Project No. A-021-PR

Dr. Adel M. Kamel

Final Report

MOVEMENT OF NON CONSERVATIVE POLLUTANTS BY WAVES AND CURRENTS IN PROTECTED WATERS

Water Resources Research Institute
University of Puerto Rico
Mayaguez, Puerto Rico

July, 1969

OWRR Project No. A-021-PR

Annual Allotment Or Matching
Grant Agreement No. 14-01-0001-1859

FCST Research Category: 5V

PROJECT TITLE:

Movement of Non-Conservative Pollutants by Waves and Currents in Protected Waters

Name and Location of University Where Project is Being Conducted:

University of Puerto Rico, Mayaguez, Puerto Rico

Project Began—Month: March Year: 1969 Scheduled Completion-Month:

Principal Investigator	Degree	Discipline
Adel M. Kamel	Ph.D.	Civil Engineering

Student Assistants 1/	Degree Held If Any	Discipline or Academic Background

Anticipation and Recommendation

The study started on March 69 on a 1/8 (one eighth) time basis by the principal investigator. This report describes work accomplished during the period March-May 69; anticipated accomplishments until Dec. 69, and recommendations.

WORK ACCOMPLISHED

During the period March-May 69, a literature review was made to outline the analytical background for dispersion process for a dissolved, colloidally-suspended, or suspended contaminants. These analytical considerations are outlined in the following paragraphs.

The Continuity Equation

Applying the conservation of mass to an incremental volume of flow, the continuity equation for a transferable scaler quantity per unit mass of fluid is:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial (u_x \ \zeta)}{\partial x} + \frac{\partial}{\partial y}(u_y \ \zeta) + \frac{\partial}{\partial z}(u_z \ \zeta) - \frac{\partial}{\partial x}\left(E_x \frac{\partial \zeta}{\partial x}\right) - \frac{\partial}{\partial y}\left(E_y \frac{\partial \zeta}{\partial y}\right) - \frac{\partial}{\partial z}\left(E_z \frac{\partial \zeta}{\partial z}\right) = 0 \qquad (1)$$

where

F = driving force

t = time

u_x, u_y, u_z = velocity of the transferable scaler quantity in the x, y, and z directions, respectively.

ζ = transferable scaler quantity

E = molecular diffusion coefficient

Assuming:

a- incompressible flow

b- origin of coordinate system is at bottom of flow with coordinate x in direction of primary flow; coordinate y normal to bottom (positive upward); and coordinate z horizontal normal to primary flow.

c- molecular diffusion is negligable in comparison to turbulent transfer (Elder, 1959).

d- for turbulent flow conditions

$$u = \bar{u} + u', \ \zeta = \bar{\zeta} + \zeta'$$

where u, ζ = instantaneous term

$\bar{u}, \bar{\zeta}$ = time average terms

u', ζ' = fluctuating terms

$$\bar{u} = \frac{1}{\tau} \int_0^\tau u dt$$

$$\bar{\zeta} = \frac{1}{\tau} \int_0^\tau \zeta \, dt$$

being long enough to mask the turbulent fluctuations but not so long to damp other variations of quantities with time

e - the percentage of volume occupied by the dispersant particles is negligable

f - off diagonal terms of diffusion tensor are zero when coordinate system is set-up as indicated in b above (Pai, 1957)

g - transferable scaler quantity is a dispersant which can be described by a concentration

with the above assumptions and neglecting the driving force, the continuity equation takes the form;

$$\frac{\partial \bar{c}}{\partial t} + \frac{\partial \bar{u}_x \bar{c}}{\partial x} + \frac{\partial \bar{u}_y \bar{c}}{\partial y} + \frac{\partial \bar{u}_z \bar{c}}{\partial z} - \frac{\partial}{\partial x}\left(E_x \frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial y}\left(E_y \frac{\partial c}{\partial y}\right) - \frac{\partial}{\partial z}\left(E_z \frac{\partial c}{\partial z}\right) = 0 \quad - (2)$$

where:

$\bar{c}$ = time average concentration of dispersant

$\bar{u}_x, \bar{u}_y, \bar{u}_z$ = time average values of dispersant velocities in x, y, and z directions, respectively.

E_x, E_y, E_z = turbulent transfer coefficients in the x, y, and z directions, respectively.

In equation (2), the first term is the substantive derivative of concentration with respect to time; the second term is convective transfer for convection in longitudinal direction and describes combined convection due to both wave orbital velocity and mass transfer velocity in the x-direction; the third term is convective transport for convection in the y-direction and describes convection due to particle fall velocity and the vertical component of both wave orbital velocity and secondary currents;

the fourth term is convective transport for horizontal convection due to secondary currents; and the last three terms are turbulent transport in the x, y, and z directions, respectively.

Dissolved or colloidally-suspended contaminants which behave like particles of the ambient fluid are transported at the mean flow velocity and dispersed longitudinally by the combined action of turbulent diffusion and differential convection due to the variation of velocity with respect to position in the cross section.

On the other hand contaminated particles which are transported mainly in suspension (like silt and sand) behave like fluid or colloidally-suspended particles except that they tend to settle and be deposited on the bed.

Substituting in equation (2), the values of $\bar{u}_x = u =$ instantaneous fluid particle velocity in the x-direction; $u_y = v - V$, $v =$ instantaneous fluid particle velocity in the y-direction (vertical component of both orbital velocity and secondary currents); $V =$ particle fall velocity (positive upward); and $u_z = w =$ secondary current velocity in the z-direction; we obtain

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial uc}{\partial y} + \frac{\partial wc}{\partial z} - \frac{V\partial c}{\partial y} - \frac{\partial}{\partial x}\left(\frac{E_x \partial c}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{E_y \partial c}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{E_z \partial c}{\partial z}\right) = 0 \qquad - \quad (3)$$

No solution to equation (3) is available however, the solution of few well selected cases would be developed in this study.

(I) <u>Dispersion process for a dissolved or colloidally-suspended contaminants under conditions of uniform flow in a straight waterway of constant cross-section.</u>

Considering transport longitudinally by convection and laterally and vertically by diffusion (i.e. neglecting longitudinal turbulent transport and secondary convective currents), equation (3) for negligable wave action reduces to

$$\frac{\partial c}{\partial t} + u\,\frac{\partial c}{\partial x} - \frac{\partial}{\partial y}\,\frac{(E_y \partial c)}{\partial y} - \frac{\partial}{\partial z}\,\frac{(E_z \partial c)}{\partial z} = 0 \qquad (4)$$

In equation (4) all temporal variations due to turbulence have been averaged out.

Letting:

$$u\,(y,z) = \bar{u} + u''(y,z)$$

and $$c\,(x,y,z) = \bar{c}\,(x) + c''(x,y,z)$$

where u'' = spatial variation of velocity

c'' = spatial variation of concentration c

and assuming that $\frac{\partial c}{\partial t}$ and $\frac{\partial c''}{\partial x}$ (the longitudinal derivative of concentration variation), are much smaller than the other terms in equation 4, Taylor (1954), obtained;

$$u''\,\frac{\partial \bar{c}}{\partial x} - \frac{\partial}{\partial y}\,(E_y \frac{\partial c''}{\partial y}) - \frac{\partial}{\partial z}\,(E_z \frac{\partial c''}{\partial z}) = 0 \qquad (5)$$

For known values of E_y and E_x, equation (5), can be solved for the distribution of c''. The solution would apply only after an initial period in which convective movements dominate the dispersion patterns (Fisher 1967). To describe the flow during that period would require a numerical solution to the basic equation of convective diffusion.

(2) Dispersion process for suspended contaminant for uniform flow in a straight waterway of constant cross-section.

The Aris moment equation method (Aris, 1956) appears to be more promising than the various analytical approaches to longitudinal dispersion in open channel flow. Using me set of transformations given in eq. 6 (Sayre, 1968); defining the local velocity by eq. 7; and the local eddy diffusivity by eq. 8; eq. 3, for two-dimensional flow in a waterway in which all $\partial/\partial z$ terms equal zero, takes the form given by eq. 9.

$$
\begin{aligned}
\xi &= \frac{x-\bar{u}t}{y_n} \\
\eta &= y/y_n \\
\zeta &= z/y_n \\
\tau &= Et/y_n^2 \\
\mu &= \bar{u}y_n/E \\
\nu_s &= Vy_n/E
\end{aligned}
\qquad (6)
$$

where y_n is the normal depth and t is the dispersion time.

$$u(y,z) \equiv u\,[\,1+\chi\,(y,z)\,] \qquad (7)$$

where $\chi\,(y,z)$ is a function describing the variation of velocity in the cross section.

$$E_x = E_y = E_z = E \quad \psi\,(y,z) \qquad (8)$$

where E is the average value of the eddy diffusivity in the cross section and $\psi\,(y,z)$ is a function describing the distribution of eddy diffusivity. ψ and χ are properties of the flow and are independent on the properties of the dispersant.

$$\frac{\partial c}{\partial t} + \mu\chi\,\frac{\partial c}{\partial \xi} - \frac{\partial}{\partial \eta}\left(\psi\,\frac{\partial c}{\partial \eta} + \nu_s c\right) \mp \psi\,\frac{\partial^2 c}{\partial \xi^2} = 0 \qquad (9)$$

Equation 9 is the Eulerian dispersion equation for suspended sediment where the concentration, $c\,(\xi,\eta,\tau)$, refers only to that component of dispersant which is entrained in the flow. The initial and boundary conditions for eq. 9 are (Sayre, 1968);

a- Initial condition corresponding to an instantaneous uniformly-distributed plane source at the origin is;

$$
\tau = 0 \qquad
\begin{aligned}
&c(\xi,\eta,0) = 0 \quad \text{for } \xi \neq 0 \\
&c(\xi,\eta,0) = \infty \quad \text{at } \xi = 0
\end{aligned}
\qquad (10)
$$

$$\int_{-\infty}^{\infty} c\,(\xi,\,\eta,0)\,d\xi = 1$$

b- Boundary condition due to the fall velocity term indicating no transport or dispersion across the water surface.

$$\eta=1, \quad \psi \frac{\partial c}{\partial \eta} + \nu_s\, c = 0$$

$$\text{c-} \quad -\eta = 0, \quad \psi \frac{\partial c}{\partial \eta} + (1-\partial)\ \nu_s\, c + \gamma W = 0 \qquad (11)$$

Eq. II permits the bed to behave either as an absorbing or a reflecting barrier and also permits temporary storage of the dispersant. In eq. (II); α = bed absorbing coefficient, $W=W\ (\xi,\tau)$ = amount of dispersant stored per unit area of bed surface; and γ = entrainment coefficient.

$$\text{d-} \quad \frac{\partial W}{\partial \tau} - \mu \frac{\partial W}{\partial \xi} = \alpha \nu_s\ c\ (\xi, 0, \tau) - \gamma W \qquad (12)$$

Eq. (12) is a statement of the conservation of mass for the bed and describes the deposition distribution function W (ξ,τ).

A computer program for the numerical solution of equations 9 and 12 would be prepared. Although a single run might require hours of computer time, such program would be most helpful when the boundary conditions and input data are carefully selected.

(3) Dispersion process for a dissolved or colloidally -suspended contaminants under wave action in a straight waterway of constant cross-section

Considering transport longitudinally and vertically by both convection and diffusion, eq. (3) reduces to;

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} - \frac{\partial}{\partial x}\left(E_x \frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial y}\left(E_y \frac{\partial c}{\partial y}\right) - \frac{\partial}{\partial z}\left(E_z \frac{\partial c}{\partial z}\right) = 0 \qquad (13)$$

In eq. (13) u is the mean value of the horizontal component of particle orbital velocity plus the mass transport velocity and v is the mean value of the vertical component of particle orbital velocity. For shallow water condition in which we are particularly interested, the term $v\frac{\partial >}{\partial y}$ would be of negligable magnitude compared to $u\,\frac{\partial c}{\partial x}$.

When the injection of the dispersant is adjusted with respect to z to match the flux of water in the particular vertical, $\frac{\partial c}{\partial z} = 0$ and eq. (13) takes the form;

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - \frac{\partial}{\partial x} (E_x \frac{\partial c}{\partial x}) - \frac{\partial}{\partial y} (E_y \frac{\partial c}{\partial y}) = 0 \qquad (14)$$

Eq. (14) cannot be solved since this would require the perior knowledge of E_x and E_y. No conclusive investigation of lateral and longitudinal diffusion under wave action has as yet been made.

ANTICIPATED ACCOMPLISHMENTS (June-December 1969)

During this period a numerical solution will be prepared and field experiments will be conducted for dispersion of dissolved or colloidally-suspended contaminants under uniform flow of a straight waterway of constant cross-sections. In this case dispersion is described by:

$$u' \frac{\partial \bar{c}}{\partial x_i} = \frac{\partial}{\partial y} (E_y \frac{\partial c'}{\partial y}) + \frac{\partial}{\partial z} (E_z \frac{\partial c'}{\partial z}) \qquad (5)$$

For known valves of E_y and E_z, eq. (13) can be solved for the distribution of c'. The solution would apply only after an initial period in which convective movements dominate the dispersion pattern. To describe the flow during that period a solution is needed to the basic equation of convective dispersion:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = \frac{\partial}{\partial y} (E_y \frac{\partial c}{\partial y}) + \frac{\partial}{\partial t} (E_z \frac{\partial c}{\partial z}) \qquad (15)$$

A numerical solution for (15) would require knowledge of the dispersion coefficients E_y and E_z, (assuming that dispersion is a diffusion process). In this analysis the following valves will be used: (Elder, 1959).

$$E_y = 5.9 \; du_* \qquad (16)$$

$$E_z = 0.23 \; du_* \qquad (17)$$

Field experiments will be carried out in the AÑASCO River. At a typical section of the river, the velocity profile will be measured and dye would be injected at that section. The dispersing dye cloud would be traced through several miles of river flow.

(2) Flume studies on the dispersion of a dissolved or colloidally-suspended contaminants under wave action, will be conducted utilizing an existing flume. The purpose of the experiments would be to examine the change in vertical and longitudinal diffusivities in eq. (14) with the energy wave spectrum.

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - \frac{\partial}{\partial x} \left(E_x \frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial y} \left(E_y \frac{\partial c}{\partial y}\right) = 0 \qquad (14)$$

Dye would be injected at the surface and along the width of the flume. The dispersing dye cloud would be traced through the length of the wave flume by sampling from selected stations. The experiments would be conducted for a constant water depth and a varying wave period and height.

The anticipated accomplishments above would complete the first phase of this project.

RECOMMENDATIONS

Since a wave flume equipped with a sediment supply and sampling system will not be available in the foreseeable future, it is recommended that completion of the investigation be limited to its first phase. This first phase deals with colloidally-suspended contaminants which behave as particles of the ambient fluid thus allowing the use of dye as a dispersant. Two cases will be considered namely;

(1) The behavior of the mass transfer coefficient in natural streams where it is believed that transverse gradients of velocity overwhelm all other effects in producing dispersion. The AÑASCO River will be selected for the study. Dye will be injected at a typical section of the river and the dispersing dye cloud will be traced through the river flow.

(2) The behavior of the mass transfer coefficient under oscillatory wave motion. In this case it is believed that vertical and lateral diffusivities increase with the energy in the wave spectrum. This phase will be investigated experimentally in an available flume after its modification at a modest cost, into a wave channel.

References

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Elder, J. W., 1959, "The dispersion of marked fluid in turbulent shear flow", Journal of Fluid Mechanics, Vol. 5, No. 4, pp. 544-560.

Fischer, H. B., 1967, "The mechanics of dispersion in natural streams", ASCE Environmental Engineering Conference, Dallas, Texas.

Jobson, H. E., 1968, "Vertical mass transfer in open channel flow, "U. S. Geological Survey, open-file report.

Pai, Shih-I, 1957, "Viscous flow theory, II-Turbulent flow", D. Van Nostrand Company, Inc., New York, 275 p.

Sayre, W. W., 1968, "Dispersion of mass in open-channel flow", Hydraulic Paper No. 3, Colorado State University, Fort Collins, Colorado

Taylor, G. I., 1954, "The dispersion of matter in turbulent flow through a pipe, "Proc. Royal Society of London 223, pp. 446-468.

Revised Financial Plan:

A. Prototype Study

1- Salaries and Wages

i) Principle Investigator

(equivalent to full time for 1 1/2 month) $ 2,062.50

ii) Graduate Student

(equivalent to full time for 2 weeks) 250.00

iii) Chemist 300.00

iv) Field Technician

(equivalent to full time for 2 weeks) 125.00

2. Expendable Equipment

Dye injection system 60.00

Sub-total 2,797.50

B. Flume Study

1. Salaries and Wages

i) Principal Investigator

(equivalent to full time for 1 1/2 month) 2,062.50

ii) Undergraduate Student

(equivalent to full time for 1 1/2 month)

iii) Laboratory Technician 300.00

(equivalent to full time for 2 weeks 110 .00

2- Non-expendable equipment

Wave generator 540.00

Wave absorber 30.00

3- Expendable equipment

Dye injection system 40.00

Sampling system 90.00

Sub-total 3,182.50

4- Preparation of final report 500.00