THE OPTIMUM DISTRIBUTION OF URBAN WATER SUPPLIES IN PUERTO RICO

TITLE 1 (PL 88-379)

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ABSTRACT

Increasing costs of supplying water to urban centers necessitate the rational analysis of such water distribution systems, in an effort to hold the water supply costs to a minimum. Optimization of urban water distribution systems may be investigated by applying linear programming techniques to minimize the costs between water supply and demand areas.

The water distribution system of the city of Mayaguez, P.R. was the subject of the present investigation. The city is supplied by two water treatment plants of combined capacity 16 million gallons per day. Treated water is supplied by gravity to twenty-two demand areas within the city, through a distribution system consisting of approximately 100 miles of main, ranging from 2 inches to 20 inches in diameter. The cost of distributing water from the supply to the demand areas include pumping raw water to plants, water treatment, water loses, and system maintenance.

The distribution system was first analyzed as a classic transportation problem in which demand areas should be supplied in such a way as to minimize the total distribution cost, under the constraints that the total amount supplied by each supply area must not exceed its capacity, and that the amount demanded by an area must be fully satisfied. In addition to the transportation analysis, a network model was developed for the Mayaguez distribution system. The network analysis maintains the same objective function, i.e. it minimizes the total distribution cost, however, different constraints are obtained by applying a material balance of water in, out, and consumed at each network node.

INTRODUCTION

A reliable supply of water of satisfactory quality and quantity forms the basis of the existence and growth of every urban community. The distribution of such a vital resource would be expected to follow principles of rational and efficient planning designed to derive the highest possible benefits from the utilization of this clearly limited and irreplaceable commodity. However, with almost all urban water distribution systems this does not appear to be the case. "Efficiency" or even "conservation" have not been seriously considered in planning urban water distribution systems in contrast to the planning of other urban utility systems such as electric power, telephone service, or transportation.

Optimization studies for water resources systems have been appearing in the pertinent literature with increasing frequency; however, to a great extent, these studies are concerned with non-urban uses of water from multipurpose, multiunit reservoir systems for flood control, hydro-electric power, navigation, and irrigation. The basic reason for the appalling lack of systematic analysis of urban water distribution systems appears to be a generally prevailing misconception among residents of urban and metropolitan communities, including those responsible for the water distribution, that water is God's gift to humanity to use or, more often, to abuse, as they see fit.

Granted that the total quantity of water in all its forms, including sea water, icecaps and glaciers, as well as atmospheric humidity, is estimated to be over three hundred million cubic miles; however, less than one-third of a percent of the above quantity of fresh water is presently available for human use. Furthermore, the distribution of this available fresh water often does not favor the needs of a given urban or metropolitan community.

The above mentioned misconception has been kept alive up to the present by the fact that the marginal cost of water has been very low, compared to that of other urban utilities. As long as the marginal cost of water supply has been low, it has appeared reasonable that consumption should be encouraged. Consequently, many municipalities supply water free of charge, while others charge a small flat rate which often decreases by steps as the consumption increases above certain limits.

This picture is changing rapidly. The frightening rate of population increase is placing ever increasing demands on water supply, while at the same time, increased water pollution causes a decrease of the available water fit for consumption. The situatic appears worse for urban communities. Suffice it to mention that of the total population of the United States, it has been estimated that between 80 to 90 per cent will be locate in urban centers by the year 2,000. Similarly, the 1970 census revealed that during the past decade the population of urban centers in Puerto Rico has increased substantially at the expense of rural communities. The same trend has been documented in the U.S.S.R.

Many urban centers are already facing problems of water shortage, and more are expected to face such problems within the immediate future, unless water distribution policy changes from the existing, to one that is based on a rational analysis of costs of supply. Water shortages make necessary construction of more production and distribution facilities. Meanwhile, sources of supply tend to be further away from the consumption points increasing pumping and conveyance costs. Water pollution, at the same time necessitates expensive new treatment facilities. Marginal costs of water supply, therefore, are continuously increasing making the traditional procedures of distributing water and establishing consumer charges unacceptably wasteful.

The ultimate goal of a rational analysis of an urban water distribution system should be the optimization of the system, in such a manner that total costs of supplying the water to the consumer should be a minimum. The expected savings from a rational cost analysis can subsequently be used to locate and develop needed new sources of water, to construct new facilities for treatment and distribution, or to upgrade the existing facilities. Unconventional sources of water such as desalinization and reuse after treatment, may also be investigated as they become financially feasible. Finally, the analysis can yield a more reasonable schedule of rates to be charged to the consumer, than the outmoded flat rate.

The greater Mayaguez area was selected for our study. The rivers Yaguez, Cañas and Añasco serve as the raw water sources for the Mayaguez area. Water from Río Y... is delivered by gravity to the "Old" treatment plant where it is treated and filtered. T... capacity of this plant is 3.0 mgd. Raw water from the Cañas and the Añasco Rivers is pumped to the "Old" treatment plant, as well as to the Miradero plant, the capacity o... which was recently increased from 5 mgd in 1965, to 14 mgd at present. Treated wate... from the plants is distributed by gravity through a distribution system consisting of appro... ximately 100 miles of main, ranging from 2 inches to 20 inches in diameter.

The establishment of the supply areas and their capacities was straightforward. How... ever, turning to the demand areas, the problem was how and what to choose as a basis for grouping. A single house would result in an impossibly large project. Therefore, some grouping of houses was needed to make up a demand area. The problem was solved by observing the fire flow protection needed for a given area. The Mayaguez water system can be conveniently divided into twenty-two fire flow areas based on population density, as shown in figure 1.

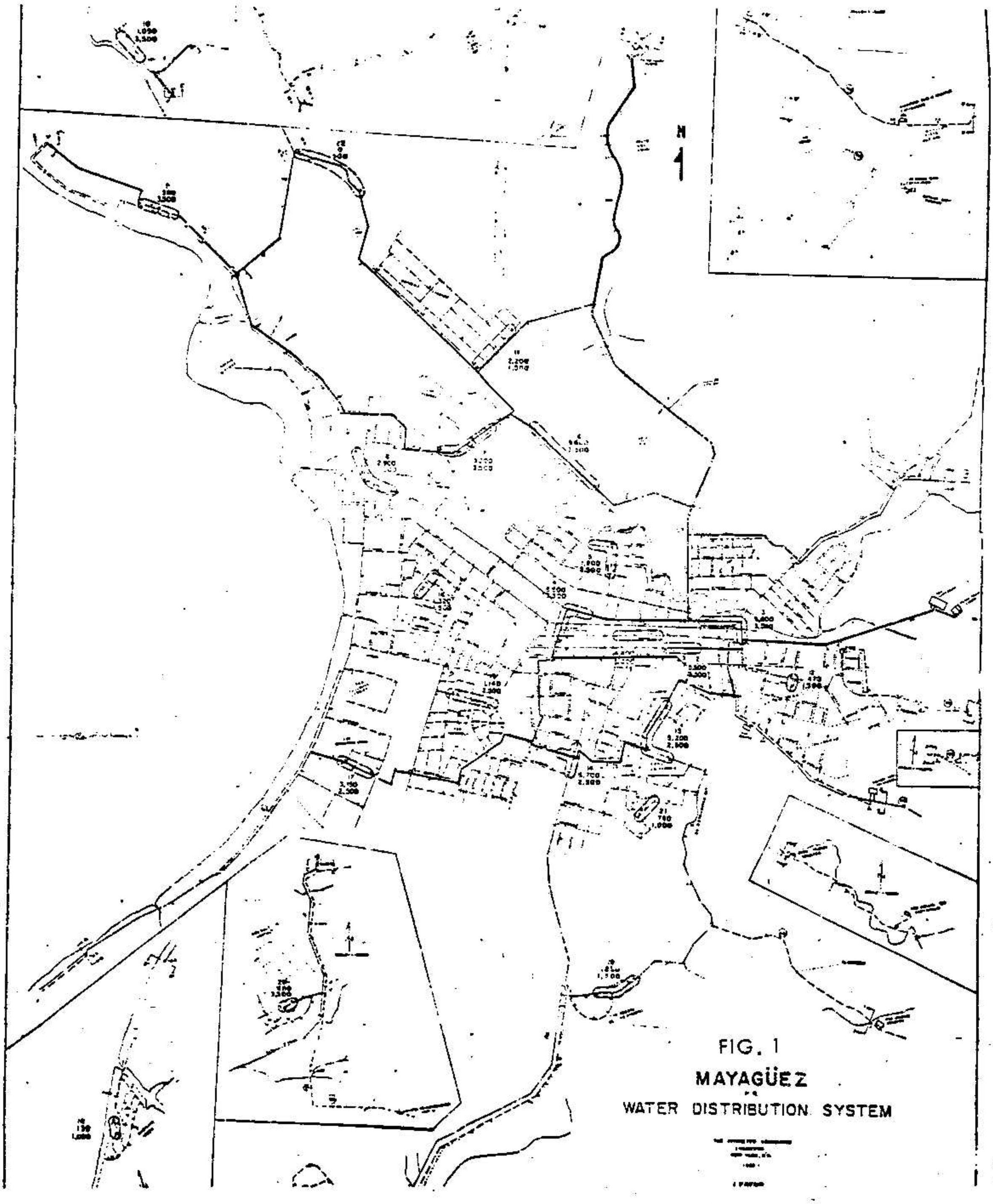

FIG. 1

MAYAGÜEZ

WATER DISTRIBUTION SYSTEM

Then, knowing the fire flow requirements for each of the areas, we were able to backcalculate to obtain the daily average usage for the given demand area. In such a manner, the supply and demand areas have been qualitatively and quantitatively defined.

THEORETICAL CONSIDERATIONS

Our analysis began with a consideration of the distribution costs involved. In general, such costs may include the following components:

1) pipeline construction cost,

2) pipeline operation, maintenance, and repair cost,

3) easement (right of way) cost,

4) pumping station cost,

5) pumping station operation, maintenance, and repair cost,

6) pumping costs,

7) insurance amortization and tax costs.

In case where actual cost data of each of the above were not available, generalized equations were developed and tested. These were used with reasonable accuracy to predict distribution costs. It was then possible to know the cost of supplying water from a known supply area to a known demand area. This cost can be expressed in dollars per unit gallon supplied.

Consider now the general problem of minimizing the cost for a total urban system and its surrounding areas, where there exists multiple supply and demand areas which can be supplied as shown in figure 2. It is clear that the demand areas should be supplied in such a way as to minimize the total cost of distribution for the given distribution system. This total cost can be expressed as follows:

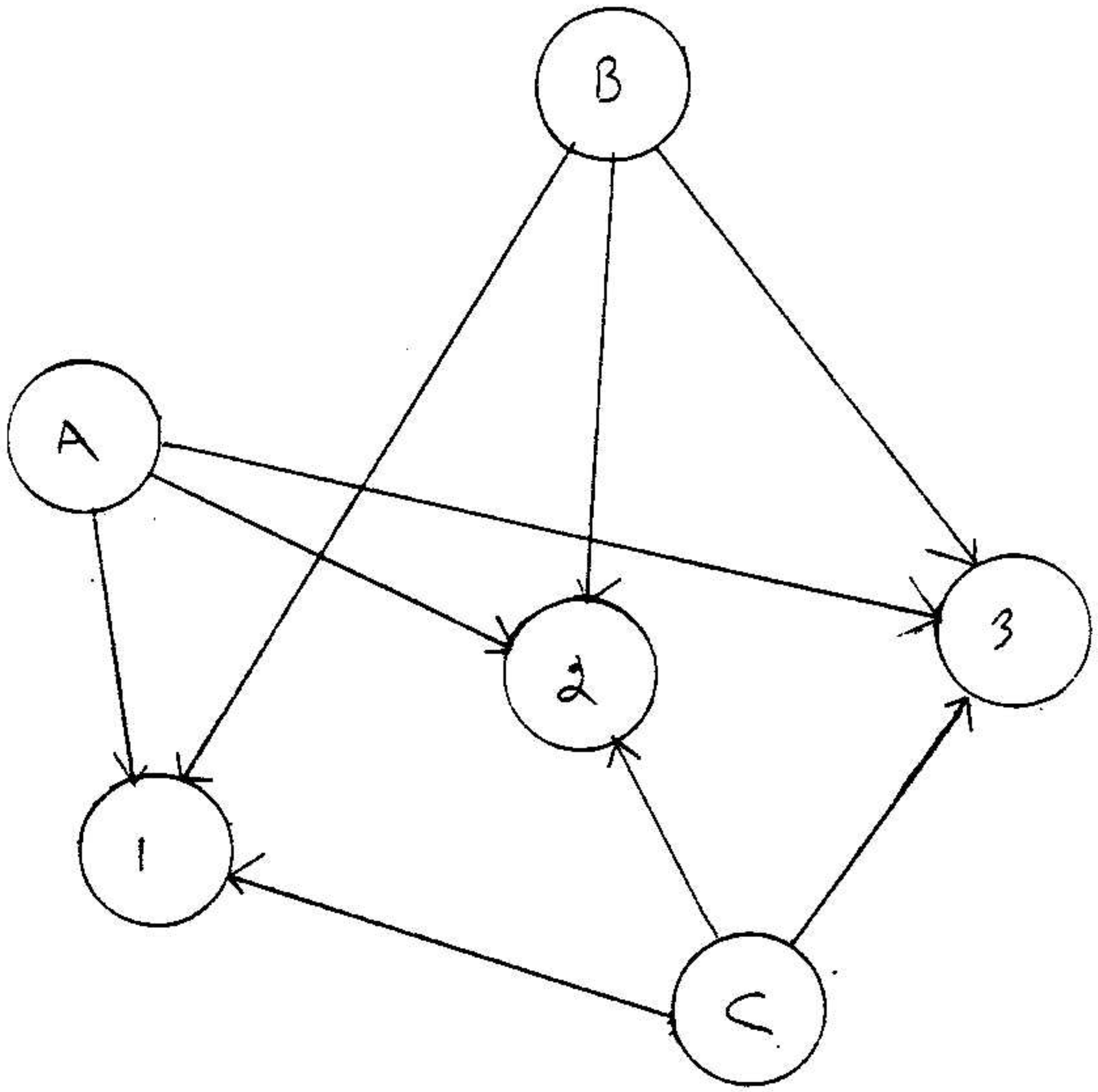

FIGURE 2: TRANSPORTATION MODEL
A, B, C denote supply areas
1, 2, 3 denote demand areas.

$$\text{Total distribution cost} = \frac{\$A1}{Gal} \times a_1 + \frac{\$A2}{Gal} \times a_2 + \frac{\$A3}{Gal} \times a_3 + \ldots\ldots$$

where $\frac{\$A1}{Gal}$ is the unit cost of supplying water from supply area A to demand area 1, and a_1 is the volume of water to be supplied. If the unit costs are considered constants the only variables which are free to be adjusted to minimize the total distribution cost are the amounts of water supplied.

The constraints on these variables are: 1) that the total amount supplied by each supply area must not exceed its capacity and 2) that the amount demanded by an area must be fully satisfied.

If all the equations are linear, or can be linearly approximated, the minimum cost of distribution can be found by using linear programming to solve the following optimization problem expressed in matrix notation.

It is desired to minimize the total cost distribution function $Z = C^T X$ subject to:

$$AX \leq B$$
$$DX \geq F$$

$$X \geq 0$$

C^T is the constant unit cost matrix, where T denotes transposition.

X is the volume of water to be distributed between the supply and the demand area to be adjusted such that the equation is minimized. A, B, D and F are constants.

The above mentioned problem is often referred to as a classic transportation problem in operations research. Of course, this model is only an approximation of a water system since direct pipe connections between each supply and demand area do not always exist. The actual system is a network as show in figure 3. This problem can be solved similarly. The objective function remains the same, that is, to minimize the total cost. However, the

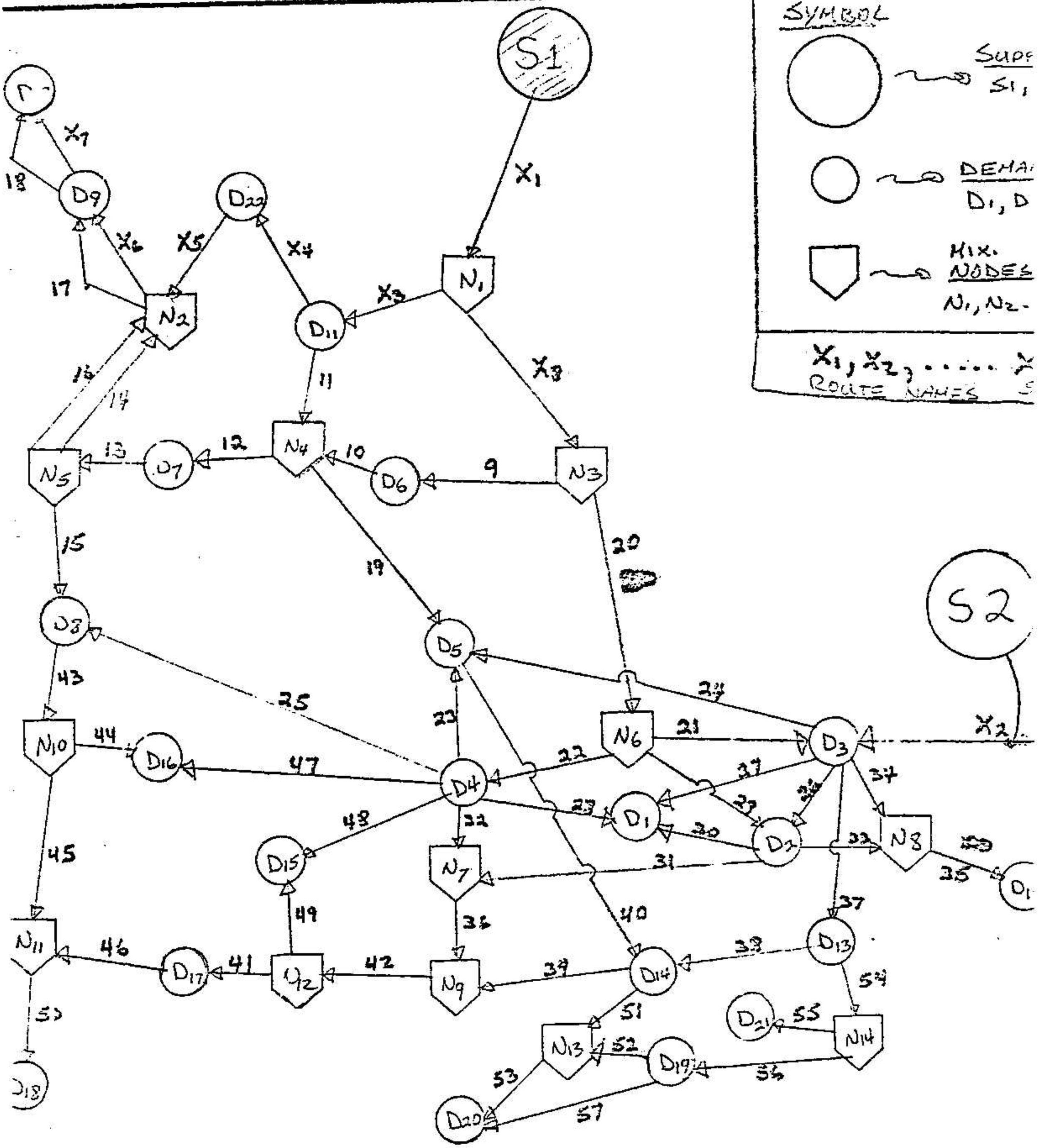

FIGURE 3: Network Model

constraints are different. Around each node, a material balance of flows in, out, and consumed is written. These material balances are the constraints of the following minimization problem:

minimize $Z = C^T X$

subject to $AX \geq B$ for a demand area

$DX = F$ for a node

$X \geq 0$

A, B, D, and F are constants, X is the flow rate, C^T is the constant unit cost matrix, where T denotes transposition. Both the transportation and the network models were studied. The results are given later in the report.

COST EQUATIONS:

The costs involved in suppling water for urban consumption within the Mayaguez area consist of the following:

1- pumping raw water to plants;

2- water treatment;

3- water loses; and

4- system maintenance.

These costs must be expressed mathematically to help compose the total cost objective function, which is to be minimized. Since we are going to minimize the total cost with respect to the flow rates between all possible supply and demand points, all the individual costs between such points need to be expressed in terms of dollars per gallon for a given time period.

For example :

$$\text{Minimize (Total Cost)} = \text{Cost}_{1,2,A} \times Q_{1,2,A,} + \cdots + \text{(all other possible paths of flow)}$$

where

$Cost_{1,2,A}$ is the dollars per gallon cost for distribution between supply points 1 and demand point 2, for a given path, A, and

$Q_{1,2,A}$ is the flow rate in gallons per day, which is to be calculated by the optimization process, between supply 1 and demand 2 over a given path, A.

Each cost between a supply and demand point in Mayaguez, as mentioned above, is composed of pumping costs to the treatment plants, water treatment, water losses, and system maintenance. All of these must be expresed in dollars per gallon for a given period of time;

1. Consider first the pumping cost to the treatment plants. This cost per gallon can be derived by fundamental principles of fluid mechanics and can be expressed as follows:

$$C_p = \frac{A^{2.73}\, Q^{1.84}\, \rho^{0.84}\, \mu^{.16}\, K(1+J)\, HL}{D_i^{4.84}\, E} + \frac{B^1}{Q}$$

where: C_p = dollars per gallon per day

Q = flow rate, cfs

ρ = fluid density, lb/ft^3

μ = fluid viscosity, centipoises

K = cost of electrical energy, \$/kwhr

H = hours of operation /year

E = efficiency of motor and pump expressed as a fraction

B^1 = a constant, independent of Q

A = conversion factor for cfs to gallons per day

L = length of pipe, ft.

2- The cost of water treatment has been obtained from several sources, and is as follows:

Plant capacity (mgd)	Treatment cost ($/million gallons), C_T
2	100-140
5	75-100
50	30-45

Therefore, the cost per gallon is defined for a given size treatment plant.

3- Water losses in the Mayaguez area amount to about 16% of the total consumption. Therefore this cost can be expressed as

$$C_L = 0.16\ C_T$$

where C_L = dollars/gallon for water losses

C_T = dollars/gallon for treatment costs

A more representative cost figure which considers the length of pipe (an important consideration in the optimization) can be developed. This is:

$$C_L = \frac{10{,}000\ C_T\ L}{\bar{Q}}$$

with L the route length in miles, and $\bar{Q}$ the average flowrate through the route, in gallons per day.

4- System maintenance costs including pipeline operation, maintenance and repairs are estimated as follows:

$$C_m^1 = \$\ 10\ D/\text{mile}$$

where D = pipe diameter in inches

The cost per gallon is found by dividing C^1_m by the average flow rate through a given path,

$$C_m = (\text{cost per gallon}) = \frac{C_m^1\ L}{\bar{Q}}$$

where C_m^1 = maintenance cost per foot of pipe

L = length of pipe in feet

$\bar{Q}$ = average flow rate in gallons per day

Another method of determining the maintenance costs is based on total system size as follows:

Capacity of system	Cost
1 mgd	21.82¢ / 1000 gal
5 mgd	14.8¢ / 1000 gal
10 mgd	13.24¢ / 1000 gal

The total cost per gallon for each path between a supply and demand point (or between nodes or demand areas for the network analysis) then becomes the sum of costs for pumping to the treatment plants, the water treatment cost, the water losses costs, and the system maintenance cost as follows:

$$C = C_p + C_T + C_L + C_m \quad \text{for each path.}$$

For the transportation model there are 44 possible paths between the supply and demand areas and for the network model there are 57 paths between nodes, supply, and demand areas.

COMPUTER PROGRAM :

The computer program consists of the following:

1- MAIN CALLING PROGRAM

Calls subrcutines "Transp" & "Netwrk"

2- SUBROUTINE "TRANSP"

a) Reads in Matrix

b) Calculates coefficients (costs)

c) Calls subroutine " LINPRO" for solution

3- SUBROUTINE "NETWRK"

a) Sets up problem and reads in matrix

b) Calculates cost coefficients

c) Calls subroutine "LINPRO" for solution

4- SUBROUTINE "LINPRO"

a) Searches for a feasible solution

b) Simplex Method Algorithm for optimum solution

5- DATA:

a) Lengths of pipes for different diameter and parts

b) Matrix of known demand and cost elements.

SUBROUTINE "LINPRO"

* Linear Programing Subroutine will solve the following problem

$$\text{Min } (A_{11} X_1 + A_{12} X_2 + A_{13} X_3 + \ldots\ldots A_{1n} X_n)$$

Subject to:

$$A_{21} X_1 + A_{22} X_2 + \ldots\ldots\ldots = A_{2,(N+1)}$$

$$A_{31} X_1 + A_{32} X_2 + \ldots\ldots\ldots = A_{3,(N+1)}$$

$$\vdots$$

$$A_{M1} X_1 + A_{M2} X_2 + \ldots\ldots\ldots = A_{M,(N+1)}$$

* The Constraints may be equality, greater than, or less than constraints.

* Input: The subroutine needs as Input the matrix size (M X (N + 1)) for a problem with (M - 1) contraints and N unknowns, and the matrix itself in which the first row corresponds to the objective function.

$$A_{1,1} \quad A_{1,2} \text{ ---------- } A_{1,(N+1)}$$

$$A_{2,1} \quad A_{2,2} \text{ ---------- } A_{2,(N+1)}$$

$$A_{M,1} \quad A_{M,2} \text{ ---------- } A_{M,(N+1)}$$

The subroutine will search for a feasible solution and if it finds one the simplex method takes place until the optimum extremum point is found.

* Varibbles (Fortran) and identification that subroutine Linpro needs:

VARIABLE		IDENTIFICATION
A (I, J)		Coefficient Number (ROW)
		I Equation Number (ROW)
		J Unknown Variable Number (Column)
L (I)		An array with the following values:
		0, If constraint is an equal to or "greater than"
	OR	Slack Var. Num., If contraint is "less than"
II		Total Number of Rows in Matrix (# of contraints + 1)
JJ		Total Number of Columns.in Matrix (# of Unknown Variables + 1)
III	=	II + 1

Our programs were run on a DEC - 1 Computer.

given in the appendix). Only data cards for each READ statement need be punched. Format statements are listed with each READ statement.

The first READ statement is on page 2 of the program listing in subroutine TRANSP. It is the size of the matrix of the coefficients of the left-hand-side of the linear equation matrix (I, J). This is followed by reading in the known left-hand-side matrix elements (see page 2, Appendix). This is accomplished by punching the I, J position of the element and then its value according to the format indicated. The right hand side elements are then read in the same manner. The program proceeds to calculate the coefficients for the objective function after reading in the value of the distances between the points concerned (see page 3, Appendix). The I, J identification of the two points is controlled by a DO loop along with the K identifications of the various pipe diameters, so that only the total distance between the two points need be punched. This is all the data that is needed for the transportation analysis of the problem. All cost data are included in the program itself.

For the subroutine NETWRK which analyzes the problem using a network analysis approach, the READ statements are clearly indicated (see pages 6 and 7) and require the same data as the subroutine TRANSP using different formats. These formats are clearly indicated below the READ statements.

RESULTS

Both the transportation and network models were applied to the Mayaguez water distribution system, for the 1965 conditions, with total supply of 8 mgd. The computer program showed that feasible solutions to both models were possible. The optimum solutions to both models were then computed. The transportation model gave a minimum cost of $1762.07/day, eliminating twenty routes while optimizing the distribution throughout the rest. The network analysis model gave a minimum cost of $1533.91/day, eliminating six routes while optimizing the distribution through the remaining routes.

The network model was then applied to the present supply-demand conditions, with the Old Plant supplying 3 mgd and the Miradero Plant 13 mgd. The computed minimum cost was $3743.06/day. In order to estimate savings, the program was modified to give the maximum cost of distribution, which was found to be $4521.42/day. This represents potential savings of about $780 per day or about $280,000 per year. This potential savings could be realized by revalving the flows as indicated by the computer program results. Cost for revalving should be minimal since it is believed that most of the valves needed already exist and no new pipes need be installed, since all the pipes have sufficient capacity.

The model was also used to check the distribution costs had the present demand been satisfied by increasing the Old Plant capacity to 8 mgd while maintaining the Miradero Plant at 8 mgd. The computed minimum cost was $3445.69/day, which represents savings about $300 per day over the system that was actually adopted.

The complete results of both cases considered are shown in tables 1 and 2.

TABLE 1
1965 Conditions
$Q_1 = 5$ mgd ; $Q_2 = 3$ mgd

Transportation		Network	
Functional	$1762.08 /DAY	Functional	$1533.91 /DAY
Variable	Value	Variable	Value
x 21	204	x 58	97
x 45	97	x 57	477
x 28	341	x 30	681
x 30	341	x 26	820
x 33	339	x 2	3000
x 38	204	x 22	1976
x 7	477	x 23	477
x 11	477	x 9	477
x 13	477	x 12	1840
x 15	409	x 15	409
x 17	477	x 6	954
x 19	477	x 7	477
x 34	2	x 3	2112
x 24	204	x 35	204
x 1	681	x 37	1499
x 4	477	x 38	341
x 6	477	x 48	341
x 31	204	x 47	204
x 42	136	x 41	477
x 35	136	x 50	136
x 26	341	x 56	681
x 40	477	x 29	338
x 9	477	x 55	136
x 43	68	x 4	68
		x 1	4903
		x 14	954
		x 8	2791
		x 11	1840
		x 13	1363
		x 20	2314
		x 32	477
		x 34	204
		x 36	477
		x 46	136
		x 44	0
		x 42	477
		x 51	0
		x 54	817

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APPENDIX

COMPUTER PROGRAM

```
C                    VICTOR/ FOR MULTIPLE RUNS
C
C
      COMMON A(45,90),W(45),L(45),III,JJ,II
      DO 10 I=1,25
      WRITE(3,1)
   10 CONTINUE
      CALL TRANSP
      WRITE(3,3)
    3 FORMAT(1H1)
      DO 20 I=1,25
      WRITE(3,2)
   20 CONTINUE
      CALL NETWRK
      CALL EXIT
    1 FORMAT(1H ,23X,'APPROACH # 1 ... TRANSPORTATION PR
    2 FORMAT(1H ,24X,'APPROACH # 2 ... NETWORK PROBLEM',/)
      END
```

F4? '26(14) 27-SEP-74 14:47 PAGE 2

```
      SUBROUTINE TRANSP
C     MINIMIZATION OF COST OF DISTRIBUTION
C          USING LINEAR PROGRAMMING
C
C                    TRANSPORTATION APPROACH
C
C
C    MAIN PROGRAM FOR READING OF KNOWN COEFFICIENTS (OF THE CONSTRAINTS)
C
C         AND FOR CALCULATION OF THOSE WHICH ARE UNKNOWN  (OBJECTIVE FCT,
C
      DIMENSION O(7),D(2,30,7),B(2,30),DIAM(7),J1(68)
      COMMON A(45,90),W(45),L(45),III,JJ,II
C    READ IN MATRIX SIZE AND INITIATE
      READ(2,1002)II,JJ
 1002 FORMAT(2I2,F8.4)
      III=II+1
      DO 1010 I=1,III
      W(I)=0.0
      L(I)=0
      DO 1010 J=1,JJ
 1010 A(I,J)=0.0
C    READ IN KNOWN MATRIX ELEMENTS
      DO 80 J=1,46
      IJX=J/2
      XJX=J/2.0
      IF(XJX-IJX) 90,90,91
   91 A(2,J)=1.0
      GO TO 80
   90 A(3,J)=1.0
   80 CONTINUE
      I=0
      IN3=46
      DO 89 I=4,25
      IN=IN+1
      IN3=IN3+1
      A(I,IN)=1.0
      IN=IN+1
      A(I,IN)=1.0
   89 A(I,IN3)=-1.0
 1011 READ(2,1002) I,J,X
      IF(I) 1016,1016,1018
 1018 A(I,J)=X
      GO TO 1011
 1016 J=JJ
C    READ IN RIGHT HAND SIDE  ELEMENTS
 1017 READ(2,1002)I,JK,X
      IF(I) 1020,1020,1019
 1019 A(I,JJ)=2.0E2*X
      L(I)=JK
      GO TO 1017
 1020 CONTINUE
C
```

```
F4?     ?26(14) 27-SEP-74        14:47   PAGE 3

C    PROCEED NOW TO CALCULATE COEFFICIENTS OF OBJECTIVE FCT,
C            READ IN DISTANCES FIRST
C         INCLUDE HERE   N= # OF DEMAND AREAS
      N=22
      NPATH=0
      WRITE(3,333)
  333 FORMAT(1H1)
      WRITE(3,699)
  699 FORMAT(17X,'MAP LENTHS (CENTIMETERS) OF POSSIBLE ROUTES',/,17X,
     1'BY SECTIONS OF DIFFERENT DIAMETERS (INCHES)',///)
      WRITE(3,700)
  700 FORMAT(2X,'ROUTE / DIAM',5X,'20',6X,'16',6X,'14',6X,'12',
     16X,'10',6X,' 8',6X,' 6',//)
      DO 10 J=1,N
      DO 10 I=1,2
      READ(2,11) (D(I,J,K),K=1,7)
   11 FORMAT(7F10.3)
      NPATH=NPATH+1
      WRITE(3,111) NPATH,(D(I,J,K),K=1,7)
  111 FORMAT(3X,'(',I2,')',6X,7F5.1)
   10 CONTINUE
      DO 12 I=1,2
      DO 12 J=1,N
   12 B(I,J)=0.0
C       SOME KNOWN DATA
C                        DIAM(I) .......DIAMETER OF TUBES IN INCHES
C                      CT1 , CT2 ........TREATMENT COST
C                     CL       .. WATER LOSSES COST (DOLLARS/1000 GALLONS)
C                    CM   .......MAINTAINANCE COST  (DOLLARS/1000 GALLONS)
C                     Q(I)  ......., FLOW FOR USE IN MAINTANACE COST EQ,
C                     FACT     .....   CONVERSION FACTOR (UNITS)
      DIAM(1)=20.0
      DIAM(2)=16.0
      DIAM(3)=14.0
      DIAM(4)=12.0
      DIAM(5)=10.0
      DIAM(6)=8.0
      DIAM(7)=6.0
      CT1=1.5E-4
      CT2=1.27E-4
      XQ=0.1247*3600.0*24.0
      Q(1)=600.0*XQ
      Q(2)=400.0*XQ
      Q(3)=270.0*XQ
      Q(4)=200.0*XQ
      Q(5)=144.0*XQ
      Q(6)=90.0*XQ
      Q(7)=45.0*XQ
      FACT=400.0/(3.12*1609.3)
C     CALCULATION PROCEDURE
      NPATH=0
      WRITE(3,749)
  749 FORMAT(1H1,11X,'CALCULATED COST COEFF. OF OBJ. FCT. (DOLLARS/1000
```

```
     1GAL)',/////,11X,'ROUTE',8X,'CT',13X,'CL',13X,'CM',14X,'COEFF,=CT+C
     2L+CM',//)
      DO 20 J=1,N
      DO 20 I=1,2
      IF(I-1) 21,21,22
  21  CT=CT1*1000.
      GO TO 23
  22  CT=CT2*1000.
  23  CL=0.16*CT
      CM=0.0
      DO 25 K=1,7
  25  CM=CM+(10.0*DIAM(K)*FACT )*(D(I,J,K)/Q(K))
      C1=CM*1200.
      B(I,J)=CT+CL+CM
      NPATH=NPATH+1
      WRITE(3,750) NPATH,CT,CL,CM,B(I,J)
 750  FORMAT(14X,I2,3F15.6,F20.6)
  20  CONTINUE
C     NOW ASSIGN VALUES OF COEFFICIENTS OF OBJECTIVE FCT,
C                  TO THE FIRST ROW OF MATRIX.
      MX=0
      DO 30 J=1,N
      DO 30 I=1,2
      MX=MX+1
  30  A(1,MX)=B(I,J)
C     LIST PROBLEM TO BE SOLVED BY LINEAR PROGRAMMING
      WRITE(3,500)
 500  FORMAT(1H1,' THE PROBLEM TO BE SOLVED IS THE FOLLOWING ...',////)
      WRITE(3,501)
 501  FORMAT(10X,'OBJECTIVE FUNCTION TO MINIMIZE',//)
      WRITE(3,555)
      WRITE(3,502) ((A(1,J),J),J=1,44)
 555  FORMAT(24X,'UNITS ARE   COEF. = (DOLLARS/1000 GALLONS)'/,40X,
     1'VARIABLES = (1000 GALLONS/DAY)',///)
 502  FORMAT(5(1X,F6.3,' *X',I2,'  +'))
      WRITE(3,503)
 503  FORMAT(2X,///, 5X,' SUBJECT TO')
      DO 504 J=1,68
 504  J1(J)=J
      WRITE(3,505) J1(2),(J1(J),J=1,45,2),A(2,JJ)
      WRITE(3,505) J1(3),(J1(J),J=2,46,2),A(3,JJ)
 505  FORMAT(4X,I2,3X,11('X',I2,' + '),/,10X,11('X',I2,' + '),/,
     110X,'XS',I2,'  =  ',F8.2,/)
      IN1=-1
      IN=0
      IN3=46
      DO 506 I=4,25
      IN1=IN1+2
      IN=IN+2
      IN3=IN3+1
      WRITE(3,507) J1(I),IN1,IN,IN3,A(I,JJ)
 507  FORMAT(4X,I2,9X,'X',I2,'  +  ','X',I2,'  =  ','XS',I2,' =',F8,2)
 506  CONTINUE
```

```
C       NOW GO TO  SUBROUTINE  LIN PRO FOR SOLUTION
        CALL LINPRO
        RETURN
        END
```

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```
      SUBROUTINE NETWRK
C          MINIMIZATION OF COST OF DISTRIBUTION
C               USING LINEAR PROGRAMMING
C
C               NETWORK APPROACH
C
C         MAIN PROGRAM FOR READING OF KNOWN COEFFICIENTS (CONSTRAINTS)
C         AND CALCULATION OF THOSE WHICH ARE UNKNOWN (OBJ. FCT.)
      DIMENSION Q(7),JCOL(8),DIAM(7),VAL(8)
      COMMON A(45,90),W(45),L(45),III,JJ,II
C  READ IN MATRIX SIZE AND INITIATE
      READ(2,1002) II,JJ
 1002 FORMAT(2I2,F8.4)
      III=II+1
      DO 1010 I=1,III
      W(I)=0.0
      L(I)=0
      DO 1010 J=1,JJ
 1010 A(I,J)=0.0
C  READ IN KNOWN MATRIX ELEMENTS
 1011 READ(2,1902) I,((JCOL(IX),VAL(IX)),IX=1,8)
 1902 FORMAT(I2,8(I4,F4.2))
      IF(I) 1016,1016,1018
 1018 DO 300 IX=1,8
      IF(JCOL(IX)) 1011,1011,301
  301 J=JCOL(IX)
  300 A(I,J)=VAL(IX)
      GO TO 1011
 1016 J=JJ
C  READ IN RIGHT HAND SIDE ELEMENTS
 1017 READ(2,1002) I,JK,X
      IF(I) 1020,1020,1019
 1019 A(I,JJ)=2.0*X
      L(I)=JK
      GO TO 1017
 1020 CONTINUE
C
C  PROCEED NOW TO CALCULATE COEFFICIENTS OF OBJ. FCT.
C      READ IN DISTANCES FIRST
      WRITE(3,700)
  700 FORMAT(1H1,11X,'CALCULATED COST COEFF. OF OBJ. FCT. (DOLLARS/1000
     1GAL)',/,12X,'AND MAP DISTANCES (CM.) ALONG TUBE DIAMETER (INCHES)'
     2,/,26X,'OF THE 57 POSSIBLE ROUTES',//,' ROUTE',6X,'DIST',6X,'DIAM'
     3,7X,'CT',11X,'CL',11X,'CM',3X,'COEFF.=CT+CL+CM',/)
C
C
C  SOME KNOWN DATA
C                    DIAM(I) .......DIAMETER OF TUBES IN INCHES
C                 CT1 , CT2 .........TREATMENT COST
C                CL       .. WATER LOSSES COST (DOLLARS/1000 GALLONS)
C               CM    .......MAINTANANCE  COST  (DOLLARS/1000 GALLONS)
C                 Q(I)  ....... FLOW FOR USE IN MAINTANACE COST EQ.
C                 FACT    .....   CONVERSION FACTOR (UNITS)
```

```
      DIAM(1)=24.0
      DIAM(2)=16.0
      DIAM(3)=14.0
      DIAM(4)=12.0
      DIAM(5)=10.0
      DIAM(6)=8.0
      DIAM(7)=6.0
      CT1=0.15
      CT2=0.127
      XQ=0.1247*3600.0*24.0
      Q(1)=600.0*XQ
      Q(2)=400.0*XQ
      Q(3)=270.0*XQ
      Q(4)=230.0*XQ
      Q(5)=140.0*XQ
      Q(6)=90.0*XQ
      Q(7)=45.0*XQ
      FACT=402.2/(3.12*1609.3)
C     CALCULATION PROCEDURE
      DO 750 I=1,57
      READ(2,710) IX,DIST
  710 FORMAT(I1,F8.4)
      IF(I-2) 725,726,727
  725 CT=CT1
      CL=10000.*CT*DIST*FACT/Q(IX)
      GO TO 788
  726 CT=CT2
      CL=10000.*CT*DIST*FACT/Q(IX)
      GO TO 788
  727 CT=(CT1+CT2)/2.0
       CL=10000.*CT*DIST*FACT/Q(IX)
      CT=0.0
  788 CM=10000.*DIAM(IX)*DIST*FACT/Q(IX)
      A(1,I)=CT+CL+CM
      WRITE(3,720) I,DIST,DIAM(IX),CT,CL,CM,A(1,I)
  720 FORMAT(4X,I2,2F10.2,4F12.6)
  750 CONTINUE
      CALL LINPRO
      RETURN
      END
```

```
      SUBROUTINE LINPRO
C     LINEAR PROGRAMMING SUBROUTINE
      COMMON A(45,90),W(45),L(45),III,JJ,II
      WRITE(3 ,131)
131   FORMAT(35H1 ITER    FUNCTIONAL          VAR. IN)
      KKK=0
22    I=1
23    I=I+1
      IF(I-III)24,40,40
24    IF(L(I))23,25,23
25    DO 27 J=1,JJ
      IF(A(I,J))26,27,26
26    A(III,J)=A(III,J)-A(I,J)
27    CONTINUE
      GO TO 23
40    K=III
44    J=1
      W(K)=0.0
      L(K)=0
42    J=J+1
      IF(J-JJ)41,45,45
41    IF(A(K,J))43,42,42
43    IF(W(K)-A(K,J))42,42,47
47    W(K)=A(K,J)
      L(K)=J
      GO TO 42
45    IF(L(K))46,62,46
46    KJ=L(K)
      DO 120 I=2,II
      IF(A(I,KJ))120,120,121
120   CONTINUE
      WRITE(3 ,130)
130   FORMAT(10H UNBOUNDED)
C     SOLUTION IS UNBOUNDED .......STOP
      RETURN
121   I=1
      JK=0
50    I=I+1
      IF(I-II)52,52,56
52    IF(A(I,KJ))50,50,51
51    X=A(I,JJ)/A(I,KJ)
      IF(JK)55,53,55
55    IF(X-XMIN)53,50,50
53    XMIN=X
      JK=I
      GO TO 50
56    X=A(JK,KJ)
      L(JK)=KJ
      DO 57 I=1,III
57    W(I)=A(I,KJ)
      IJ=JK-1
      DO 59 I=1,IJ
      DO 59 J=1,JJ
```

```
      IJ=JK+1
      DO 61 I=IJ,III
      DO 61 J=1,JJ
      IF(A(JK,J))60,61,60
   60 IF(W(I))600,61,600
  600 A(I,J)=A(I,J)-W(I)*(A(JK,J)/X)
   61 CONTINUE
      DO 205 J=1,JJ
  205 A(JK,J)=A(JK,J)/X
      KKK=KKK+1
      AOUT=-A(K,JJ)
      WRITE(3,125) KKK,AOUT,L(JK)
  125 FORMAT(I4,F15.2,10X,I4)
      GO TO 44
   62 IF(K-1)70,70,63
   63 IJ=JJ-1
      DO 65 J=1,IJ
      IF(A(K,J)-.0001)65,65,66
   65 CONTINUE
      WRITE(3 ,103)
  103 FORMAT(9H FEASIBLE)
      DO 131 J=1,JJ
  131 A(III,J)=0.0
      K=1
      KKK=0
      GO TO 44
   66 WRITE(3 ,6)
    6 FORMAT(11H INFEASIBLE)
C   SOLUTION IS INFEASIBLE ....STOP *
      RETURN
   70 WRITE(3,999)
  999 FORMAT(1H1,5X,'SOLUTION IS'//)
      AOUT2=-A(1,JJ)
      WRITE(3,8) AOUT2
    8 FORMAT(11H FUNCTIONAL,F20.8,//)
      WRITE(3,7)
    7 FORMAT(24H VARIABLE              VALUE)
      DO 71 I=2,II
   71 WRITE(3,5) L(I),A(I,JJ)
    5 FORMAT(2H X,I3,F20.5)
      RETURN
      END
```

CONSTANTS

0 1036433334272

COMMON

APPROACH # 1 ... TRANSPORTATION PROBLEM
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MAP LENGTHS (CENTIMETERS) OF POSSIBLE ROUTES
BY SECTIONS OF DIFFERENT DIAMETERS (INCHES)

ROUTE / DIAM	20	16	14	12	10	8	6
(1)	23.0	0.0	3.1	0.0	0.0	0.6	0.0
(2)	7.4	0.0	0.0	0.5	0.0	2.7	0.0
(3)	23.0	0.0	0.3	0.0	0.0	1.0	0.0
(4)	7.4	0.0	0.0	1.8	0.0	0.0	0.0
(5)	23.0	0.0	0.4	0.0	0.0	0.0	0.0
(6)	7.4	0.0	0.0	0.0	0.0	0.0	0.0
(7)	23.0	0.0	2.0	1.8	0.0	0.0	0.0
(8)	7.4	0.0	4.4	1.3	0.0	0.0	0.0
(9)	23.0	0.0	2.0	1.3	0.0	1.5	0.0
(10)	7.4	0.0	4.4	1.3	0.0	1.5	0.0
(11)	9.4	7.2	0.0	0.0	0.0	0.0	0.0
(12)	7.4	0.0	4.4	1.3	0.0	5.1	0.0
(13)	9.4	7.7	0.0	0.0	0.0	0.0	0.0
(14)	10.7	0.7	0.0	0.0	0.0	0.0	0.0
(15)	9.4	9.0	0.0	0.0	0.0	0.0	1.5
(16)	7.4	0.0	4.4	2.7	0.0	7.7	0.0
(17)	9.4	9.0	0.0	12.6	0.0	0.0	2.9
(18)	7.4	0.0	4.4	2.7	0.0	15.0	2.9
(19)	9.4	9.0	0.0	12.0	0.0	0.0	10.0
(20)	7.4	0.0	4.4	2.7	0.0	10.0	10.0
(21)	9.4	4.0	0.0	0.0	0.0	0.0	0.0
(22)	7.4	0.0	4.4	1.3	0.0	9.2	2.4
(23)	23.0	0.0	3.0	0.0	0.0	2.4	0.0
(24)	7.4	0.0	0.0	1.3	0.0	2.3	0.0
(25)	23.0	0.0	3.0	5.4	0.0	0.0	0.0
(26)	7.4	0.0	0.0	5.4	0.0	0.0	0.0
(27)	23.0	0.0	3.0	10.6	0.0	0.0	0.0
(28)	7.4	0.0	0.0	10.6	0.0	0.0	0.0
(29)	23.0	0.0	3.0	18.0	0.0	3.5	0.0
(30)	23.0	0.0	1.8	3.5	0.0	3.9	0.0
(31)	23.0	0.0	2.0	3.3	0.0	4.7	0.0
(32)	7.4	0.0	4.4	3.3	0.0	4.7	0.0
(33)	23.0	0.0	3.0	16.6	0.0	0.0	0.0
(34)	7.4	0.0	0.0	20.0	0.0	0.0	0.0
(35)	23.0	0.0	3.0	19.0	0.0	23.0	0.0
(36)	7.4	0.0	0.0	22.5	0.0	23.0	0.0
(37)	23.0	0.0	3.0	11.0	0.0	6.5	5.9
(38)	7.4	0.0	0.0	1.3	12.1	0.0	0.0
(39)	23.0	0.0	3.0	11.0	0.0	6.5	27.4
(40)	7.4	0.0	0.0	7.3	40.0	0.0	0.0
(41)	23.0	0.0	1.8	9.7	0.0	1.7	4.0
(42)	7.4	0.0	7.4	0.0	3.0	0.0	1.9
(43)	9.4	0.0	0.0	8.5	0.0	0.0	0.0
(44)	7.4	2.5	4.4	9.8	0.0	9.2	0.0

CALCULATED COST COEFF. OF OBJ. FCT. (DOLLARS/1000 GAL)

ROUTE	CT	CL	CM	COEFF.=CT+CL+CM
1	0.150000	0.024000	0.072517	0.246517
2	0.127000	0.020320	0.038203	0.185523
3	0.150000	0.024000	0.064411	0.238411
4	0.127000	0.020320	0.026225	0.173545
5	0.150000	0.024000	0.058222	0.232222
6	0.127000	0.020320	0.018239	0.165559
7	0.150000	0.024000	0.072342	0.246342
8	0.127000	0.020320	0.043094	0.190414
9	0.150000	0.024000	0.079983	0.253983
10	0.127000	0.020320	0.050735	0.198055
11	0.150000	0.024000	0.048013	0.222013
12	0.127000	0.020320	0.074396	0.221716
13	0.150000	0.024000	0.045942	0.219942
14	0.127000	0.020320	0.083652	0.230972
15	0.150000	0.024000	0.090114	0.264114
16	0.127000	0.020320	0.097696	0.245016
17	0.150000	0.024000	0.136644	0.310644
18	0.127000	0.020320	0.193984	0.341304
19	0.150000	0.024000	0.211571	0.385571
20	0.127000	0.020320	0.268912	0.416232
21	0.150000	0.024000	0.034999	0.208999
22	0.127000	0.020320	0.125005	0.272325
23	0.150000	0.024000	0.083965	0.257965
24	0.127000	0.020320	0.039123	0.186443
25	0.150000	0.024000	0.092140	0.266140
26	0.127000	0.020320	0.042196	0.189516
27	0.150000	0.024000	0.115217	0.289217
28	0.127000	0.020320	0.065266	0.212586
29	0.150000	0.024000	0.171051	0.345051
30	0.127000	0.020320	0.104750	0.252070
31	0.150000	0.024000	0.118761	0.292761
32	0.127000	0.020320	0.089513	0.236833
33	0.150000	0.024000	0.141836	0.315836
34	0.127000	0.020320	0.106969	0.254289
35	0.150000	0.024000	0.303653	0.477653
36	0.127000	0.020320	0.269229	0.416549
37	0.150000	0.024000	0.217881	0.391881
38	0.127000	0.020320	0.087913	0.235233
39	0.150000	0.024000	0.429847	0.603847
40	0.127000	0.020320	0.261887	0.409207
41	0.150000	0.024000	0.157232	0.331232
42	0.127000	0.020320	0.081187	0.228507
43	0.150000	0.024000	0.075667	0.249667
44	0.127000	0.020320	0.146448	0.293768

THE PROBLEM TO BE SOLVED IS THE FOLLOWING ...

OBJECTIVE FUNCTION TO MINIMIZE

UNITS ARE COEF. = (DOLLARS/1000 GALLONS)
VARIABLES = (1000 GALLONS/DAY)

0.247 *X 1 + 0.186 *X 2 + 0.238 *X 3 + 0.174 *X 4 + 0.232 *X 5 +
0.156 *X 6 + 0.246 *X 7 + 0.190 *X 8 + 0.254 *X 9 + 0.198 *X10 +
0.222 *X11 + 0.222 *X12 + 0.220 *X13 + 0.231 *X14 + 0.264 *X15 +
0.245 *X16 + 0.311 *X17 + 0.341 *X18 + 0.386 *X19 + 0.416 *X20 +
0.289 *X21 + 0.272 *X22 + 0.258 *X23 + 0.186 *X24 + 0.266 *X25 +
0.198 *X26 + 0.259 *X27 + 0.213 *X28 + 0.345 *X29 + 0.252 *X30 +
0.295 *X31 + 0.237 *X32 + 0.316 *X33 + 0.254 *X34 + 0.478 *X35 +
0.417 *X36 + 0.342 *X37 + 0.235 *X38 + 0.504 *X39 + 0.429 *X40 +
0.331 *X41 + 0.229 *X42 + 0.253 *X43 + 0.294 *X44 +

SUBJECT TO

2 X 1 + X 3 + X 5 + X 7 + X 9 + X11 + X13 + X15 + X17 + X19 + X21 +
X23 + X25 + X27 + X29 + X31 + X33 + X35 + X37 + X39 + X41 + X43 +
XS45 = 1320.00

3 X 2 + X 4 + X 6 + X 8 + X10 + X12 + X14 + X16 + X18 + X20 + X22 +
X24 + X26 + X28 + X30 + X32 + X34 + X36 + X38 + X40 + X42 + X44 +
XS46 = 3960.00

4 X 1 + X 2 - XS47 = 1362.00
5 X 3 + X 4 - XS48 = 954.00
6 X 5 + X 6 - XS49 = 954.00
7 X 7 + X 8 - XS50 = 954.00
8 X 9 + X10 - XS51 = 954.00
9 X11 + X12 - XS52 = 954.00
10 X13 + X14 - XS53 = 954.00
11 X15 + X16 - XS54 = 818.00
12 X17 + X18 - XS55 = 954.00
13 X19 + X20 - XS56 = 954.00
14 X21 + X22 - XS57 = 408.00
15 X23 + X24 - XS58 = 408.00
16 X25 + X26 - XS59 = 682.00
17 X27 + X28 - XS60 = 682.00
18 X29 + X30 - XS61 = 682.00
19 X31 + X32 - XS62 = 408.00
20 X33 + X34 - XS63 = 682.00
21 X35 + X36 - XS64 = 272.00
22 X37 + X38 - XS65 = 408.00
23 X39 + X40 - XS66 = 954.00
24 X41 + X42 - XS67 = 272.00
25 X43 + X44 - XS68 = 136.00

ITER	FUNCTIONAL	VAR. IN
1	14444.00	1
2	13493.00	3
3	12536.00	5
4	11582.00	7
5	10628.00	9
6	9674.00	11
7	8720.00	13
8	7982.00	15
9	6948.00	17
10	5994.00	19
11	5586.00	21
12	5178.00	23
13	4496.00	25
14	3814.00	27
15	3132.00	29
16	2846.00	31
17	2724.00	2
18	2642.00	33
19	1773.00	35
20	1444.00	37
21	1382.00	4
22	491.00	39
23	485.00	6
24	155.00	41
25	2.00	43
FEASIBLE		
1	4452.00	40
2	4415.20	45
3	4379.11	1
4	4340.09	38
5	4328.73	42
6	4315.50	36
7	4307.97	3
8	4299.94	28
9	4299.92	26

X 45	194.00000
X 38	408.00000
X 1	1362.00000
X 5	954.00000
X 7	954.00000
X 9	954.00000
X 11	954.00000
X 13	954.00000
X 15	817.99999
X 17	954.00000
X 19	954.00000
X 21	408.00000
X 23	408.00000
X 25	680.00000
X 26	680.00000
X 3	954.00000
X 32	680.00000
X 33	680.00000
X 35	272.00000
X 26	2.00000
X 40	954.00000
X 42	272.00000
X 43	136.00000

APPROACH # 2 ... NETWORK PROBLEM
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CALCULATED COST COEFF. OF OBJ. FCT. (DOLLARS/1000 GAL)
AND MAP DISTANCES (CM.) ALONG TUBE DIAMETER (INCHES)
OF THE 57 POSSIBLE ROUTES

ROUTE	DIST	DIAM	CT	CL	CM	COEFF.=CT+CL+CM
1	9,40	20.00	0.150000	0,000174	0,023168	0,173342
2	7,40	20,00	0,127000	0,000116	0,018239	0,145355
3	4,00	16,00	0,000000	0,000102	0,011831	0,011933
4	8,50	12,00	0,000000	0,000435	0,037710	0,038145
5	5,20	12,00	0,000000	0,000266	0,023070	0,023336
6	3,00	12,00	0,000000	0,000154	0,013309	0,013463
7	5,50	12,00	0,000000	0,000282	0,024401	0,024682
8	9,30	20,00	0,000000	0,000159	0,022922	0,023081
9	3,90	12,00	0,000000	0,000200	0,017302	0,017502
10	0,80	12,00	0,000000	0,000041	0,003549	0,003590
11	2,10	16,00	0,000000	0,000054	0,006211	0,006265
12	0,70	16,00	0,000000	0,000018	0,002070	0,002088
13	4,50	12,00	0,000000	0,000230	0,019964	0,020195
14	6,60	12,00	0,000000	0,000338	0,029281	0,029619
15	1,40	8,00	0,000000	0,000159	0,009202	0,009361
16	5,40	8,00	0,000000	0,000956	0,055210	0,056165
17	3,30	8,00	0,000000	0,000341	0,019718	0,020059
18	6,90	8,00	0,000000	0,001572	0,068026	0,069596
19	6,40	8,00	0,000000	0,000728	0,042064	0,042793
20	4,20	20,00	0,000000	0,000072	0,010352	0,010423
21	0,50	14,00	0,000000	0,000011	0,001150	0,001162
22	3,70	14,00	0,000000	0,000140	0,014186	0,014326
23	1,50	8,00	0,000000	0,000171	0,009859	0,010030
24	4,20	8,00	0,000000	0,000478	0,027605	0,028083
25	7,70	8,00	0,000000	0,000876	0,050609	0,051485
26	1,70	12,00	0,000000	0,000087	0,007542	0,007629
27	2,70	8,00	0,000000	0,000307	0,017746	0,018053
28	1,80	8,00	0,000000	0,000205	0,011831	0,012035
29	0,80	8,00	0,000000	0,000091	0,005258	0,005349
30	1,20	8,00	0,000000	0,000137	0,007887	0,008024
31	4,40	12,00	0,000000	0,000225	0,019521	0,019746
32	0,60	12,00	0,000000	0,000031	0,002662	0,002693
33	2,50	8,00	0,000000	0,000284	0,016431	0,016716
34	0,60	8,00	0,000000	0,000068	0,003944	0,004012
35	1,70	8,00	0,000000	0,000193	0,011173	0,011367
36	3,80	12,00	0,000000	0,000195	0,016859	0,017053
37	5,40	12,00	0,000000	0,000277	0,023957	0,024234
38	2,80	12,00	0,000000	0,000143	0,012422	0,012566
39	1,10	12,00	0,000000	0,000056	0,004880	0,004936
40	5,20	8,00	0,000000	0,000705	0,040750	0,041455
41	1,80	12,00	0,000000	0,000092	0,007986	0,008078
42	5,20	12,00	0,000000	0,000266	0,023070	0,023336
43	4,40	8,00	0,000000	0,000455	0,026290	0,026745
44	2,40	8,00	0,000000	0,000228	0,013145	0,013373
45	6,80	8,00	0,000000	0,000683	0,039435	0,040119
46	1,00	12,00	0,000000	0,000051	0,004436	0,004488
47	5,60	8,00	0,000000	0,000637	0,036806	0,037444
48	3,90	8,00	0,000000	0,000444	0,025633	0,026077
49	3,50	8,00	0,000000	0,000398	0,023004	0,023402
50	23,00	8,00	0,000000	0,002617	0,151169	0,153786
51	8,00	8,00	0,000000	0,000910	0,052581	0,053491
52	1,00	12,00	0,000000	0,000073	0,005282	0,005355
53	27,40	6,00	0,000000	0,006236	0,270133	0,276368
54	2,00	12,00	0,000000	0,000146	0,010563	0,010709
55	1,80	6,00	0,000000	0,000410	0,017746	0,018156

56	10,62	10,00	0,000000	0,004775	0,055984	0,056762
57	25,47	10,00	0,000000	0,021931	0,139433	0,141364

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SOLUTION IS

FUNCTIONAL 3743.[illegible]

VARIABLE	VALUE
X 58	194.[illegible]
X 30	1362.[illegible]
X 21	1364.[illegible]
X 29	2316.[illegible]
X 2	3000.000000
X 22	3952.000000
X 23	954.000000
X 9	954.000000
X 12	3680.000000
X 15	818.000000
X 6	1920.000000
X 7	954.000000
X 3	4224.000000
X 35	408.000000
X 37	2994.000000
X 38	682.000000
X 46	682.000000
X 47	408.000000
X 41	954.000000
X 52	272.000000
X 56	1362.000000
X 57	954.000000
X 55	272.000000
X 4	136.000000
X 1	12826.000000
X 14	1928.000000
X 8	5582.000000
X 11	3634.000000
X 13	2726.000000
X 20	7628.000000
X 32	954.000000
X 34	420.000000
X 36	954.000000
X 46	272.000000
X 44	0.000000
X 42	954.000000
X 51	0.000000
X 54	1634.000000