Project No. A-021-PR
Dr. Adel M. Kamel

Final Report
MOVEMENT OF NON CONSERVATIVE POLLUTANTS BY WAVES AND CURRENTS IN PROTECTED WATERS

Water Resources Research Institute
University of Puerto Rico
Mayaguez, Puerto Rico

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OWRR Project No. A-021-PR

PROJECT TITLE:

Annual Allotment Or Matching Grant Agreement No. 14-01-0001-1859 Movement of Non-Conservative Pollutants by Waves and Currents in Protected Waters

FCST Research Category: 5V

Name and Location of University Where Project is Being Conducted:

University of Puerto Rico, Mayaguez, Puerto Rico

Project Began—Month: March; Year: 1969 Scheduled Completion—Month:

Principal Investigator
Adel M. Kamel

Degree
Ph.D.

Discipline
Civil Engineering

Student Assistants 1

Degree Held
If Any

Discipline or Academic Background

Anticipation and Recommendation

The study started on March 69 on a 1/8 (one eighth) time basis by the principal investigator. This report describes work accomplished during the period March-May 69; anticipated accomplishments until Dec. 69; and recommendations.

WORK ACCOMPLISHED

During the period March-May 69, a literature review was made to outline the analytical background for dispersion process for a dissolved, colloidally-suspended, or suspended contaminants. These analytical considerations are outlined in the following paragraphs.
The Continuity Equation

Applying the conservation of mass to an incremental volume of flow, the continuity equation for a transferable scalar quantity per unit mass of fluid is:

$$\frac{\partial}{\partial t} \left( \frac{\partial \xi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \xi}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial \xi}{\partial z} \right) - \frac{\partial}{\partial x} \left( \frac{\partial E_x}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{\partial E_y}{\partial y} \right) - \frac{\partial}{\partial z} \left( \frac{\partial E_z}{\partial z} \right) = 0$$

(1)

where

- \( F \) = driving force
- \( t \) = time

- \( u_x, u_y, u_z \) = velocity of the transferable scalar quantity in the \( x, y, \) and \( z \) directions, respectively.
- \( \xi \) = transferable scalar quantity
- \( E \) = molecular diffusion coefficient

Assuming:

a- incompressible flow
b- origin of coordinate system is at bottom of flow with coordinate \( x \) in direction of primary flow; coordinate \( y \) normal to bottom (positive upward); and coordinate \( z \) horizontal normal to primary flow.
c- molecular diffusion is negligible in comparison to turbulent transfer (Elder, 1939).
d- for turbulent flow conditions

\[ u = u + u', \xi = \xi + \xi' \]

where \( u, \xi \) = instantaneous term
\[ \bar{u}, \bar{\xi} \] = time average terms
\[ u', \xi' \] = fluctuating terms

\[ \bar{u} = \frac{1}{T} \int_0^T u \, dt \]

\[ \xi = \frac{1}{T} \int_0^T \xi \, u \, dt \]
being long enough to mask the turbulent fluctuations but not so long to damp other variations of quantities with time

\( e \) - the percentage of volume occupied by the dispersant particles is negligible

\( f \) - off diagonal terms of diffusion tensor are zero when coordinate system is set-up as indicated in \( b \) above (Pai, 1957)

\( g \) - transferable scalar quantity is a dispersant which can be described by a concentration

with the above assumptions and neglecting the driving force, the continuity equation takes the form:

\[
\frac{\partial \bar{c}}{\partial \tau} + \frac{\partial \bar{u}_x \bar{c}}{\partial x} + \frac{\partial \bar{u}_y \bar{c}}{\partial y} + \frac{\partial \bar{u}_z \bar{c}}{\partial z} = 3 \left( \frac{\partial}{\partial x} \frac{\partial \bar{c}}{\partial x} \right) - 2 \left( \frac{\partial}{\partial y} \frac{\partial \bar{c}}{\partial y} \right) - 2 \left( \frac{\partial}{\partial z} \frac{\partial \bar{c}}{\partial z} \right) = 0 \quad (2)
\]

where:

\( \bar{c} \) = time average concentration of dispersant

\( \bar{u}_x, \bar{u}_y, \bar{u}_z \) = time average values of dispersant velocities in \( x, y, \) and \( z \) directions, respectively.

\( E_x, E_y, E_z \) = turbulent transfer coefficients in the \( x, y, \) and \( z \) directions, respectively.

In equation (2), the first term is the substantive derivative of concentration with respect to time; the second term is convective transfer for convection in longitudinal direction and describes combined convection due to both wave orbital velocity and mass transfer velocity in the \( x \)-direction; the third term is convective transport for convection in the \( y \)-direction and describes convection due to particle fall velocity and the vertical component of both wave orbital velocity and secondary currents;
the fourth term is convective transport for horizontal convection due to secondary currents; and the last three terms are turbulent transport in the x, y, and z directions, respectively.

Dissolved or colloiddally-suspended contaminants which behave like particles of the ambient fluid are transported at the mean flow velocity and dispersed longitudinally by the combined action of turbulent diffusion and differential convection due to the variation of velocity with respect to position in the cross section.

On the other hand contaminated particles which are transported mainly in suspension (like silt and sand) behave like fluid or colloiddally-suspended particles except that they tend to settle and be deposited on the bed.

Substituting in equation (2), the values of $\bar{u}_x = u = \text{instantaneous fluid particle velocity in the } x \text{-direction}$; $u_y = v = \text{instantaneous fluid particle velocity in the } y \text{-direction (vertical component of both orbital velocity and secondary currents)}$; $v = \text{particle fall velocity (positive upward)}$; and $u_z = w = \text{secondary current velocity in the } z \text{-direction}$; we obtain

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z} = \nabla \cdot \left( D \nabla c \right) + u \left( \frac{\partial c}{\partial x} \right) + v \left( \frac{\partial c}{\partial y} \right) + w \left( \frac{\partial c}{\partial z} \right)$$

No solution to equation (3) is available however, the solution of few well selected cases would be developed in this study.

(1) Dispersion process for dissolved or colloiddally-suspended contaminants under conditions of uniform flow in a straight waterway of constant cross-section.

Considering transport longitudinally by convection and laterally and vertically by diffusion (i.e. neglecting longitudinal turbulent transport and secondary
convective currents), equation (3) for negligible wave action reduces to

\[
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - \frac{\partial}{\partial y} \left( E_y \frac{\partial c}{\partial y} \right) - \frac{\partial}{\partial z} \left( E_z \frac{\partial c}{\partial z} \right) = 0
\]  \hspace{1cm} (4)

In equation (4) all temporal variations due to turbulence have been averaged out.

Letting:

\[
u (y, z) = \tilde{u} + u^\prime(y, z)
\]

and

\[
\frac{\partial c}{\partial x} (x, y, z) = \tilde{c} (x) + c^\prime(x, y, z)
\]

where \( u^\prime \) = spatial variation of velocity

\( c^\prime \) = spatial variation of concentration \( c \)

and assuming that \( \frac{\partial c}{\partial t} \) and \( \frac{\partial}{\partial x} \) (the longitudinal derivative of concentration variation),

are much smaller than the other terms in equation 4, Taylor (1954), obtained;

\[
u \frac{\partial \tilde{c}}{\partial x} - \frac{\partial}{\partial y} \left( E_y \frac{\partial \tilde{c}}{\partial y} \right) - \frac{\partial}{\partial z} \left( E_z \frac{\partial \tilde{c}}{\partial z} \right) = 0
\]  \hspace{1cm} (5)

For known values of \( E_y \) and \( E_z \), equation (5), can be solved for the distribution of \( c^\prime \). The solution would apply only after an initial period in which convective movements dominate the dispersion patterns (Fisher 1967). To describe the flow during that period would require a numerical solution to the basic equation of convective diffusion.

(2) Dispersion process for suspended contaminant for uniform flow in a straight waterway of constant cross-section.

The Aris moment equation method (Aris, 1956) appears to be more promising than the various analytical approaches to longitudinal dispersion in open channel flow. Using me set of transformations given in eq. 6 (Soyre, 1968); defining the local
velocity by eq. 7; and the local eddy diffusivity by eq. 8; eq. 3, for two-dimensional flow in a waterway in which all \( \partial / \partial \xi \) terms equal zero, takes the form given by eq. 9.

\[
\begin{align*}
\xi &= \frac{x-x}{y_n} \\
\eta &= \frac{y}{y_n} \\
\zeta &= \frac{z}{y_n} \\
\tau &= \frac{t}{y_n^2} \\
\mu &= \frac{\bar{u}y_n}{E} \\
\nu &= \frac{v y_n}{E}
\end{align*}
\] (6)

where \( y_n \) is the normal depth and \( t \) is the dispersion time.

\[
u(y, z) = u(1 + \chi(y, z)) + u(7)
\]

where \( \chi(y, z) \) is a function describing the variation of velocity in the cross section.

\[
E_x = E_y = E_z = E \psi(y, z)
\] (8)

where \( E \) is the average value of the eddy diffusivity in the cross section and \( \psi(y, z) \) is a function describing the distribution of eddy diffusivity. \( \psi \) and \( \chi \) are properties of the flow and are independent on the properties of the dispersant.

\[
\frac{\partial \xi}{\partial t} + u \xi \frac{\partial \xi}{\partial \eta} + \frac{\partial^2 \xi}{\partial \zeta^2} \frac{\partial \xi}{\partial \eta} + \psi \frac{\partial^2 \xi}{\partial \zeta^2} = 0
\] (9)

Equation 9 is the Eulerian-dispersion equation for suspended sediment where the concentration, \( c(\xi, \eta, \tau) \), refers only to that component of dispersant which is entrained in the flow. The initial and boundary conditions for eq. 9 are (Sayre, 1966):

a- Initial condition corresponding to an instantaneous uniformly-distributed plane source at the origin is;

\[
\begin{align*}
\tau &= 0 & c(\xi, \eta, 0) &= 0 \text{ for } \xi \neq 0 \\
c(\xi, \eta, 0) &= \infty \text{ at } \xi = 0 \\
\int_{-\infty}^{\infty} c(\xi, \eta, 0) \, d\xi &= 1
\end{align*}
\] (10)
b- Boundary condition due to the fall velocity term indicating no
transport or dispersion across the water surface.

\[ \eta = 1, \psi \frac{\partial c}{\partial \eta} + \nu_B c = 0 \]

\[ \eta = 0, \psi \frac{\partial c}{\partial \eta} + (1-\eta) \nu_B c + \gamma W = 0 \]  

(11)

Eq. (11) permits the bed to behave either as an absorbing or a reflecting barrier and
also permits temporary storage of the dispersant. In eq. (11); \( \nu_B = \) bed absorbing
coefficient, \( W = W(\xi, \tau) \) = amount of dispersant stored per unit area of bed surface;
and \( \gamma = \) entrainment coefficient.

\[ \frac{\partial W}{\partial \tau} - \nu_B \frac{\partial W}{\partial c} = \alpha \nu_B c (\xi, 0, \tau) - \gamma W \]  

(12)

Eq. (12) is a statement of the conservation of mass for the bed and describes the
deposition distribution function \( W(\xi, \tau) \).

A computer program for the numerical solution of equations 9 and 12
would be prepared. Although a single run might require hours of computer time, such
program would be most helpful when the boundary conditions and input data are care-
fully selected.

(3) Dispersion process for a dissolved or colloidal-suspended contami-
nants under wave action in a straight waterway of constant cross-section.

Considering transport longitudinally and vertically by both convection
and diffusion, eq. (3) reduces to;

\[ \frac{\partial c}{\partial t} + \nu \frac{\partial c}{\partial x} + \nu \frac{\partial c}{\partial y} - \frac{\partial}{\partial x} (E_x \frac{\partial c}{\partial x}) - \frac{\partial}{\partial y} (E_y \frac{\partial c}{\partial y}) - \frac{\partial}{\partial z} (E_z \frac{\partial c}{\partial z}) = 0 \]  

(13)

In eq. (13) \( u \) is the mean value of the horizontal component of particle orbital
velocity plus the mass transport velocity and \( v \) is the mean value of the vertical
component of particle orbital velocity. For shallow water condition in which we are
particularly interested, the term \( \nu \frac{\partial c}{\partial y} \) would be of negligible magnitude compared
to \( u \frac{\partial c}{\partial x} \).
When the injection of the dispersant is adjusted with respect to z to match the
flux of water in the particular vertical, $\frac{\partial c}{\partial z} = 0$ and eq. (13) takes the
form:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - \frac{\partial}{\partial x} \left( E_x \frac{\partial c}{\partial x} \right) - \frac{\partial}{\partial y} \left( E_y \frac{\partial c}{\partial y} \right) = 0$$

(14)

Eq. (14) cannot be solved since this would require the prior knowledge
of $E_x$ and $E_y$. No conclusive investigation of lateral and longitudinal diffusion
under wave action has as yet been made.

**ANTICIPATED ACCOMPLISHMENTS**

(June-December 1969)

During this period a numerical solution will be prepared and field
experiments will be conducted for dispersion of dissolved or colloidally-suspended
contaminants under uniform flow of a straight waterway of constant cross-sections.

In this case, dispersion is described by:

$$u \frac{\partial c}{\partial x} = \frac{\partial}{\partial x} \left( E_x \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( E_y \frac{\partial c}{\partial y} \right)$$

(5)

For known values of $E_y$ and $E_z$, eq. (13) can be solved for the distribution of $c$.

The solution would apply only after an initial period in which convective mo-
mements dominate the dispersion pattern. To describe the flow during that period
a solution is needed to the basic equation of convective dispersion:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = \frac{\partial}{\partial x} \left( E_x \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( E_y \frac{\partial c}{\partial y} \right)$$

(15)

A numerical solution for (15) would require knowledge of the dispersion
coefficients $E_y$ and $E_z$ (assuming that dispersion is a diffusion process). In
this analysis the following values will be used: (Elder, 1959).

$$E_y = 5.9 \text{ du}_x$$

(16)

$$E_z = 0.23 \text{ du}_z$$

(17)
Field experiments will be carried out in the Añasco River. At a typical section of the river, the velocity profile will be measured and dye would be injected at that section. The dispersing dye cloud would be traced through several miles of river flow.

(2) Flume studies on the dispersion of a dissolved or colloidally-suspended contaminants under wave action, will be conducted utilizing an existing flume. The purpose of the experiments would be to examine the change in vertical and longitudinal diffusivities in eq. (14) with the energy wave spectrum.

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - \frac{\partial}{\partial x} \left( E_x \frac{\partial c}{\partial x} \right) - \frac{\partial}{\partial y} \left( E_y \frac{\partial c}{\partial y} \right) = 0 \quad (14)$$

Dye would be injected at the surface and along the width of the flume. The dispersing dye cloud would be traced through the length of the wave flume by sampling from selected stations. The experiments would be conducted for a constant water depth and a varying wave period and height.

The anticipated accomplishments above would complete the first phase of this project.
RECOMMENDATIONS

Since a wave flume equipped with a sediment supply and sampling system will not be available in the foreseeable future, it is recommended that completion of the investigation be limited to its first phase. This first phase deals with colloidal-dally-suspended contaminants which behave as particles of the ambient fluid thus allowing the use of dye as a dispersant. Two cases will be considered namely;

(1) The behavior of the mass transfer coefficient in natural streams where it is believed that transverse gradients of velocity overwhelm all other effects in producing dispersion. The ARASCO River will be selected for the study. Dye will be injected at a typical section of the river and the dispersing dye cloud will be traced through the river flow.

(2) The behavior of the mass transfer coefficient under oscillatory wave motion. In this case it is believed that vertical and lateral diffusivities increase with the energy in the wave spectrum. This phase will be investigated experimentally in an available flume after its modification at a modest cost, into a wave channel.
References


Revised Financial Plan:

A. Prototype Study

1. Salaries and Wages
   i) Principal Investigator
      (equivalent to full time for 1 1/2 month) $ 2,062.50
   ii) Graduate Student
      (equivalent to full time for 2 weeks) 250.00
   iii) Chemist
      300.00
   iv) Field Technician
      (equivalent to full time for 2 weeks) 125.00

2. Expandable Equipment
   Dye injection system 60.00

Sub-total 2,797.50

B. Flume Study

1. Salaries and Wages
   i) Principal Investigator
      (equivalent to full time for 1 1/2 month) 2,062.50
   ii) Undergraduate Student
      (equivalent to full time for 1 1/2 month)
   iii) Laboratory Technician
      (equivalent to full time for 2 weeks) 110.00

2. Non-expandable equipment
   Wave generator 540.00
   Wave absorber 30.00

3. Expandable equipment
   Dye injection system 40.00
   Sampling system 90.00

Sub-total 3,182.50

4. Preparation of final report
   500.00

Sub-total 3,682.50