eq.5 for the energy and continuity equation, respectively. The vector of flow depth and discharge corrections $\{\Delta\}$ provides the corrections of the flow depth and discharge for all the sections of all channel. This vector of solutions will be updated at each iteration until the corrections are smaller than a certain tolerance.

$$\frac{\partial F_{i,k}}{\partial y_{i,j}} = -1 + Q_{i,j}|Q_{i,j}|(\frac{\alpha_i T_{i,j}}{g{A_{i,j}}^3} - \frac{2{n_i}^2\left(x_{i,j+1} - x_{i,j}\right)}{3{C_o}^2{A_{i,j}}^2{R_{i,j}}^{2.33}} \times \frac{dR_{i,j}}{dy_{i,j}} - \frac{{n_i}^2 T_{i,j}\left(x_{i,j+1} - x_{i,j}\right)}{3{C_o}^2{A_{i,j}}^3{R_{i,j}}^{1.33}}) \quad \text{Eq. 4}$$

$$\frac{\partial F_{i,k}}{\partial Q_{i,j}} = 2Q_{i,j}(-\frac{\alpha_i}{2gA_{i,j}{}^2} + \frac{n_i{}^2(x_{i,j+1} - x_{i,j})}{2C_o{}^2 A_{i,j}{}^2 R_{i,j}{}^{1.33}}$$

$$\frac{\partial F_{i,k}}{\partial y_{i,j+1}} = 1 - Q_{i,j+1}|Q_{i,j+1}|(\frac{\alpha_i T_{i,j+1}}{gA_{i,j+1}{}^3} - \frac{2n_i{}^2(x_{i,j+1} - x_{i,j})}{3C_o{}^2 A_{i,j+1}{}^2 R_{i,j+1}{}^{2.33}} \times \frac{dR_{i,j+1}}{dy_{i,j+1}} - \frac{n_i{}^2 T_{i,j+1}(x_{i,j+1} - x_{i,j})}{3C_o{}^2 A_{i,j+1}{}^3 R_{i,j+1}{}^{1.33}})$$

$$\frac{\partial F_{i,k}}{\partial Q_{i,j+1}} = 2Q_{i,j+1}(-\frac{\alpha_i}{2gA_{i,j+1}{}^2} + \frac{n_i{}^2(x_{i,j+1} - x_{i,j})}{2C_o{}^2 A_{i,j}{}^2 R_{i,j}{}^{1.33}}$$

$$\frac{\partial F_{i,k+1}}{\partial Q_{i,j}} = -1$$

Eq. 5

$$\frac{\partial F_{i,k+1}}{\partial Q_{i,j+1}} = 1$$

where:

T = Top width of flow area (Longitude).

The energy and continuity equation for all the N_i reaches of the four channels of the looped network, gives a total of 2(N_i + N_{i+1} + N_{i+2} + N_{i+3}) equations [6]. Since, the flow depth and the discharge are unknowns at each reach, a total of 2(N_i + N_{i+1} + N_{i+2} + N_{i+3} +4) unknowns are needed to be solved. Therefore, for obtaining and unique solution for the system, eight additional equations are needed, which are provided by the boundary and end conditions. The upstream or the downstream end condition provides two equations, one for flow depth and another for discharge. For this procedure, the upstream end condition will be selected (Eq. 6).

$$F_{i,1} = y_{i,1} - y_u = 0$$

Eq. 6

$$F_{i,2} = Q_{i,1} - Q_u = 0$$

where:

y_u = Specified flow depth at the upstream end channel i (Longitude), and

Q_u = Specified discharge at the upstream end channel *i* (Longitude).

The remaining six equations are provided by boundary conditions at both junction of the looped network. The upstream junction (jn_1) provides three equations, in which one is from the continuity equation and two are from the energy equation (Eq. 7) (see Figure 3a). In a similar manner, the downstream junction (jn_2) provides the last three equations needed (Eq. 8) (see Figure 3b).

$$F_{jn_1,1} = Q_{i,N_i+1} - Q_{i+1,1} - Q_{i+2,1} = 0$$

$$F_{jn_1,2} = z_{i,N_i+1} - z_{i+1,1} + y_{i,N_i+1} - y_{i+1,1} + \frac{Q_{i,N_i+1}\left|Q_{i,N_i+1}\right|}{2gA_{i,N_i+1}{}^2} - (\alpha_{i+1} + \varsigma)\frac{Q_{i+1,1}\left|Q_{i+1,1}\right|}{2gA_{i+1,1}{}^2} = 0 \qquad \text{Eq. 7}$$

$$F_{jn_1,3} = z_{i,N_i+1} - z_{i+2,1} + y_{i,N_i+1} - y_{i+2,1} + \frac{Q_{i,N_i+1}\left|Q_{i,N_i+1}\right|}{2gA_{i,N_i+1}{}^2} - (\alpha_{i+2} + \varsigma)\frac{Q_{i+2,1}\left|Q_{i+2,1}\right|}{2gA_{i+2,1}{}^2} = 0$$

$$F_{jn_2,1} = Q_{i+3,1} - Q_{i+1,N_{i+1}+1} - Q_{i+2,N_{i+2}+1} = 0$$

$$F_{jn_2,2} = z_{i+1,N_{i+1}+1} - z_{i+3,1} + y_{i+1,N_{i+1}+1} - y_{i+3,1} + \frac{Q_{i+1,N_{i+1}+1}\left|Q_{i+1,N_{i+1}+1}\right|}{2gA_{i+1,N_{i+1}+1}{}^2} - (\alpha_{i+3} + \varsigma)\frac{Q_{i+3,1}\left|Q_{i+3,1}\right|}{2gA_{i+3,1}{}^2} = 0 \qquad \text{Eq. 8}$$

$$F_{jn_2,3} = z_{i+2,N_{i+2}+1} - z_{i+3,1} + y_{i+2,N_{i+2}+1} - y_{i+3,1} + \frac{Q_{i+2,N_{i+2}+1}\left|Q_{i+2,N_{i+2}+1}\right|}{2gA_{i+2,N_{i+2}+1}{}^2} - (\alpha_{i+3} + \varsigma)\frac{Q_{i+3,1}\left|Q_{i+3,1}\right|}{2gA_{i+3,1}{}^2} = 0$$

where:

ς = Head-loss coefficient (dimensionless).

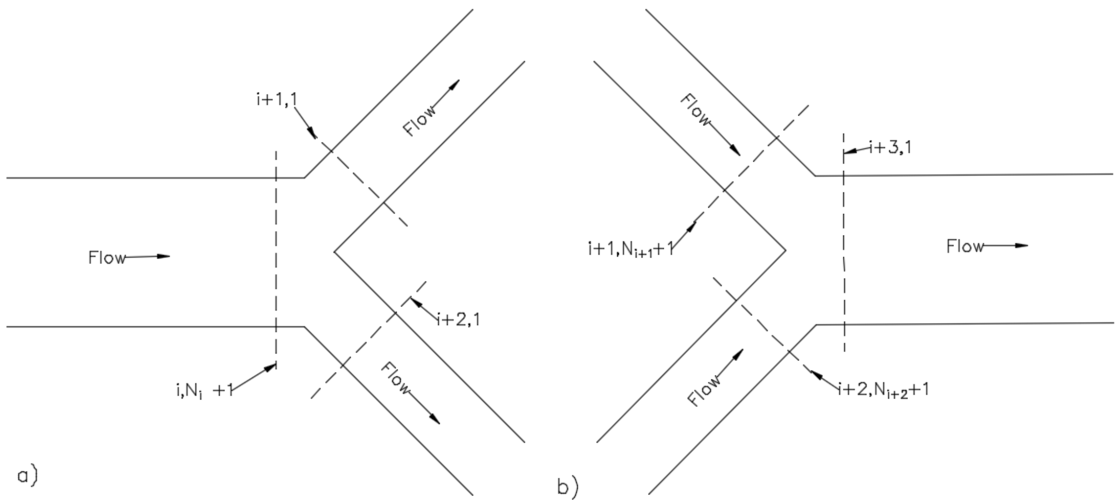

Figure 3: Definition sketch of a channel junctions in a looped network. a) Upstream junction (jn_1); and b) downstream junction (jn_2).

If the equations for a channel network are arbitrarily arranged, then all the nonzero elements of the Jacobian matrix may not necessarily lie on or near the principal diagonal [13]. This results in increased storage requirements, increased computer time, and, most probably, reduced accuracy [13]. For a parallel-channel system with M parallel channels, this arrangement of equations results in a Jacobian of bandwidth $3M + 1$ [13]. In more complex networks, the previous equations must be included for branching junctions of three channels. Also, for complex channel networks, there is not a generalized procedure for arranging the equations to produce a Jacobian matrix of minimum bandwidth, since the system is asymmetric.

Lateral weir design

The lateral weir design depends on the flow depth at the weir. To determine the appropriate height of the crest (P_w) it's recommend that as an initial estimate for this value the ratio of the wetted area to the top width ($P_w = A_w/T_w$) [14]. Also it's recommended that the height of the crest of a suppressed rectangular weir (Figure 4) should be at least equal to three times the maximum head (h_{max}) at the weir [15]. Also the sidewalls of the weir must extend at least a distance of 0.3 h_{max}.

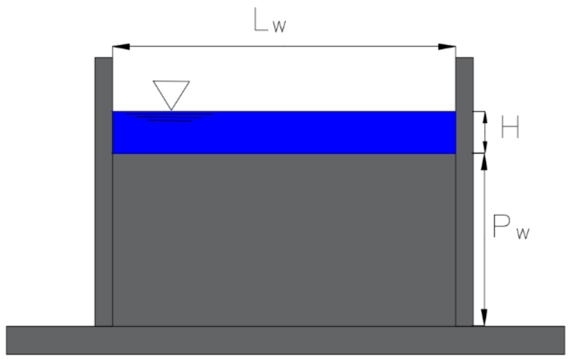

Figure 4: Cross section of a suppressed rectangular weir.

The head at the crest of the weir is obtained by subtracting the height of the crest from flow depth (H=Y_w -P_w). For the SM the flow depth at the weir is considered to be the average flow depth between the section preceding and following section of the weir. This head of water above the crest is related to the discharge; therefore a higher head of water means an increase of the flow throughout the weir. To determine the discharge that passes through the weir, the following equations are used (Eq. 9 to Eq. 11). This flow can be a maximum of 25% of the original discharge that flows through the main channel [16].

$$Q_w = C_e L_w H^{3/2} \qquad \text{Eq. 9}$$

$$C_e = \frac{2}{3} C_d \sqrt{2g} \qquad \text{Eq. 10}$$

$$C_d = 0.485\ (\frac{2 - Fr^2}{2 + 3Fr^2})^{0.5} \qquad \text{Eq. 11}$$

where:

L_w = Length of the weir (Longitude),

H = Head above the weir (Longitude),

C_e = Effective discharge coefficient (dimensionless),

C_d = Discharge coefficient (dimensionless), and

Fr = Froude number (dimensionless).

When a lateral weir is incorporated to the series of channel, in the SSM, produces a division of the existing channel into two new channels, with the same geometric properties of the existing channel, joined by the lateral weir. This new junction can be represented as series junction criteria, since the flow through the weir will reduce the flow through the main channel and will modify the energy equation vector and the Jacobian matrix. The main modification is the addition of the weir flow to the continuity equation (Eq. 2), since a value is now available and there will be a reduction on the flow at the main channel. The design criteria for the lateral weir consists of determining the length of the weir necessary to produce a specific discharge throughout the weir that will irrigate a specific parcel. Therefore, the weir flow desired will be given to the algorithm, as a percent of the discharge upstream of the weir location. Also, an initial estimate of the height of the crest is given to the algorithm. It is important to verify with the solution if the height of the crest meets the minimum criteria proposed by the U.S. Bureau of Reclamation. With the flow depth calculated, the coefficient of discharge and effective coefficient of discharge is obtained, to finally determine the length of the weir from Eq. 9.

Inverted syphon design

Inverted siphons (sometimes called sag culverts or sag lines) are used to convey water by gravity under roads, railroads, other structures, various types of drainage channels and depressions [17]. It is defined as a closed conduit designed to run full and under pressure [17]. The siphon profile (see Figure 5) is design in such a way to satisfy certain requirements of cover, siphon slopes, bend angles and submergence of inlet and outlet. One of the most important design criteria is the siphon velocities. According to the Utah Department of Transportation (UDT), these velocities should range between 3.5 to 10-ft/s and depends on the available head, economic considerations and siphon length [17]. The siphon velocity criteria will determine the minimum siphon diameter in the following manner [17]:

1. 3.5-ft/s or less for a short siphon not located under a highway with only earth transitions provided at entrance and exit,
2. 5-ft/s or less for a short siphon located under a highway with either a concrete transition or control structure provided at the inlet and a concrete transition provided at the outlet, and

3. 10-ft/s or less for a long > 200-ft siphon with either a concrete transition or control structure provided at the inlet and a concrete transition provided at the outlet.

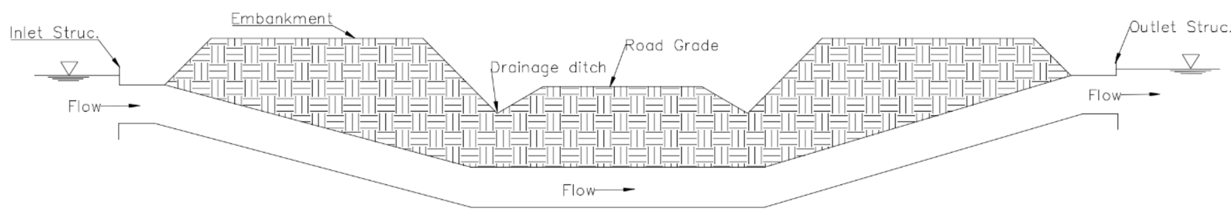

Figure 5: Example of a siphon profile that crosses a road. (Adapted from [17])

The head losses that should be included on the syphon design are [17]:

1. convergence loss in the inlet transition,
2. losses for a check structure where a check is installed in the inlet,
3. control structure losses where a control is installed in the inlet,
4. friction and bend losses in the siphon,
5. divergence loss in the outlet transition,
6. transition friction only in special or very long transitions, and
7. convergence and divergence head losses in earth transitions where required between an unlined canal and concrete transition are usually small and are thus ignored.

The first step on the design procedure, provided by the UDT, of a siphon inverted is to determine the inlet and outlet structures and approximate the syphon size. Next, select a preliminary transition geometry and create a preliminary siphon profile. Then, compute the siphon head losses and compared it with the available head. If the computed losses are greater than the difference in upstream and downstream canal water surface, the siphon will probably cause backwater in the canal upstream from the siphon, and therefore the siphon size should increase or the canal profile can be changed. In the other hand, if the computed losses are appreciably less than the difference in upstream and downstream canal water surface, it may be possible to decrease the size of siphon so that the available head is approximately the same as the head losses. Finally, determine the final transition geometry, compute actual head losses and prepare the final siphon profile.

Sluice gates design

A sluice gate is an opening in a hydraulic structure used for controlling the discharge [18]. Figure 6 shows the definition sketch for free flow and submerged flow sluice gate. Downstream free flow occurs at a (relatively) large ratio of upstream depth to the gate-opening height [18]. However, submerged flow at the downstream would occur for low values of this ratio [18]. The conventional sluice gate discharge equation is written in the following form (Eq.12):

$$Q_s = C'_d ab\sqrt{2gy_0} \qquad \text{Eq. 12}$$

where:

Q_s	=	Sluice gate discharge (Volume/time),
C'_d	=	Sluice gate discharge coefficient (depends on the flow condition) (dimensionless),
a	=	Sluice gate opening (Longitude),
b	=	Sluice gate length (Longitude), and
y_0	=	Upstream water depth (Longitude).

The free flow condition can be defined as Eq. 13 and the submerged flow conditions can be defined as Eq. 14 [18]. Depending on the flow conditions the sluice gate discharge coefficient can be defined as Eq. 15 or Eq. 16 for free flow and submerged flow conditions, respectively.

$$y_0 \geq 0.81y_2(\frac{y_2}{a})^{0.72} \qquad \text{Eq. 13}$$

$$y_2 \leq y_0 \leq 0.81y_2(\frac{y_2}{a})^{0.72} \qquad \text{Eq. 14}$$

$$C'_d = 0.611(\frac{y_0 - a}{y_0 + 15a})^{0.072} \qquad \text{Eq. 15}$$

$$C'_d = 0.611\left(\frac{y_0 - a}{y_0 + 15a}\right)^{0.072}(y_0 - y_2)^{0.7}\{0.32\left[0.81y_2\left(\frac{y_2}{a}\right)^{0.72} - y_0\right]^{0.7} + (y_0 - y_2)^{0.7}\}^{-1} \qquad \text{Eq. 16}$$

where:

y_2 = Tailwater Depth (Longitude).

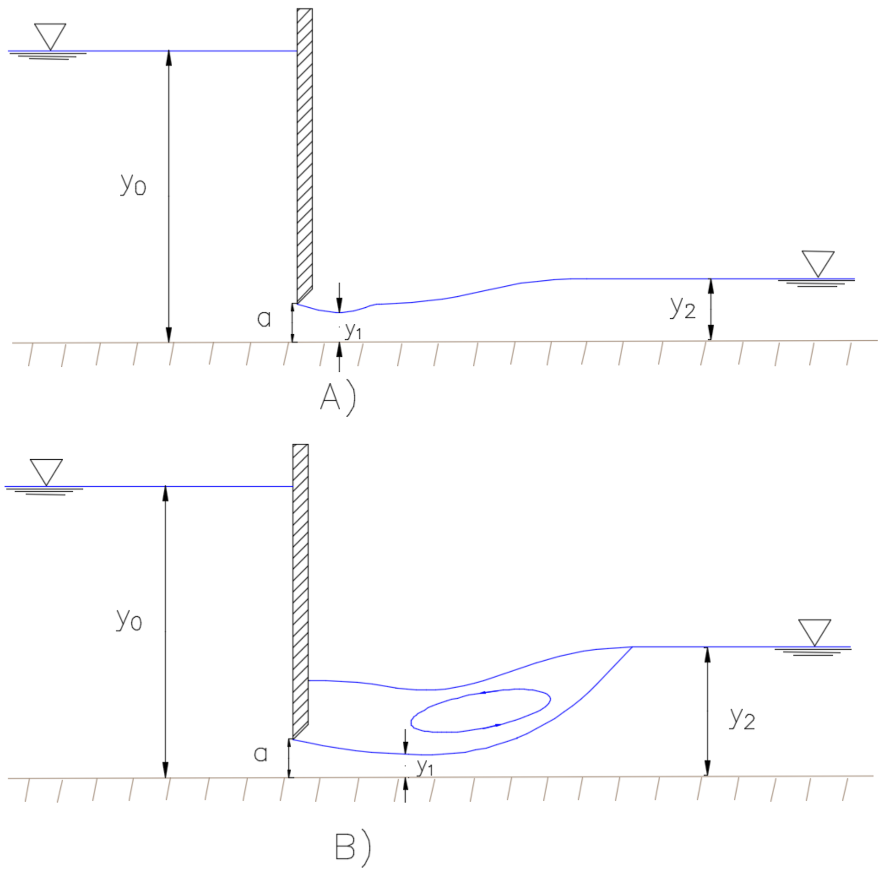

Figure 6: Sluice gate definition sketch. A) Free flow condition; B) Submerged flow condition.

Numerical solution of system

The numerical solution of the system of non-linear equations is based on the Newton-Raphson iterative procedure. This procedure is based on given an initial guess of the unknown variables, flow depths and discharges, in all the section of the channel network for the first iteration. Then the matrix system, shown in Eq. 3, will be created and solved using a numerical solver. The results from this, will be used for correcting the flow depths and discharge, previously assumed. This corrected flow depths and discharges will be used, on the second iteration, for computing the new values of the matrix system and therefore solving the matrix system. This iterative procedure is repeated until the flow depths and discharge corrections (solutions from the matrix system of Eq. 3) are less than a given tolerance.

The numerical solver selected was the Bi-conjugated Gradient Stabilizer with Preconditioner method (BiCGSTAB). BiCGSTAB is a variation of the Conjugate Gradient Squared method, which is based on squaring the residual polynomial and may lead to substantial build-up of rounding errors [19]. The BiCGSTAB is design to solve non-symmetric positive definite system that are large and sparse linear system, such as our system. Since our matrix system is ill-conditioned and is non-diagonally dominant, the preconditioner used with BiCGSTAB was the incomplete LU factorization (iLU) with threshold and pivoting. iLU produces a unit lower triangular matrix, and an upper triangular matrix, in which the ceros on the original matrix are preserve on the produced matrices; preserving the sparsity of the system. The pivoting of the iLU helps to avoid that an element close to the main diagonal is cero, which can produce numerical destabilization.

Preliminary results

Some preliminary results have been already developed from the proposed algorithm. The algorithm is capable of solving for the flow depths and discharges of looped channel networks with analysis and/or design of lateral weir. Figure 7 shows a sketch of the proposed case study of a looped channel network that contains five lateral weirs and 11 trapezoidal channels that have different geometric properties and amount of reaches. Table 1 shows details of the geometric and roughness properties for the channel system used as a case study.

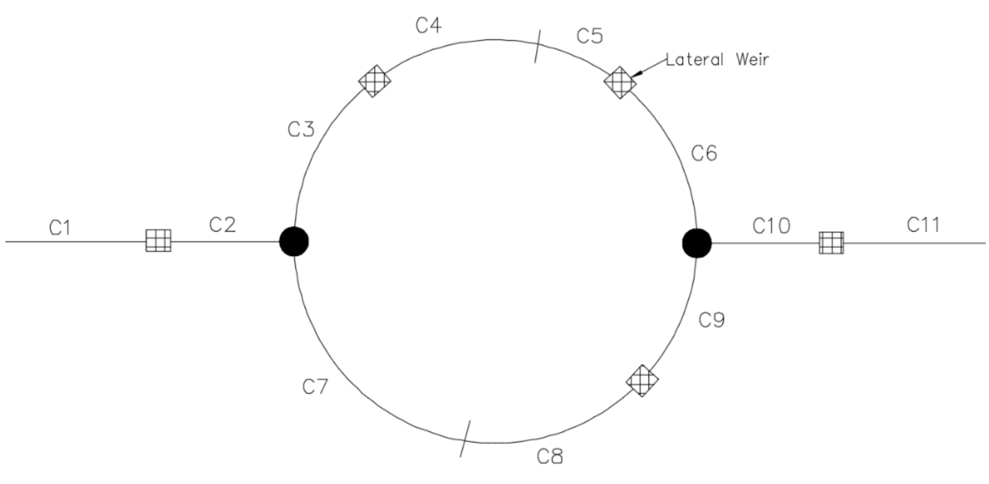

Figure 7: Case study of a looped channel network with lateral weirs.

Table 1: Geometric and roughness properties of the channel system

Channel ID	Reach Length (m)	Bottom Slope (m/m)	Manning's Roughness Coefficient	Bottom Width(m)	Lateral Slope (m/m)	Number of Reaches
C1	100	0.0001	0.013	50	1:1.5	2
C2	100	0.0001	0.013	45	1:1.5	2
C3	200	0.0005	0.012	30	1:1.5	4
C4	100	0.0005	0.012	25	1:1.5	2
C5	100	0.0005	0.012	20	1:1.5	2
C6	100	0.0005	0.014	20	1:1.5	2
C7	200	0.0005	0.013	40	1:1.5	5
C8	200	0.0005	0.014	35	1:1.5	4
C9	100	0.0005	0.014	30	1:1.5	2
C10	100	0.0001	0.015	20	1:1.5	2
C11	100	0.0001	0.015	20	1:2.0	2

The results from the SSM were compared with the ones from an identical model developed on the Hydrologic Engineering Center- River Analysis System software (HEC-RAS), which is from the U.S. Army Corps of Engineers. HEC-RAS uses the standard step method (StdSM) for solving the flow depths and discharges. The StdSM computes the solution one reach at a time, starting from the downstream end until reaching the upper limit of the network. To be able to compare both methods results, SSM and StdSM, the length of each reach was computed with the Direct Step Method (DSM) using flow depths and discharges obtained from SSM and StdSM. The computed reach lengths, from both methods were compared with the given reach lengths of each channel. The relative error in reach lengths obtained with this comparison is shown in Figure 8. The error calculations was as follows (Eq.17):

$$E = \left| \frac{L_{comp} - L_{given}}{L_{given}} \right| \times 100 \qquad \text{Eq. 17}$$

where:

L_{comp} = Computed reach length using the DSM,

L_{given} = Reach length given to both methods (SSM and StdSM), and

E = Percent of error for the reach length of each channel.

The SSM have significant lower percent of error for reach length than HEC-RAS- StdSM, giving confidence that the proposed algorithm is accurate in calculating the flow depth and discharges in looped channel networks with lateral weirs.

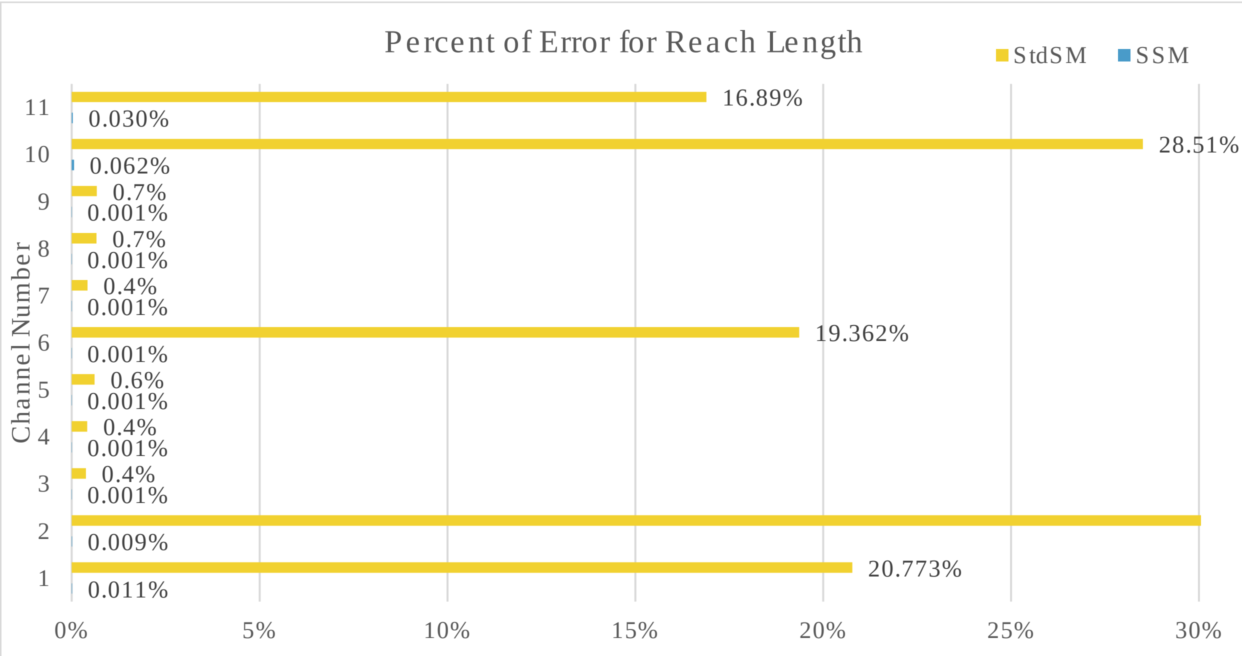

Figure 8: Percent of error for reach length of both method results for the case study of the looped channel network.

References

[1] Irrigation Museum (2015). *Irrigation Timeline* [Online]. Available: http://www.irrigationmuseum.org/exhibit2.aspx [March 22, 2016].

[2] A.M. Michael, "Utilization of Water Resources and Irrigation Development" in *Irrigation: Theory and Principles*, 2nd ed. Jangpura, India: Vikas Publishing House, 2008, ch. 1, pp. 1-6.

[3] United Nations Sustainable Development, "Agenda 21", in *United Nations Conference on Environmental and Development*, Rio de Janeiro, Brazil, 1992, pp. 18.1-18.90.

[4] Departamento de Recursos Naturales de Puerto Rico (2008). *Plan Integral de Aguas de Puerto Rico* [Online]. Available: http://www.recursosaguapuertorico.com/Cuencas-Principales-en-PR.html [March 22, 2016].

[5] M. H. Chaudhry and A. M. Schulte, "Computation of steady-state, gradually varied flow in parallel channels", in *Canadian Journal of Civil Engineering*, vol. 13, pp. 39-45, 1986.

[6] M. H. Chaudhry and A. M. Schulte, "Gradually-varied flows in open-channel network", in *Journal of Hydraulic Research*, vol. 25, pp. 357-371, 1987.

[7] B. J. Naidu *et al.*, "GVF Computation in Tree-type Channel Networks", in *Journal of Hydraulic Engineering*, vol. 131, pp. 457-465, Aug. 1997.

[8] H. P. Reddy and S. M. Bhallamudi, "Gradually Varied Flow Computations in Cyclic Looped Channel Networks", in *Journal of Irrigation and Drainage Engineering*, vol. 130, pp. 424–431, Oct. 2004.

[9] D. J. Sen and N. K. Garg, "Efficient Algorithm for Gradually Varied Flows in Channel Networks", in *Journal of Irrigation and Drainage Engineering*, vol. 128, pp. 351-357, Nov. 2002.

[10] A. Islam *et al.*, "Comparison of Gradually Varied Flow Computation Algorithms for Open-Channel Network", in *Journal of Irrigation and Drainage Engineering,* vol. 123, pp. 457-465, Sept. 2005.

[11] A. Islam *et al.*, "Development and Application of Hydraulic Simulation Model for Irrigation Canal Networks", in *Journal of Irrigation and Drainage Engineering*, vol. 134, pp. 49-59, Feb. 2008.

[12] D. Zhu *et al.*, "Simple, Robust, and Efficient Algorithm for Gradually Varied Subcritical Flow Simulation in General Channel Networks", in *Journal of Hydraulic Engineering*, vol. 137, pp. 766-774, July 2011.

[13] H. M. Chaudhry, "Computation of Gradually Varied Flow" in *Open-Channel Flow*, 2nd ed. New York, NY: Springer Science, 2008, ch.6, pp.150-197.

[14] R. W. P. May *et al.*, "Design Considerations" in *Hydraulic Design of Side Weirs*, 1st ed. Reston, VA: Thomas Telford Publishing, 2003, ch. 3, pp. 15-23

[15] U. S. Department of the Interior: Bureau of Reclamation, "Weirs" in *Water Measurement Manual*, 3rd ed. Washington, DC: U.S. Government Printing Office, 2001, ch. 7, pp. 7.1-7.30

[16] V. M. Vargas, "Hydraulic Design of an Overland Flow Distribution System Using Lateral Weirs", M.S. thesis, Dept. Civil Engineering and Surveying, Univ. of Puerto Rico, Mayagüez, PR, 2014.

[17] Utah Department of Transportation (2004). *Manual of Instruction: Roadway Drainage* [Online]. Available: https://www.udot.utah.gov/main/f?p=100:pg:0:::1:T,V:826, [July 10, 2016].

[18] P.K. Swamee, "Sluice-Gate Discharge Equations", in *Journal of Irrigation and Drainage Engineering*, vol. 118, pp. 56-60, Feb. 1992.

[19] Y. Saad, "Krylov Subspace Methods part II", in *Iterative Methods for Sparse Linear System,* 2nd ed. Philadelphia, PA: SIAM, 2003, ch. 7, pp. 229-258.