AUTOMATION STUDY OF ACTIVATED SLUDGE PROCESSES

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Automation Study of Activated Sludge Processes

Activated sludge is one of the most common wastewater treatment processes. The performance of the process largely depends on the experience of the plant operator, his ingenuity, and the circumstances under which he works. For this reason, failure to produce consistently high quality effluents is very common in sewage treatment plants. To improve the situation, the use of process analysis and modern control techniques is needed.

This study is divided into the following parts: (1) a summary of the existing control strategies, (2) an evaluation of the control strategy selected for detailed analysis, and (3) the development of a fully automatic control system for the activated sludge biological treatment processes.

Control Strategies

Control strategies in the operation of an activated sludge process include both steady-state control and dynamic-state control. Steady-state control is to maintain the long-term process objectives and is applied in response to seasonal changes in influent conditions. Dynamic-state control, on the other hand, is to maintain short-term process stability and is applied in response to diurnal flow variations.

A. Methods of Steady-state Control

There are five methods commonly applied to steady-state control of activated sludge processes. The typical activated sludge process and the process parameters with their
descriptions are given in Figure 1. The control principle and measurement requirements for each method are described as follows:

1. Suspended Solids Method

For steady condition, the activated sludge is best controlled by the use of the biological solids retention time (BSRT) as the control parameter. The relationship between the BSRT and the food to microorganism (F/M) ratio can be expressed as

\[ \frac{1}{\text{BSRT}} = Y(F/M) - k_d \]  

where

\[ Y = \text{cell yield coefficient} \]
\[ k_d = \text{endogenous decay coefficient} \]

It is clear in this equation that a constant F/M ratio can be obtained by maintaining a constant BSRT because both Y and k_d are constants. A constant F/M ratio of a biological process indicates that the microorganism feeding rate in the process is constant. The result will have a constant microbial growth rate, stable solids-liquid separation characteristics in the clarifier and thickener, and relatively constant effluent quality.

The constant BSRT can be obtained by varying the wasted sludge flow rate (Q_w) and the knowledge of both the mixed liquor suspended solids concentration (C_0) and the underflow sludge concentration (C_u). The mathematical equation of their relationship is expressed as
V = VOLUME OF AERATION TANK
C_o = MIXED LIQUOR SUSPENDED SOLIDS CONC.
Q = PLANT FLOW RATE
Q_r = RECYCLED SLUDGE FLOW RATE
Q_w = WASTED SLUDGE FLOW RATE
C_u = UNDERFLOW SLUDGE CONC.

FIGURE 1  A TYPICAL ACTIVATED SLUDGE PROCESS
\[ Q_w = \frac{VC_o}{(BSRT)C_u} \] (2)

This method of control requires the constant measurement of \( C_o \) and \( C_u \). This is why the method is called "suspended solids method." The measurement of suspended solids concentration in the laboratory is quite time consuming. Plant operators find the task of continuous measurement rather tedious.

2. Hydraulic Method with Underflow Wasting

This method eliminates the need to measure the suspended solids concentrations in the laboratory. Only the flow rate measurements are required. The method which has been reported in the literature\(^3\) is based on the following theory:

A mass balance of sludge at the steady state around the aeration tank as shown in Figure 1 gives

\[ Q_r C_u - (Q + Q_r) C_o + VC_o / BSRT = 0 \] (3)

This equation can be rearranged into

\[ \frac{C_o}{C_u} = \frac{Q_r}{Q + Q_r - V / BSRT} \] (4)

However, the BSRT can be expressed as

\[ BSRT = \frac{VC_o}{Q_w C_u} \] (5)

Therefore, by uniting equations 4 and 5, the wasted sludge rate for maintaining a desired BSRT can be calculated by

\[ Q_w = \frac{VQ_r}{BSRT(Q + Q_r) - V} \] (5a)
It is clear in this equation that for a given BSRT and a known V value, the wasted sludge flow rate (Q_w) changes only with the variations of the plant flow (Q) and the recycled sludge flow (Q_r). Flow rates can easily be measured. Continuous flow information can be fed into equation 5a for the wasted sludge flow determination on a continuous basis.

3. Hydraulic Method with Mixed-Liquor Wasting

This hydraulic method is even simpler than the hydraulic method with underflow wasting. Sludge wasting is discharged directly from the aeration tank instead of the sludge recycle line. Therefore, the BSRT can be expressed as

\[ \text{BSRT} = \frac{VC_o}{Q_w C_o} \]  \hspace{1cm} (6)

Hence, the sludge wasting rate is

\[ Q_w = \frac{V}{\text{BSRT}} \]  \hspace{1cm} (7)

In equation 7, no measurement is needed for determining the sludge wasting rate. For any given BSRT, the sludge wasting rate is fixed. This method is apparently easy on the process control, however, the requirement of an additional solids-liquid separator makes the method uneconomical.

4. Process Loading Method

This is usually referred to as the food to microorganism (F/M) ratio method. The process loading (or F/M) is defined as

\[ \frac{F}{M} = \frac{Q_{\text{BOD, infl.}}}{V X_a} \]  \hspace{1cm} (8)
where

\[ EOD_{\text{infl.}} = \text{BOD}_5 \text{ concentration of the sewage entering} \]
\[ \text{into the aeration tank (ML}^{-3}) \]
\[ X_a = \text{active microorganism concentration in the} \]
\[ \text{aeration tank (ML}^{-3}) \]

Constant microorganism feeding rate in the aeration tank produces a sludge with consistent settling characteristics which in turn result in better effluent quality.

This control method requires the measurement not only of the flow rate but also of the \text{BOD}_5 concentration of the sewage entering the aeration tank and the active microorganism concentration in the aeration tank. Both \text{BOD}_5 concentration and active microorganism concentration are difficult to obtain. The measurements give only estimated values. Besides, it takes 5 days to obtain \text{BOD}_5 values; this period of time is too long to be suitable for use in process control.

5. Constant MLSS Concentration Method

This is the most common control method used in the field, yet it is most unreliable. In using this method of control, a value of the mixed liquor suspended solids concentration is selected. The selection is based upon the experience in operation of a given plant. Maintaining this solids concentration results in satisfactory removal of substrate. If the influent characteristics do not vary significantly from their average values, then the control method is considered reasonably satisfactory. The only measurement required is that of the mixed liquor suspended
solids concentration in the aeration tank. The amount of recycled sludge to be wasted is determined largely by experience with the plant.

If the influent characteristics vary significantly from their average values, overloading or underloading is bound to occur; this, in turn, will alter the sludge settling characteristics and the effluent quality. This control method is inadequate to cope with the situation.

Table 1 shows all five methods for the steady state control of activated sludge processes. The control equation and the parameters to be measured are included in the table.

B. Methods of Dynamic-state Control

Dynamic-state control methods are applied to prevent solids overload in the settling tank and to reduce variations in mixed liquor suspended solids concentrations in the aeration tank. In domestic sewage treatment, diurnal flow variations and the surge flows during storms are occasions when the dynamic-state control is needed. Methods for dynamic-state control have not been fully developed and employed. Many plants employ only the steady-state control and ignore the dynamic-state control. Others employ recycle control for the dynamic-state control; the decision is made entirely on the basis of the plant’s experience in operation. In recent years, some progress has been made in unifying the sludge settling characteristics as represented by a batch solids-flux curve together with the flow variations of the treatment plant. Both the empirical method and the more rational recent method
Table 1 Methods for the Steady-state Control of Activated Sludge Processes

<table>
<thead>
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<th>Control Equation</th>
<th>Given Parameters</th>
<th>Parameter to be Measured</th>
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<tr>
<td>1. Suspended Solids</td>
<td>$Q_w = \frac{V(MLSS)}{\theta_c X_r}$</td>
<td>$V, \theta_c$</td>
<td>MLSS, $X_I$</td>
</tr>
<tr>
<td>2. Hydraulic method with Underflow Wasting</td>
<td>$Q_w = \frac{VQ_r}{\theta_c (Q+Q_r)-V}$</td>
<td>$V, \theta_c$</td>
<td>$Q, Q_r$</td>
</tr>
<tr>
<td>3. Hydraulic Method with Mixed-liquor wasting</td>
<td>$Q_w = \frac{V}{\theta_c}$</td>
<td>$V, \theta_c$</td>
<td>none</td>
</tr>
<tr>
<td>4. Process Loading</td>
<td>$\frac{Q}{M} = \frac{Q_{BOD_{infl}}}{V MLSS}$</td>
<td>$V$</td>
<td>$Q, MLSS, BOD_{infl}$</td>
</tr>
<tr>
<td>5. Constant MLSS</td>
<td>none</td>
<td>none</td>
<td>MLSS</td>
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$Q =$ treatment plant flow rate  
$Q_w =$ sludge wasting flow rate  
$V =$ volume of aeration tank  
$MLSS =$ mixed liquor suspended solids concentration  
$X_r =$ underflow suspended solids concentration  
$\theta_c =$ mean cells residence time  
$Q_r =$ recycled sludge flow rate  
$F/M =$ food to microorganism ratio
for dynamic-state control are summarized as the following:

1. Empirical Method

Most plants set their sludge recycle pump at a certain rate (e.g., 30% of the plant average flow rate) and leave it at that same rate all the time. Under this arrangement, the clarifier-thickener may be overloaded with solids during the peak flow rate period of the plant and the sludge recycle may be overpumped during the minimum flow rate period of the plant. Overloading of solids in the clarifier-thickener results in the rising of the sludge blanket. In serious cases, solids may be carried away and lost in the effluent of the clarifier-thickener. On the other hand, overpumping the recycled sludge leads to waste of electrical energy. Besides, excessive recycle flow causes more turbulent condition in the clarifier-thickener than what is necessary. The effluent quality will be poorer.

Another empirical approach is to set the sludge recycle rate proportional to the plant flow rate. This practice may alleviate the problems mentioned above. However, to set the recycle pump at a certain percentage of the plant flow rate requires the operation experience of the plant involved.

2. Recent Development

A more rational approach to dynamic-state control is to control the sludge recycle rate determined by the application of information concerning the sludge settling characteristics in the clarifier-thickener. Without taking into consideration the sludge settling characteristics in the dynamic-state control, the control strategy is incomplete. The settling
characteristics of the mixed liquor suspended solids can be represented by a batch solids flux curve.

**Batch Solids Flux Curve**

Solids flux is defined as the mass of sludge solids passing through a unit horizontal area in a unit time. The curve is constructed firstly to determine the zone settling velocity of each of the various MLSS concentrations. The solids flux is the product of the solids concentration and the zone settling velocity. The batch solids flux curve is a plot of the solids flux versus its solids concentration. The typical batch solids flux curve in the solids concentration range of interest in the study is shown in Figure 2.

**Use of Batch Solids-flux Curve**

In the clarifier-thickener, two factors contribute to the downward movement of the sludge solids: (1) gravity settling, and (2) the removal of the sludge in the underflow. Therefore, the total solids flux \( G \) in the clarifier-thickener is

\[
G = VC + UC
\]  

(9)

where

\[
V = \text{downward velocity due to sludge removal (LT}^{-1})
\]

\[
U = \text{downward velocity due to sludge settling (LT}^{-1})
\]

\[
C = \text{sludge solids concentration (ML}^{-3})
\]

The relationship between zone settling velocity and solids concentration can be expressed as\(^5\)
FIGURE 2  TYPICAL BATCH SOLIDS FLUX CURVE
\[ U = gC^{-h} \quad (10) \]

where \( g \) and \( h \) are constants.

Combining equations 9 and 10 gives

\[ G = VC + gC^{1-h} \quad (11) \]

which can be expressed graphically as shown in Figure 3. In this figure, the minimum value of the total solids flux curve can be obtained by differentiating equation 11 with respect to \( C \) and setting \( dG/dC=0 \). This value is called the limiting solids flux \((C_L)\) and the corresponding suspended solids concentration is called the limiting suspended solids concentration \((C_{L})\). The real significance of the limiting solids flux is that for a given sludge recycle rate, this flux rate exists somewhere on the pathway of the sludge settling and thickening in the clarifier-thickener. If the supplying solids flux is greater than the limiting solids flux, it will not be possible for the extra solids flux to pass through the barrier; hence, it gets accumulated above the point where the limiting flux occurs. As a result, the sludge blanket rises. The solids may eventually be lost into the overflow weir if the condition continues. On the other hand, if the supplying solids flux is smaller than the limiting solids flux, the sludge recycle rate becomes greater than that which is necessary. In this case, the sludge recycle rate should be reduced to the point of balance between the supplying solids flux and the limiting solids flux. The limiting solids flux and the suspended solids concentration at the limiting flux are expressed in the
FIGURE 3  TYPICAL TOTAL SOLIDS FLUX CURVE IN A CLARIFIER-THICKENER
following two equations, respectively:

\[
C_L = (gh^{-1})^{1/h} \left( h/(h-1) \right) \left( Qr/A \right)^{(h-1)/h} \quad (12)
\]

\[
C_L = \left( gh^{-1}A/Qr \right)^{1/h} \quad (13)
\]

**State Point Concept**

The state point concept for the operation of activated sludge systems was first introduced by McHarg. The state point is the intersection of the two operating lines which are established on the batch solids flux curve diagram. The two operating lines are the overflow operating line and the recycle operating line. A typical batch solids flux curve with the operating lines is shown in Figure 4.

The overflow operating line begins at the origin and has a slope showing the overflow rate of the clarifier-thickener. The recycle operating line is established by using the negative value of the recycle downward flow rate of the clarifier-thickener as its slope.

Yoshioka et al. demonstrated graphically that the intersection of the operating line with the vertical axis is equal to \( C_L \) and that the intersection of the operating line with the horizontal axis is equal to the suspended solids concentration in the underflow (\( C_U \)) if the recycle operating line
FIGURE 4  A TYPICAL BATCH SOLIDS FLUX CURVE WITH BOTH OVERFLOW AND RECYCLE OPERATING LINES
is tangent to the solids flux curve. The demonstration is shown in Figure 5.

In application, if the recycle operating line is tangent to the batch solids flux curve, the mixed liquor suspended solids (MLSS) concentration in the aeration tank can be obtained from the state point. In fact, the x-axis reading of the state point is equal to the MLSS concentration. This can be proved by the following analyses:

First of all, equation $G = \frac{Q}{A}C$ represents the overflow operating line and equation $G = -\frac{Q_r}{A}C + b$ represents the recycle operating line. Then b value is equal to $G_L$ when the y-axis intersection point ($C = 0, G = G_L$) is substituted into the recycle operating line equation. From the mass balance of solids around the clarifier-thickener, $G_L$ is equal to $(Q + Q_r)C_0/A$. Therefore, the recycle operating equation becomes $G = -\frac{Q_r}{A}C + (Q + Q_r)C_0/A$. This last equation together with the overflow operating line equation indicates that $C$ is equal to $C_0$.

The position between the recycle operating line and the batch solids flux curve can be used to explain the phenomenon of the process. For example, if the recycle operating line in Figure 6 is tangent to the batch solids flux curve (Case A), the solids flow from the top to the bottom of the clarifier-thickener is expected to be steady; hence, no solids will be accumulated in the clarifier-thickener. In other words, the sludge blanket is kept at a constant level.
FIGURE 5 GRAPHIC DEMONSTRATION OF YOSHIOKA TECHNIQUE
FIGURE 6  BATCH SOLIDS FLUX CURVE WITH VARIOUS RECYCLE OPERATING LINES
However, if the recycle operating line intersects the batch flux curve (Case B), the surface of the sludge blanket is expected to rise. The reason is that the sludge recycle rate is too low to remove the sludge. A lower recycle rate results in a lower limiting solids flux value in the system. Thus, the incoming solids flux exceeds the allowable passage flux rate which is established by the limiting solids flux. The solids accumulate in the clarifier-thickener and the surface of the sludge blanket rises.

In some situations, the recycle operating line is located on the left side of the batch solids flux curve (Case C). Then the clarifier-thickener is at an underloaded condition which means that the sludge removal from the bottom of the clarifier-thickener is faster than the sludge coming into the clarifier-thickener. Another way of obtaining the same limiting solids flux ($G_L$) is to construct a line tangent to the batch solids flux curve. (See the broken line in Figure 6.) The slope of the new tangent line is smaller than that of the old line. This indicates that a smaller recycle flow rate is sufficient for the case. An unnecessarily high recycle flow rate produces unnecessarily high turbulent conditions in the clarifier-thickener which may result in lower effluent quality. Besides, higher pumping rates use more electrical energy for operational purposes, a condition which is undesirable in today's energy conscious society.
Approach

Using the settling flux approach for process operation has been advocated recently by Keinath et al.\textsuperscript{8} In this study, the investigators purport to develop further this approach in terms of its use in the dynamic-state control of activated sludge processes.

The approach involves the establishment of the solids flux curve, the overflow operating line, and the recycle operating line as shown in Figure 7. The intersection of the two operating lines which is called the state point determines the MLSS in the aeration tank as indicated previously in this report. The recycle operating line is tangent to the solids flux curve. Based on these relationships, the overall governing equation is as follows: (See Appendix A for detailed derivation.)

\[
\frac{Q}{A} = \frac{g(h-1)(\frac{h}{h-1})^h(R)^{h-1}}{C_0^h (1+R)^h}
\]  

(14)

in which

- \( Q \) = plant flow rate
- \( A \) = surface area of the clarifier-thickener
- \( g, h \) = constants related to the establishment of the solids flux curve
- \( C_0 \) = MLSS in the aeration tank
- \( R \) = sludge recirculation ratio = \( Q_R/Q \)
- \( Q_R \) = recycle flow rate
FIGURE 7  TYPICAL SOLIDS FLUX CURVE AND OPERATING LINES
For a given plant flow, the recirculation ratio can be determined by equation 14. Thus, the recycle flow rate can be calculated. At this recycle flow rate, the clarifier-thickener is operated at a fully loaded condition. This is the optimal condition for the operation of the clarifier-thickener. If the clarifier-thickener is operated in an underloaded condition, the recycle rate becomes greater than that which is needed; hence, energy losses result. On the contrary, if the clarifier-thickener is operated at an overloaded condition, the sludge accumulates in the clarifier-thickener. This may result in losing sludge into the effluent.

For a given solids flux curve and a desired MLSS concentration, the recirculation ratio for each overflow rate can be determined by solving equation 14. Typical results are given in Figure 8 for overflow rates ranging from 12 m$^3$/m$^2$-d to 36 m$^3$/m$^2$-d. If the clarifier-thickener is designed by using the overflow rate of 24 m$^3$/m$^2$-d for the average daily flow of the plant, then the overflow rates 36 m$^3$/m$^2$-d and 12 m$^3$/m$^2$-d represent the maximum daily flow and the minimum daily flow of the plant, respectively. For all other in-between flows, the R values can be determined easily.

$C_0$ Value Selection

Since equation 14 is used for the dynamic-control operation, the question that arises is one of determining the value of $C_0$ to be used. Should the constant value of $C_0$ be at the steady state or should the value of $C_0$ changing with time be used? Theoretically speaking, the latter should
FIGURE 8  AN EXAMPLE OF USING SOLIDS FLUX CURVE FOR CONTROL
be selected. Then one may ask further how much effect the constant $C_0$ (as opposed to the variable $C_0$) has on the value of "R." A demonstration has been carried out in order to answer this question. The major endeavor in this study has been undertaken to demonstrate this point by way of mathematical simulation analysis.

Christensen and McCarty\textsuperscript{9}, in their biological treatment model study, consider total suspended solids in the reactor to be consisting of the inorganic portion of the suspended solids (SS), $X_{in}$, and four different kinds of organic SS: (a) biologically decomposable SS originally present in the sewage influent, $X_d$; (b) non-biodegradable or refractory SS originally present in the sewage influent, $X_r$; (c) active organisms formed during waste treatment, $X_a$; and (d) non-biodegradable remains of organisms after cellular decay, $X_i$. Hence, the MLSS concentration ($C_0$) may be expressed as follows:

$$C_0 = X_{in} + X_d + X_r + X_a + X_i \quad (15)$$

To evaluate these components of the suspended solids concentration at dynamic state, the following analyses have been made. Firstly, the concentration of SS in the biological reactor is related to $\Theta_0$. The flow of suspended solids through the sewage treatment system is illustrated in Figure 9. It is evident that the $\Theta_0$ for each of the SS components ($X_{in}$, $X_d$, $X_r$, $X_a$, $X_i$) is equal to the total mass of this component in the system divided by the rate at which the component leaves the system. The expression can be written as
FIGURE 9 COMPLETE MIX WITH SOLIDS RECYCLE
BIOLOGICAL TREATMENT SYSTEM
\[ \theta_c = \frac{X_j V}{Q^w x_j^w + Q^e x_j^e} \] (16)

\( X_j \) can be equal to \( X_{in} \), \( X_d \), \( X_r x_a \), or \( X_i \). \( V \) is the volume of the aeration tank. It is assumed that no suspended solids are present in the settling tank effluent.

Secondly, mass balance for the \( X_{in} \) of the treatment system at dynamic-state condition gives

\[ V \left( \frac{dX_{in}}{dt} \right) = Q x_{in}^o - (Q x_{in}^e + Q x_{in}^w) \] (17)

When equations 16 and 17 are combined and the differential equation is integrated, the inorganic suspended solids as a function of time becomes

\[ X_{in} = \left( \theta_c/\theta_h \right) x_{in}^o + \left( x_{in}' - \left( \theta_c/\theta_h \right) x_{in}^o \right) e^{-\frac{t}{\theta_c}} \] (18)

in which

\( X_{in}^o = \) initial concentration at time zero

By using the same method, the time-dependent formula for the refractory SS in the system becomes

\[ X_r = \left( \theta_c/\theta_h \right) x_r^o + \left( x_r' - \left( \theta_c/\theta_h \right) x_r^o \right) e^{-\frac{t}{\theta_c}} \] (19)

Thirdly, the mass balance of the active microorganism leads to the following:

\[ V \left( \frac{dx_a}{dt} \right) = \frac{k^o x_a S}{K_m + S} - k_d x_a V - (Q x_a^e + Q x_a^w) \] (20)
Combining equations 16 and 20 leads to

\[
\frac{dx_a}{dt} = \frac{k_o S}{K_m + S} - k_d - \frac{1}{\theta_c} X_a
\]  
(21)

The mass balance of the substrate is

\[
\frac{dS}{dt} = \left( \frac{1}{\theta_h} \right) (S^o - S) - \frac{k_o x_a S}{Y(K_m + S)}
\]  
(22)

At any given time, the values of \( X_a \) and \( S \) can be obtained by solving the differential equations 21 and 22 through Runge-Kutta numerical integration method.

Christensen and McCarty\(^9\) have derived the following two equations:

\[
x_d = x_d^0 \frac{S^o \theta_c}{S^o \theta_h}
\]  
(23)

\[
x_i = 0.2 k_d x_a \theta_c
\]  
(24)

Equation 23 is for the calculation of the decomposable organic SS concentration. Equation 24 is for the nonbiodegradable remains of microorganisms in the system. Both equations 23 and 24 are time-dependent because both values of \( S \) and \( X_a \) are a function of time. The steady state values of \( X_{in}, X_r, X_a, X_d \)
and \( X_i \) are used for the initial concentration at time zero. Substituting all these time-dependent values into equation 15 gives the variable \( C_0 \) which is used for calculation in the model.

The determination of the constant \( C_0 \) involves the calculating of all five forms of SS at the steady state. Equation 23 can be used for the determination of the \( X_d \) value at the steady state. However, the substrate concentration \( S \) is calculated from

\[
S = \frac{K_m(1 + k_d \theta_c)}{\theta_c(Yk_0 - k_d) - 1}
\]  
(25)

Equation 24 is also used for determining the \( X_i \) value at the steady state. The \( X_a \) in the equation is calculated by

\[
X_a = \frac{\theta_c Y(S^o - S)}{\theta_h (1 + k_d \theta_c)}
\]  
(26)

The steady state values of \( X_r \) and \( X_{in} \) are calculated from

\[
X_r = X_r^o \left( \frac{\theta_c}{\theta_h} \right)
\]  
(27)

\[
X_{in} = X_{in}^o \left( \frac{\theta_c}{\theta_h} \right)
\]  
(28)

Substituting all these steady-state values into
equation 15 gives the constant $C_o$ which is used for calculation in the model.

**Dynamic Mathematical Simulation**

A computer program is written to include the calculation of both the variable $C_o$ and the constant $C_o$. These data are fed into equation 14 to determine the $R$ value as a function of time. Figure 10 represents typical variations of flow, SS, and $BOD_5$ during a 24-hour period. A hypothetical plant with a treatment capacity of 37,860 m$^3$/d (10 MGD) at the average flow, an aeration tank of 9460 m$^3$ (2.5 MG) volume, and a clarifier-thickener of 2480 m$^2$ (26700 ft$^2$) surface area is used for the simulation run. The coefficients of the sludge settling characteristics are $g = 1.27 \times 10^{-5}$ m/d and $h = 2.65$. The kinetic constants are $k_o = 0.4$ hr$^{-1}$, $Km = 120$ mg/L, $k_d = 0.003$ hr$^{-1}$, and $Y = 0.5$. Other necessary items of input information are $S^0 = 160$ mg/L, $X^0_d = 10$ mg/L, $X^0_r = 27$ mg/L, and $X^0_{in} = 19$ mg/L. The solids retention time, $\Theta_o$, is equal to 5 days.

Figure 11 shows the result of the diurnal variation of $C_o$ in the simulation study. Although the curve appears to be widely varied, the maximum deviation is actually within 10% of the average concentration. The simulation result of the recirculation ratio is shown in Figure 12. Here the "R" values calculated from the constant $C_o$ are quite close to those calculated from the variable $C_o$. This indicates the high acceptability of using constant $C_o$ in determining $R$ values by
FIGURE 10  HOURLY VARIATION OF FLOW, BOD$_5$, AND SUSPENDED SOLIDS IN INFLUENT SEWAGE
**FIGURE 11** DIURNAL VARIATIONS OF $C_0$
FIGURE 12  DIURNAL VARIATIONS OF R VALUE IN RELATION TO $C_o$ ($\theta_c = 5$ DAYS)
equation 14. This outcome greatly facilitates the use of the settling flux approach for process control because constant \( C_0 \) is much easier to obtain than variable \( C_0' \). One can measure it directly from the aeration tank (average daily concentration) when the process has reached the steady state.

Running the program again with a change only in the \( \theta_c \) from 5 days to 10 days results in the recirculation ratio given in Figure 13. The difference between the two curves is again a small one.

**Changing Settling Characteristics**

If the sludge settling solids flux curve is used for process control, it needs to be checked periodically. Figure 14 illustrates the problem involved when the recycle operating line does not change in accordance with the new flux curve. The intersecting of the recycle operating line and the new flux curve indicates that the clarifier-thickener is overloaded and that the solids have begun to accumulate in the clarifier-thickener. If the recycle flow rate remains unchanged, the solids concentration in the aeration tank will decrease from \( C_0' \) to \( C_0'' \). If maintaining the \( C_0' \) in the aeration tank is desirable, then knowledge of an up-to-date solids flux curve is essential. However, for a well-operated plant at the steady state, the solids flux curve should not change considerably. A monthly evaluation of the flux curve is probably adequate.

**Surge Flow Remedy**

Storm flows often result in hydraulic surges. Figure 15
FIGURE 13
DIURNAL VARIATIONS OF R VALUES IN
RELATION TO $C_0$ ($\theta_c = 10$ DAYS)
FIGURE 14  PROBLEM OF CHANGING SETTLING FLUX CURVE
FIGURE 15  THE IMPACT OF SURGE FLOWS
illustrates the impact of surge flows. Assume that the state point is at "a" for any given flow $Q_1$. The recycle operating line is tangent to the flux curve. The clarifier-thickener is operated in a fully loaded condition with respect to thickening and in an underloaded condition with respect to clarification. Upon the occurrence of a surge flow, the state point moves upward from a to b and c. Since all these points are located below the flux curve. One can adjust the recycle flow according to the recycle operating line at each change of the state point. The clarifier-thickener is still operated under fully loaded condition for thickening. If the flow continues to increase to $Q_4$ and the state point moves to d which falls on the flux curve, then both clarifying and thickening aspects of the clarifier-thickener are fully loaded. If the surge flow, $Q_s$, increases to a point greater than $Q_4$, then the state point e can no longer intersect with the flux curve because it is located above the curve. The clarifying aspect of the clarifier-thickener is well overloaded. No amount of recycle flow can prevent the sludge blanket from rising and getting lost into the effluent.

If an extraordinary surge flow occurs, the calculation necessary for minimizing the loss of sludge solids involves firstly determining the new $C_o$ by

$$C_o = \left(\frac{Q_s}{A}\right)^{-1/h}$$

(29)

The new $C_o$ is the concentration reading at the intersection
point between the overflow operating line and the solids flux curve. To determine the R value, simply substitute both the new \( C_0 \) and the \( Q_s \) for the terms in equation 14.

In fact, substituting equation 29 into equation 14 and simplifying it leads to

\[
\frac{(h-1)^{h-1}}{h^h} = \frac{R^{h-1}}{(1+R)^h}
\]

(30)

It is evident that \( R \) equals \( h-1 \) in equation 30. Therefore, if equation 29 is used to handle those state points located above the solids flux curve, the recirculation ratio, \( R \), becomes equal to \( h-1 \) and will be the same for all surge flows. Figure 16 is the result of the previous simulation study (constant \( C_0 = 4000 \text{ mg/L} \), \( \theta_c = 10 \text{ days} \)) with only changes in some peak flows which serve as surge flows. The strategy of equation 29 is used for each surge flow with the "\( R \)" value being equal to 1.65 for all of them.

Figure 17 demonstrates that the minimum solids concentration in the aeration tank will be at 3300 \text{ mg/L}. If the clarifier-thickener has enough capacity to store the solids carried over by the surge flows, then the solids concentration in the aeration tank will return to 4000 \text{ mg/L} after the surge flows subside. Otherwise, it will take some time to build the solids concentration at the steady state level.

Comparison

The new control approach is compared and contrasted
FIGURE 16  SIMULATION RESULT OF SURGE FLOW
FIGURE 17  MINIMUM SOLIDS CONCENTRATION IN THE AERATION TANK AFTER SURGE FLOWS
with the traditional control approach. Solids retention time is used as the control parameter. When the hydraulic method with underflow wasting is used, the sludge wasting rate at the steady state can be expressed as

\[ Q_w = \frac{V Q_r}{e_c (Q + Q_r) - V} \]  \hspace{1cm} (31)

where

- \( Q_w \) = underflow wasting rate
- \( V \) = volume of aeration tank
- \( Q_r \) = sludge recycle rate

Furthermore, one has to know the sludge volume (\( V_1 \) in ml) through a 30-minute settling test in a one-liter cylinder, balance the solids around the settling tank, and apply equation 31. After these three steps, (see Appendix B for detailed derivation) the recirculation ratio becomes

\[ R = \frac{1 - \theta_h / \theta_c}{\frac{1000}{V_1} - 1} \]  \hspace{1cm} (32)

where

- \( \theta_h \) = hydraulic detention time = \( \frac{V}{Q} \)

In equation 32, "R" varies with changes in the flow rate.

For the comparison, both the new and the traditional control approaches use the same hypothetical data (\( \theta_c = 10 \) days, \( V = 9460 \) m\(^3\), \( C_0 = 4000 \) mg/L, etc.) as shown in the previous
example. A range of $V_1$ values (300, 400, and 500 ml) has been
selected. Figure 18 gives the results of the diurnal "R" values
and Figure 19 shows the operating lines (maximum and minimum
values) for each case together with the solids flux curve.
The diurnal variation of "R" is generally quite small in all
three cases. The reason is that the $\theta_c$ is much greater than
the "$\theta_h$" (10 days versus 3 to 6 hours). In Case A, all the
recycle operating lines are tangent to the solids flux curve.
This is the optimal operation as indicated previously. In
Case B, the maximum recycle line lies above the solids flux
curve; solids accumulation in the settling tank will result
during this period. However, if the recycle operating line lies
under the solids flux curve and is not tangent to it, the re-
cycle flow is higher than that which is necessary. Both cases
C and D indicate that the recycle flow is unnecessarily high
most of the time. This results in energy waste and poor clar-
ification in the settling tank. There is sufficient evidence
to warrant the conclusion that the new approach is superior as
a control technique for optimal utilization of clarifier-thick-
enner capacity with minimal accumulation of sludge in the
settling tank and appropriate recycle rate during diurnal flow
fluctuation.

Another point worth noting is that equation 31 can
be simplified into

$$Q_w = \frac{VR}{\theta_c(1+R) - \theta_h}$$

(33)
FIGURE 18 RESULTS OF THE DIURNAL R VALUES
FIGURE 19  MAXIMUM AND MINIMUM OPERATING LINES FOR EACH CASE STUDY
If \( \theta_c \gg \theta_h \), equation 33 can be further simplified to

\[
Q_w = \frac{VR}{\theta_c(1+R)} \quad (34)
\]

As noted earlier, the diurnal variation of "R" is generally quite small if equation 32 is used for \( R \) determination. However, the variation of \( R \) is significant if equation 14 is used for \( R \) determination. In general, using \( Q_w \) as a control parameter is for process stability at the steady state. The steady-state control variable should not be adjusted in response to short-term variation in process state. Therefore, \( Q_w \) should be adjusted only on a daily basis by using equations 32 and 34. Average daily flow rate should be used in the calculation of \( \theta_h \) in equation 32.

**Automation**

Interest in computer-control of wastewater treatment processes has been growing in recent years. Figure 20 shows the schematic control diagram of the activated sludge processes. Sewage flow rates are fed on-line into a microcomputer where \( R/Q \) values are computed. In order to avoid constant change of the recirculation, an average value of a selected time interval (e.g. 30 minutes) is fed into the pump control system where the control signal is generated. The recirculation pump is operated automatically according to the given signal.

The constants \((g\) and \(h)\) which govern the settling characteristics of the sludge are revised periodically (e.g. on a monthly basis). The method of the \( g \) and \( h \) determination
FIGURE 20  SCHEMATIC DIAGRAM OF THE AUTOMATIC CONTROL SYSTEM
can be found in journal articles\textsuperscript{4,11} and textbooks\textsuperscript{12,13}.

Conclusions

From the data gathered and analyzed, the following conclusions can be drawn:

1. The new approach for dynamic-state control of activated sludge processes is both practically feasible and theoretically sound.

2. For practical purposes, constant $C_0$ for the control equation can be considered a good approximation.

3. The solids flux curve needs to be renewed periodically; this may be done on a monthly basis. The value of $C_0$ should be modified if the curve is changed significantly.

4. A technique is offered for minimizing the loss of sludge into the effluent under extraordinary surge flow conditions. The equation can be integrated into the computer program.

5. The integration of this control approach into plant automation has a very promising future.

The following areas may be recommended for further studies:

1. A refinement of the technique for determining the solids flux curve

2. The use of a small treatment plant to test the new approach for dynamic-state operation

3. The installation in the demonstration plant of an automated system for controlling the sludge recycle rate

This is a step further toward the automation of plant operation.
Notations

A surface area of clarifier-thickener, m²
BSRT biological solids retention time, days
C sludge solids concentration, mg/L
C₀ MLSS concentration in the aeration tank, mg/L
C_L solids concentration at the limiting solids flux, mg/L
C_u underflow suspended solids concentration, mg/L
F/M food to microorganism ratio
g constant, m/d
h constant, unitless
G solids flux, kg/m²-d
G_L limiting solids flux, kg/m²-d
K_o maximum rate of substrate utilization per unit weight of microorganisms, 1/day
K_d decay rate of microorganisms, 1/day
K_m half velocity constant, mg/L
Q plant flow rate, m³/d
Q_e plant effluent flow rate, m³/d
Q_r sludge recycle rate, m³/d
Q_s surge flow rate, m³/d
Q_w underflow sludge wasting rate, m³/d
R sludge recirculation ratio, Q_r/Q
S substrate concentration in effluent, mg/L
S_c substrate concentration in influent, mg/L
t time
u downward velocity due to sludge settling, m/d
v downward velocity due to sludge removal, m/d
V volume of aeration tank, m³
V_L sludge volume through a 30-min settling test, ml
X_aae active microorganisms in aeration tank, mg/L
X_aef active microorganisms in effluent, mg/L
X_aew active microorganisms in wasting sludge, mg/L
X_d_ae influent biodegradable SS in aeration tank, mg/L
X_d_ee biodegradable SS in influent, mg/L
X_d_ee influent biodegradable SS in effluent, mg/L
X_d_w influent biodegradable SS in wasting sludge, mg/L
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$X_i$</td>
<td>inert remains of microorganisms in aeration tank, mg/L</td>
</tr>
<tr>
<td>$X_{ei}$</td>
<td>inert remains of microorganisms in effluent, mg/L</td>
</tr>
<tr>
<td>$X_{wi}$</td>
<td>inert remains of microorganisms in wasting sludge, mg/L</td>
</tr>
<tr>
<td>$X_{in}$</td>
<td>inorganic SS in aeration tank, mg/L</td>
</tr>
<tr>
<td>$X_{on}$</td>
<td>inorganic SS in influent, mg/L</td>
</tr>
<tr>
<td>$X_{en}$</td>
<td>inorganic SS in effluent, mg/L</td>
</tr>
<tr>
<td>$X_{wn}$</td>
<td>inorganic SS in wasting sludge, mg/L</td>
</tr>
<tr>
<td>$X_{in,0}$</td>
<td>inorganic SS in aeration tank at time zero, mg/L</td>
</tr>
<tr>
<td>$X_{ir}$</td>
<td>influent refractory VSS in aeration tank, mg/L</td>
</tr>
<tr>
<td>$X_{ri}$</td>
<td>refractory VSS in influent, mg/L</td>
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<tr>
<td>$X_{re}$</td>
<td>influent refractory VSS in effluent, mg/L</td>
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<tr>
<td>$X_{rw}$</td>
<td>influent refractory VSS in wasting sludge, mg/L</td>
</tr>
<tr>
<td>$X_{r}$</td>
<td>influent refractory VSS in aeration tank at time zero, mg/L</td>
</tr>
<tr>
<td>$Y_e$</td>
<td>microorganism yield coefficient</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>hydraulic detention time of aeration tank, days</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>solids retention time, days</td>
</tr>
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</table>
References


Appendix A - Detailed Derivation of the Governing Equation

The solids flux at the fully loaded condition may be expressed as

\[ G_L = \left(\frac{g(h-1)}{h/(h-1)}\right)^{1/h} \left(\frac{Q_r/A}{(h-1)/h}\right) \]

A material balance equation for solids entering and passing through the limiting flux layer in the clarifier-thickener is given by

\[ (Q + Q_r)C_o = G_L A \]

By substituting equation 1 into equation 2 and simplifying it, the following governing control equation can be obtained:

\[ \frac{Q/A}{C_o (1+R)^h} = \frac{g(h-1)(h/(h-1))^R}{C_o h^{h-1}} \]

in which

\[ R = \text{recirculation ratio} \]
Appendix B – Detailed Derivation of Equation 32

By assuming effluent suspended solids at zero, the material balance around the clarifier-thickener can be expressed as

\[(Q + Q_r)C_o = Q_r C_u + Q_w C_u\]  \hspace{1cm} (1)

The sludge wasting rate by the hydraulic method is

\[Q_w = \frac{VQ_r}{\theta_c (Q+Q_r) - V}\]  \hspace{1cm} (2)

Substituting equation 2 into equation 1 gives

\[(Q + Q_r)C_o = Q_r C_u + \frac{VQ_r}{\theta_c (Q+Q_r) - V} C_u\] \hspace{1cm} (3)

Also the value of \(C_u\) can be approximately equal to \(100 C_o / \sqrt{V_1}\); substituting this expression into equation 3 and simplifying it gives

\[R = \frac{1 - \theta_h/\theta_c}{1000 \frac{V_1}{V_1} - 1}\]